concordance=TRUE

Constrained High-Dimensional Integrated Macroeconomic Prediction & Scenario-Generation (CHIMPS) Model

Model Documentation

April 05, 2019

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1 Model Goals

The CHIMPS model is a large-scale macroeconomic model which constructs an integrated model of the macroeconomy. Potentially several hundred macroeconomic covariates may be modeled simultaneously.

The primary goals of the CHIMPS model are to:

- Generate baseline quarterly forecasts of up to 5 years for key macroeconomic variables.
- Forecast the probability distributions and the risk levels for each of these variables at all time periods within the forecast horizon.
- Answer "what-if" business questions about opportunities and challenges in the macroeconomy by generating economic scenarios, forecasting their probability of occurance, and forecasting the paths of macroeconomic variables in each scenario.

The model is built to optimize prediction accuracy of a large number of macroeconomic variables over its 0-5 year time frame. It is structured to allow for easy addition of new variables and new scenarios.

2 Model Structure

2.1 Econometric Structure

Large-scale macroeconomic models can generally be seperated into several classes: vector autoregressions (VARs), dynamic factor models (DFMs), structural econometric models (SEMs), and dynamic stochastic general equibrlium models (DSGEs). Before constructing the model, a careful analysis of existing large-scale macro models was undertaken to find a model structure most appropriate for our model goals.

A vector autoregression (VAR) model forecasts variables jointly as functions of the past values of these variables and the past values of all other variables. Vector autoregression models are pure time-series methods that require minimal economic assumptions - they are flexible and data-driven. These models tend to produce high level of forecasting accuracy, particularly for short-term forecasts (Moody's Analytics 2015, Giannoni 2016, Stock and Watson 2001, Bekiros and Paccagnini 2014). The downside of these models is that in their basic form, they are only capable of predicting causality, not correlation. Moreover, these models can be prone to overfitting so variables must be selected carefully.

Dynamic factor models (DFMs) are similar to VARs - but instead of modeling variables jointly as function of other variables, they model variables jointly as functions of unobserved latent variables (Stock and Watson 2002). These aim to preserve the benefits of VARs while aiming to reduce some of their common statistical problems.

Dynamic stochastic general equilibrum (DSGE) models which aim for complete causal interpretation of its results. These model the individual intertemporal utility-maximization functions of households and firms, then attempt to infer the relationship of macroeconomic aggregates by modeling how these households and firms trade with one another in an economic equilibrium. These models often provide poor empirical results, but their unique capacity to model causality makes them popular among central banks who wish to understand the impacts of monetary policy.

SEMs are models which assume a certain economic structure between aggregate variables in the economy, and estimate the parameters of that structure. For example, the Philips Curve and the IS-LM model are SEMs - they impose certain economic assumptions about the way aggregate macro variables relate to one another, which then allows one to estimate the parameters of the model. Econometrically, SEMs are often just VARs with restrictions placed on coefficient estimates, and causality assigned by assumption.

Most of the variables in CHIMPS are modeled using a joint DFM-VAR model, known as a factor-augmented vector autoregression (FAVAR) model. This model structure is considered to be optimal for short-term forecasting accuracy. However, some variables are modeled as SEMs when causality is necessary or when a SEM is known to have strong forecasting ability [1].

Other macroeconomic models using a similar structure include the Atlanta Fed's GDP forecasting model *GDPNow* which uses a FAVAR (Higgins 2017), the Vanguards Global Capital Markets Model which also uses a FAVAR (Davis et al. 2014), and the Moody's U.S. Macro Model which uses a mix of VARs and SEMs (Moody's 2015).

Most large-scale macro models only aim to model the forecasted mean values of variables. The CHIMPS model goes further and models the expected volatility of each variable at each point in the future, allowing us to derive a sequence of probability distributions for each variable. This allows to both measure risk levels and generate our own economic scenarios.

In the next sections we discuss the economic structure of the model.

2.2 Aggregate Supply

Aggregate supply and its determinants - population growth, technology growth, and labor and technology productivity, potential output - are considered exogenous variables to the model. This reflects the standard view that in the short run, the path of aggregate supply is fixed. Fluctuations in economic variables and other macroeconomic indicators are determined by fluctuations in aggregate demand.

We use a pre-specified forecasted growth rate for population growth, potential output, and so on, implicitly assuming that changes in the rest of the economy do not alter their path through the 5-year forecast horizon of the model. Forecasts of aggregate supply variables are provided by the U.S. Congressional Budget Office, which projects those variables using a standard neoclassical framework[2].

2.3 Aggregate Demand

Consumer spending, domestic investment, and net exports are modeled bottom-up from individual subcomponents. A full list of subcomponents used can be found at `https://fred.stlouisfed.org/release/tables?rid=53&eid=14897`. These quarterly subcomponents are modeled individually as functions of their past lags and of contemporaneous movements in prices and yields, using purely statistical methods with few assumptions from economic theory.

Government spending is specified as an exogenous variable. Historically, fiscal policy has been both procyclical or anticyclical depending on political factors which we will not estimate here. Instead, we use the

[1] For example, see the later section on modeling the federal funds rate through the Taylor Rule

[2] CBO model documentation can be found in Arnold (2018).

CBO estimates for government spending and subject it to random Gaussian shocks in simulations.

2.4 Asset Prices, Yields, and Sentiment

Fluctuations in asset prices and measures of business & consumer sentiment are assumed to be the key drivers of aggregate demand and hence of fluctuations in the economy. These variables tend to be leading indicators of economic movements and tend to display highly correlated joint dynamics, so most of these variables are modeled together in the FAVAR. We follow standard financial theory to build parsimonious and intuitive representations forecasts of yields.

2.5 Monetary Policy

Monetary policy through the nominal federal funds rate is modeled with a modified Taylor rule specification. The Taylor rule is a commonly used reference equation by the FOMC that describes how to set an optimal federal funds rate given the state of inflation and economic growth. We use a modified version of the Taylor rule that takes into account adjustment speed and the so-called zero-lower-bound on nominal interest rates, and calibrate parameters of the Taylor rule based on historical fit.

2.6 Inflation

We assume that price levels are set by firms subject to sticky prices in output markets. With price rigidity, there are periods in which factors of production, particularly labor, are underused as firms are unable to charge the correct market price for their goods. We model this relationship using the standard New Keynesian Philips Curve. The New Keynesian Philips Curve is a specification that specifies actual inflation as a function of inflation expectations, and a firm's marginal cost, which we proxy by nonfarm workers' share of total income.

3 Modeling Process

3.1 Preparing Historical Data

The `macro-model/model-input/variables.xlsx` spreadsheet contains specifications used preparing the historical data which will be described in the following sections. A full list of variables can also be found in the appendix.

Data Collection

Over 100 variables are used in the model, with data for most variables pulled from online sources via a data scraping script. Data are aggregated using end-of-period values at either a quarterly or monthly frequency, using the higher frequency available.

Data is sourced from the following agencies:

1. Federal Reserve Board - Treasury yields*, TIPS yields*, capacity utilization*, forex rates*, federal funds rate*, prime rate*, outstanding mortgage debt and subcomponents*, aggregate commercial bank balance sheet subcomponents*
2. Federal Reserve Bank of Philadelphia - business sentiment, anxious index[3]
3. Federal Reserve Bank of Cleveland - expected inflation[4]
4. Federal Reserve Bank of New York - natural real rate of interest[5]
5. Federal Reserve Bank of Chicago - national activity index*[6]
6. Congressional Budget Office - potential GDP, federal and state government spending forecasts[7]
7. U.S. Bureau of Economic Analysis - GDP and subcomponents*, personal income*, personal expenditure*, PCE deflator*, personal savings rate*, corporate profits*, vehicle sales*
8. U.S. Federal Housing Finance Agency - housing construction*, housing sales*, and related subcomponents*
9. U.S. Energy Information Administration - spot WTI crude prices*
10. U.S. Bureau of Labor Statistics - unemployment rate*, producer price indices*, nonfarm labor share*
11. U.S. Bureau of the Census - homeownership rate*
12. U.S. Department of Housing and Urban Development - housing starts*, buildings permits*
13. Yahoo! Finance - S&P 500 prices
14. Moody's - corporate bond yields*
15. Freddie Mac - mortgage rates*
16. University of Michigan - consumer sentiment*

**Collected indirectly via St. Louis Federal Reserve Economic Database (FRED) at https://fred.stlouisfed.org/.*

All variables are directly scraped from FRED or their listed source above with the exception of the natural real rate of interest, the anxious index, potential GDP, and government spending. These variables must be entered in manually in the `macro-model/model-input/user-supplied-variables.xlsx` spreadsheet.

Following data scraping, all variables are exported into the `macro-model/model-output/raw-data.xlsx` spreadsheet, with worksheets for monthly and quarterly variables. The variable keys in the spreadsheed column names correspond to the same keys as in the `key` column of the `macro-model/model-input/variables.xlsx` spreadsheet and in the R code.

[3] `https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/anxious-index`

[4] `https://www.clevelandfed.org/our-research/indicators-and-data/inflation-expectations.aspx`

[5] `https://www.newyorkfed.org/research/policy/rstar`

[6] `https://www.chicagofed.org/publications/cfnai/index`

[7] `https://www.cbo.gov/about/products/budget-economic-data`

Deflating Nominal to Real Values

Next, all variables with currency units are deflated into real variables.

Inflation is measured by quarterly changes in the personal consumption expenditure price index (PCEPI). The PCEPI is used by the FOMC for inflation targeting, and in this model, FOMC inflation targets are a key determinant on nominal interest rates. We deflate all historical values into current-day dollars using the standard formula:

$$real\_var_t = nominal\_var_t \times \frac{PCE_{base}}{PCE_t}$$

where *base* is defined below.

Historical data, projections, and forecasts for all variables with currency units are reported in terms of real U.S. dollar values of the most recent quarter with data on all variables available.

The `variables.xlsx` spreadsheet details which variables are converted from nominal to real variables in the `convert_to_real` column; these are also listed in the appendix.

Calculated Variables

In addition to the variables collected above, we generate historical data for several of our own variables. These are described in the next several sections.

Dynamic Nelson Siegal (DNS) Parametrization

In this model, we do not include the various interest rates directly in the FAVAR, but instead decompose yield curves using a variant of the widely-used Dynamic Nelson-Siegel (1987) parameterization. This models a full interest rate or yield curve as an exponential function of just three parameters.

This parameterization has several benefits. It reduces the number of variables we will need to include in the model, reducing the likelihood of overfitting as well as reducing the dimensionality of the FAVAR. The fitted yield curves from this model are economically intuitive, specifying (1) a smooth yield curve and forward curve, (2) a positive forward rate at all horizons, and (3) a discount function that starts at one and approaches zero as the time to maturity approaches infinity. This factorization also has been shown to increase yield curve forecast performance, particularly in a model with other macro covariates and during recessions (De Pooter et al. 2010, Diebold et al. 2008). Additionally, it allows us to generate forecasts for the yield curve at *any* time-to-maturity, even without historical data for bond yields with that time-to-maturity.

Note that this parameterization still allows for inversions and partial inversions of the yield curve.

We use a mathematically equivalent respecification of the Dynamic Nelson-Siegel model, the Diebold-Li (2006) model, which has the benefit of allowing us to interpret the three parameters as the level, slope, and curvature of the yield curve. We will calculate historical data for these three parameters and later feed them as inputs to the model estimation procedure.

The Diebold-Li function is given by

$$yield_t(ttm) = f_{1t} + f_{2t}\left(\frac{1-\exp^{-\lambda ttm}}{\lambda ttm}\right) + f_{3t}\left(\frac{1-\exp^{-\lambda ttm}}{\lambda ttm} - e^{-\lambda ttm}\right)$$

where f_{1t}, f_{2t}, f_{3t} can be interpreted as the yield level, slope, and curvature at time t respectively, and ttm refers to the time to maturity in months.

In this section we will use the Treasury yields for our $yield_t$ to test the DNS decomposition for model validation purposes, in order to replicate the Diebold and Li (2006) paper.

To obtain historical estimates for f_{1t}, f_{2t}, f_{3t}, we use an OLS regression on the available Treasury yield data of any available maturity lengths at time t, with λ set to 0.0609 as per the original methodology. The figure below shows the estimated time series of coefficients f_1, f_2, and f_3 and can be validated against Fig. 7 in Diebold and Li (2006).

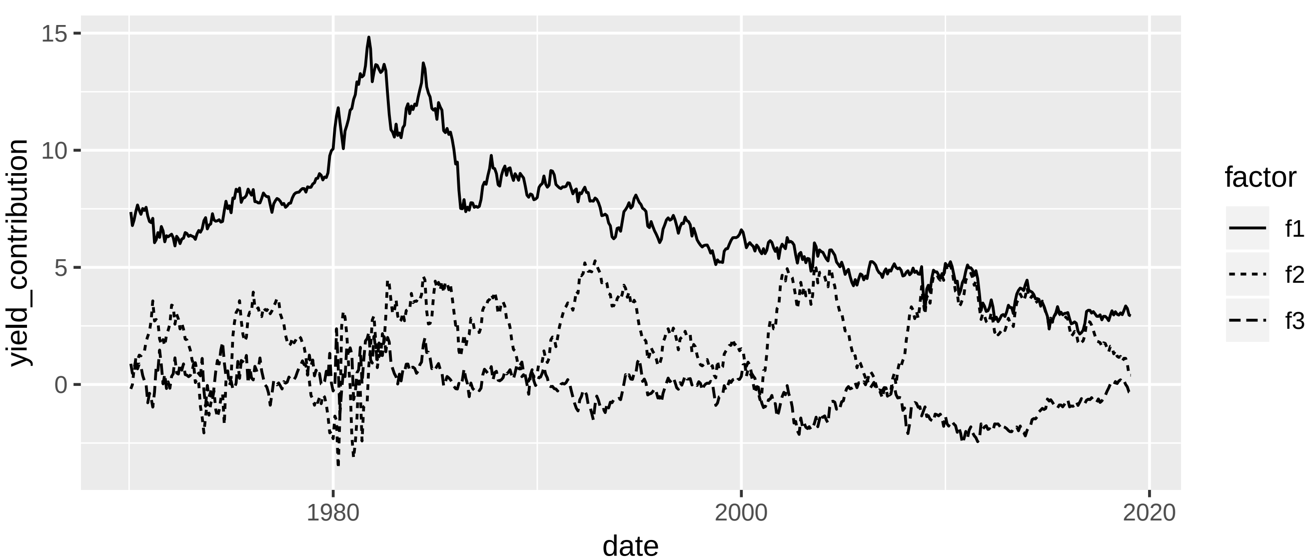

Figure 1: Treasury Yields Dynamic Nelson-Siegel Decomposition Chart

Following estimation of f_{1t}, f_{2t}, and f_{3t}, we can then calculate $yield_t(ttm)$ for any ttm by putting ttm, f_{1t}, f_{2t}, and f_{3t} into the Diebold-Li function. The chart below shows the fit of the Diebold-Li yield curve compared to the actual yield curve for the last available time period.

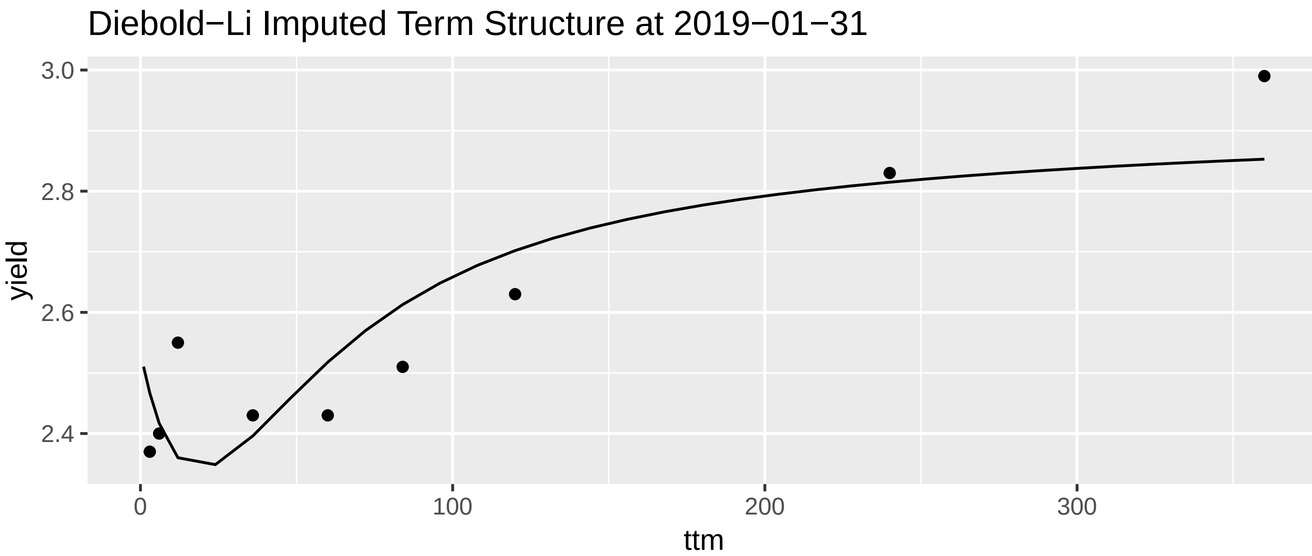

Figure 2: Treasury Yield DNS Fit for Last Data period

Expected Inflation

An analogous decomposition is conducted for the expected inflation curve. Historical data is provided by the Cleveland Fed, which estimates historical expected inflation using the spread between Treasury Inflation-Protected Securities (TIPS) and Treasuries and through inflation term swaps[8].

We compute the historical series of the three DNS parameters for expected inflation, $einf_{f1}$, $einf_{f2}$, and $einf_{f3}$ similarly to f_1, f_2, and f_3. However, instead of using the λ provided by Diebold and Li (2006), we use a λ calculated through an optimization algorithmn that minimizes SSE between the DNS expected inflation curve and the actual expected inflation curve.

Optimal λ	0.0189

The chart below shows the estimated time series values of the three parameter estimates.

[8]Model details can be found at `https://www.clevelandfed.org/our-research/indicators-and-data/inflation-expectations.aspx` with technical documentation in Haubrich et al. (2011).

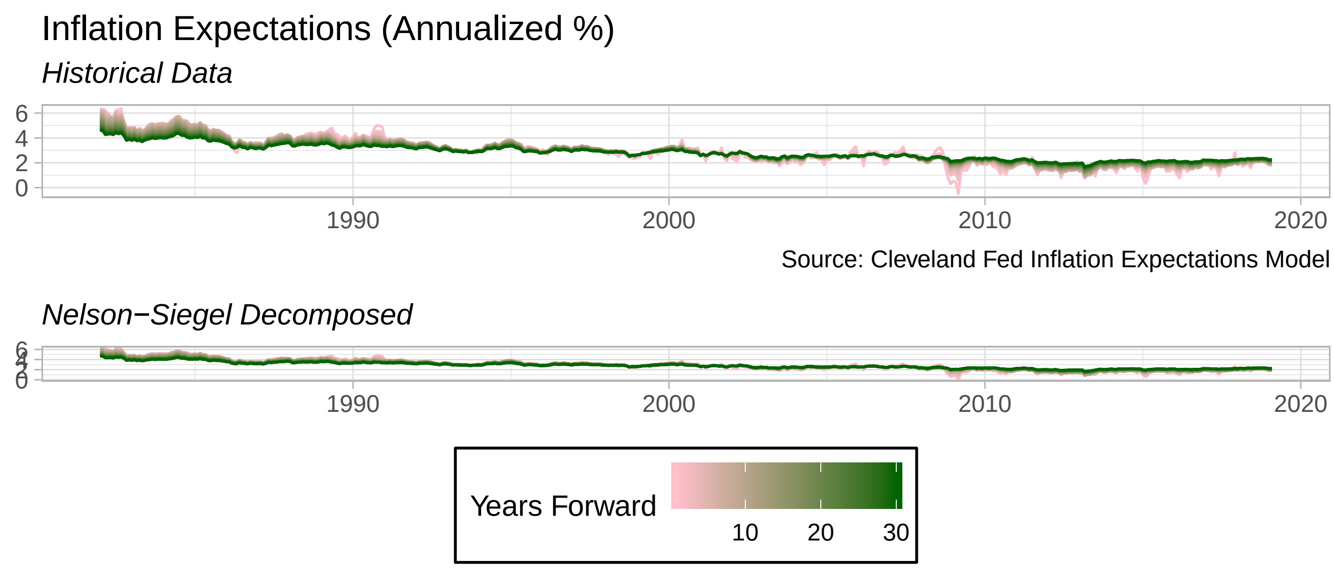

Figure 3: Expected Inflation Rate Dynamic Nelson-Siegel Decomposition Chart

We then rebuild the time series of expected inflation using our estimated time series values of $einf_{f1}$, $einf_{f2}$, and $einf_{f3}$. The graph below shows that it closely approximates the actual expected inflation curve over time.

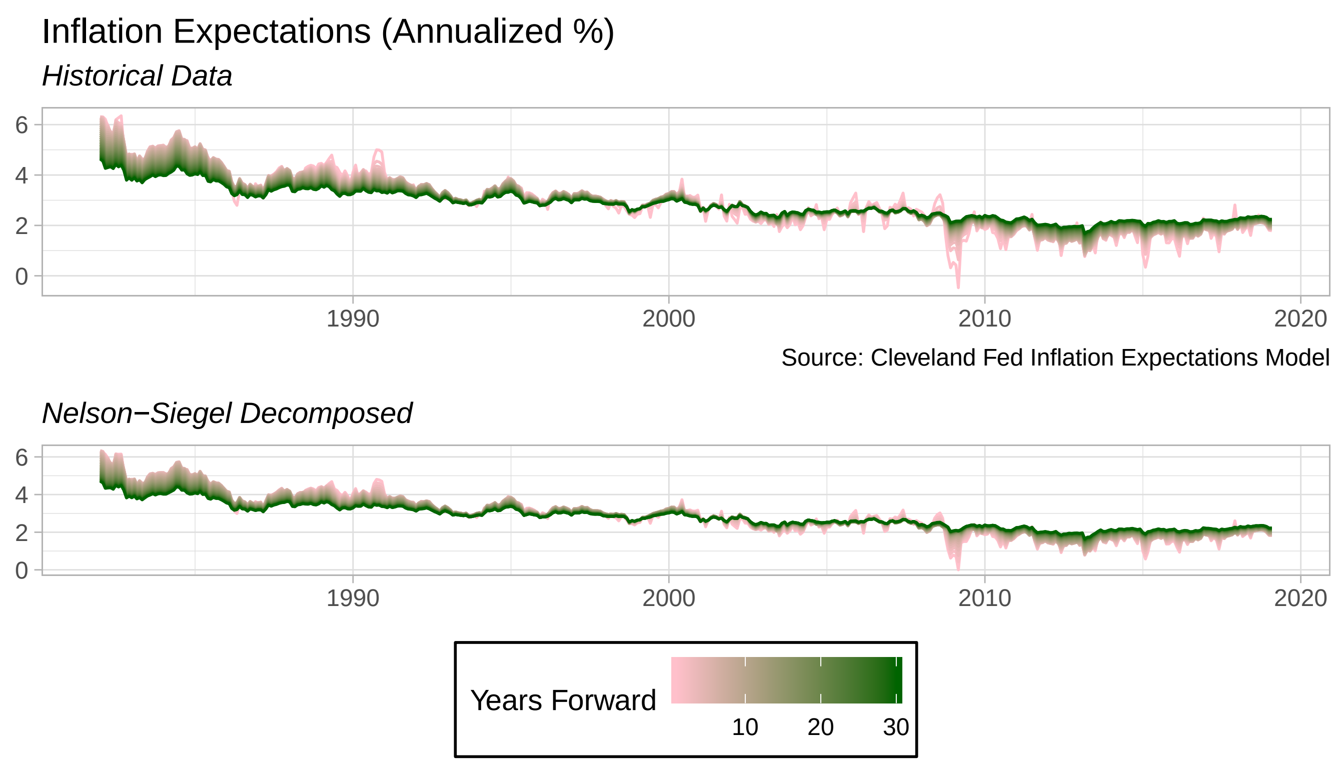

Figure 4: Expected Inflation

Yield Curve Spreads

We follow standard financial theory in decomposing nominal interest rates into

$$\begin{aligned} nominal\_rate = {} & real\_risk\_free\_rate + expected\_inflation + inflation\_risk\_premium + \\ & maturity\_risk\_premium + default\_risk\_premium + liquidity\_premium. \end{aligned}$$

We want to create an economically coherent model of interest rates. For example, if we forecast expected inflation to be 2% and the real risk-free rate to be 1%, we would expect that corporate yields would be higher than 3%. To do so, we include spreads and risk premia in the model estimation process instead of including values of interest rates directly.

We make several standard assumptions in order to decompose interest rates: (1) the default risk premium and liquidity premium for Treasuries is zero, and (2) the maturity premium is zero on 3-month T-bills.

The two assumptions combined imply the nominal risk-free rate is given by 3-month T-bills. This allows us to calculate the real risk-free rate by taking the difference between 3-month Treasury yields and expected inflation (while we do not have data on 3-month expected inflation, we can get an estimated value by using our DNS decomposition of the expected inflation curve). This allows us to generate a time series of the real risk free rate, $rrfr_t$.

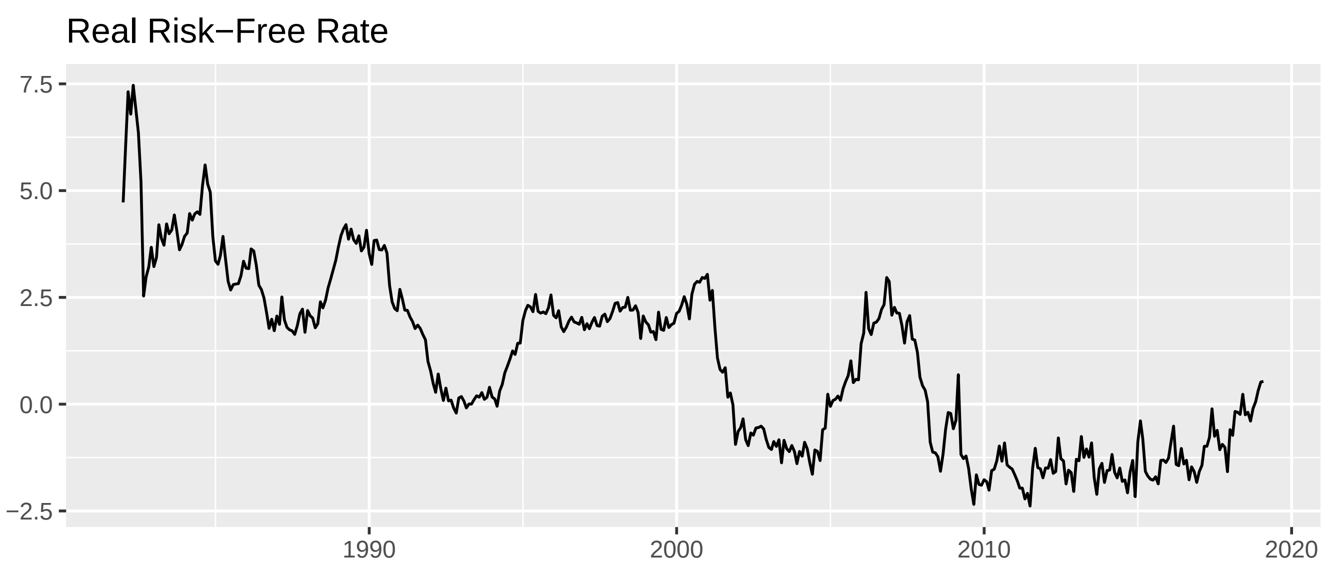

Figure 5: Calculated Historical Real Risk-Free Rate

We now use time series of our longer-dated Treasuries and the estimated risk-free rate time series to find the sum of the maturity risk premium and the inflation risk premium, which for simplicity we notate as the $rate\_premium$. The rate premium is a curve thats value depends on the time to maturity. Therefore we can convert it into a three-parameter time series using the DNS specification. Again, we find the optimal λ using an SSE minimization procedure.

Optimal λ	0.0516

The chart below shows the historical values of the parameter values.

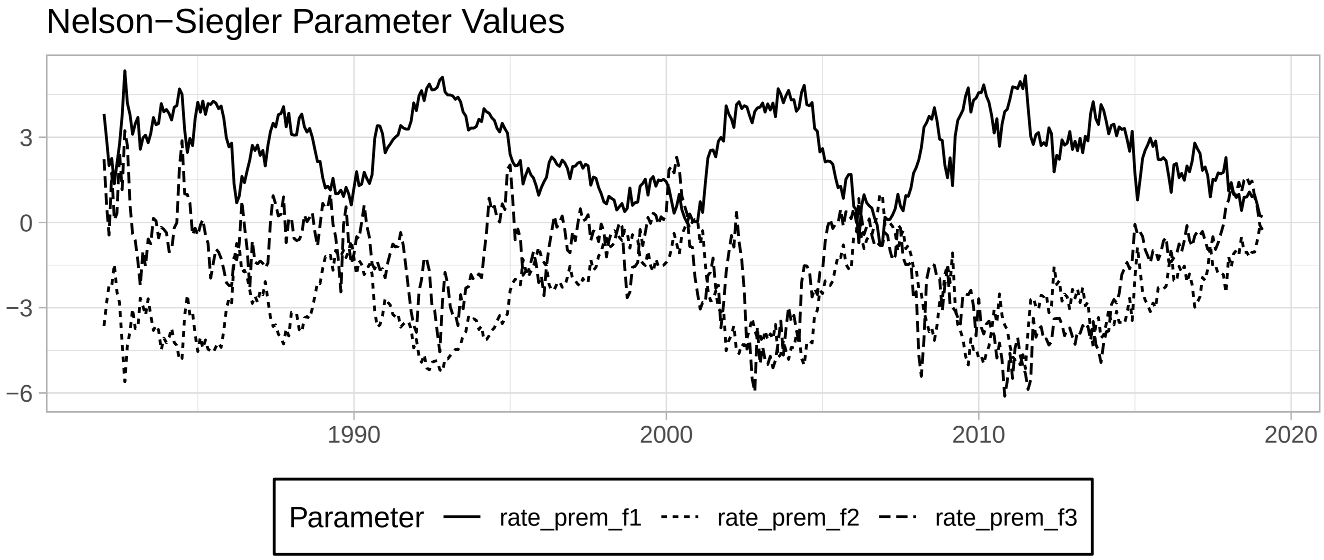

Figure 6: Historical Rate Premium Dynamic Nelson-Siegel Decomposition Chart

As before, we rebuild the rate premium time series using our calculated time series of parameter values. We graph it below in comparison with the estimated historical data to show that it serves as a close approximation.

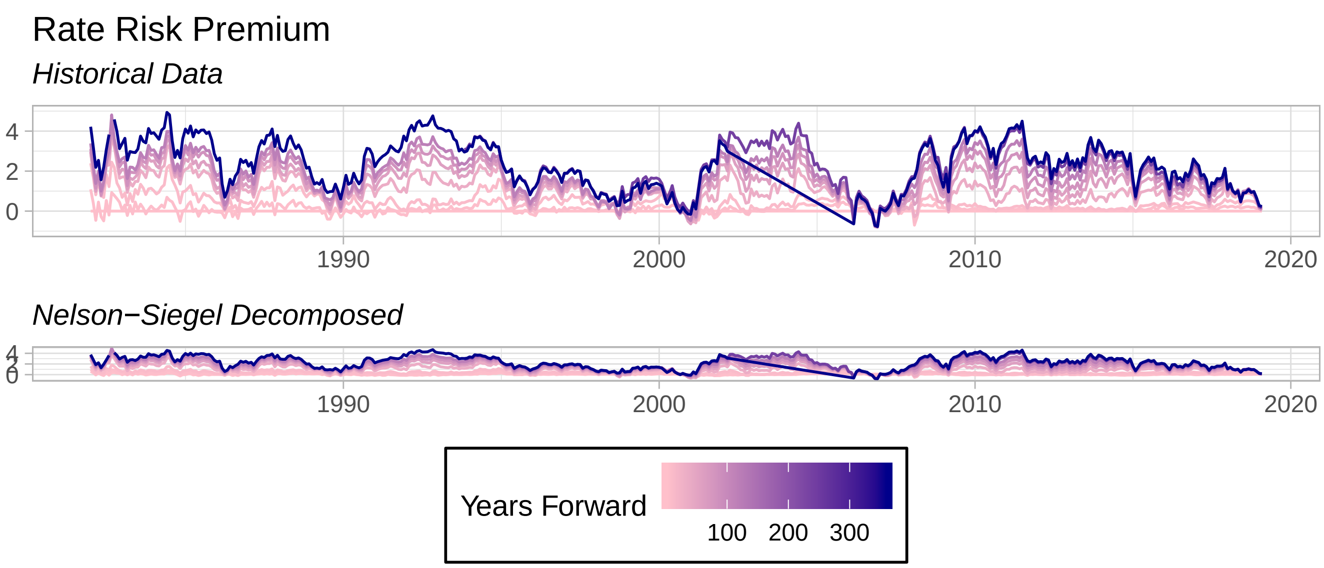

Figure 7: Rate Premium

We also show the fit of the DNS curve for the last time point with available data.

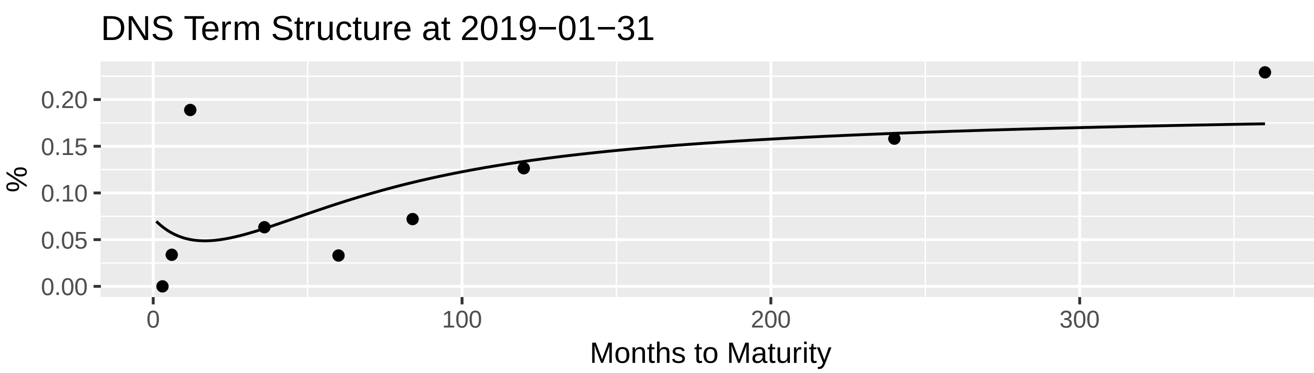

Figure 8: Last-Period DNS Curve Fit for Rate Risk Premium

With these estimates of the real risk-free rate, the three DNS parameters describing the expected inflation curve, and the three DNS parameters describing the rate-premium curve, and holding our above assumptions true, we are able to construct estimates of the Treasury yield curve. The graph below shows the reconstructed 30-year Treasury yield series using this method, compared to historical data. The two are a close fit.

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## Warning:  Removed 57 rows containing missing values (geom_path).
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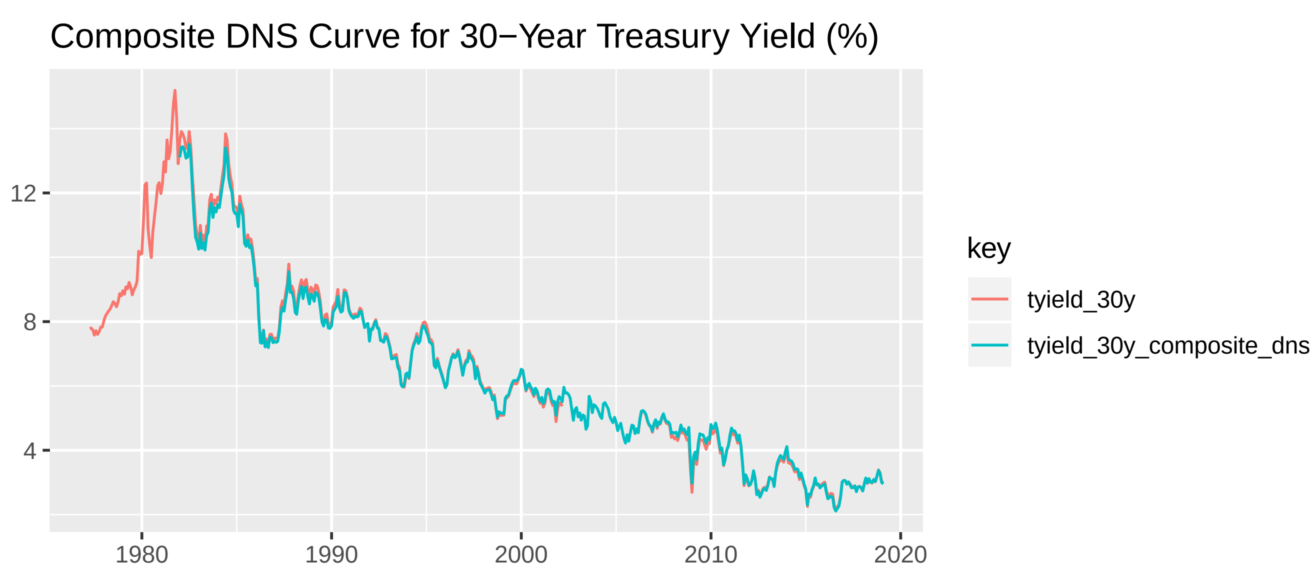

Figure 9: Reconstructed versus Historical 10-Year Treasury Yield

When we later create our forecast model, this decomposition will have the benefit of allowing us to parsimonously and cohesively estimate forecasts of the full Treasury yield curve and its subcomponents by only forecasting the seven parameters described above. Moreover, this decomposition allows us to model how changes in one variable impact specific subcomponents of the yield curve. For example, we may wish to model that a variable impacts the riskless rate but not inflation expectations, or model monetary policy actions which alter only specific pieces of the rate premium[9].

We also calculate $aaa\_spread_t = aaa\_yield_t - real\_risk\_free\_rate_t - expected\_inflation_t - rate\_premium_t$ for all t to derive a proxy for the $maturity\_risk\_premium + default\_risk\_premium$ on corporate Aaa bonds. Later, forecasting this variable out with the seven previously described variables will allow us to generate parsimonious estimates of the Aaa yields.

We repeat the process with all other interest rates in the model, excluding the federal funds rate which will be determined by the Taylor Rule, and the prime rate, which is modeled as a spread from the fed funds rate due to the two variables' close co-movement.

Monetary Policy Variables

We also calculate historical values of the GDP gap (logged actual GDP minus logged potential GDP), historical values of the inflation gap (realized inflation minus the inflation target), and historical Taylor Rule targets (see the Taylor Rule section).

Transformations

Finally, after converting variables into real variables when necessary and adding the variables previously discussed, we transform variables into forms needed for later time-series modeling.

Most data series are transformed by taking a natural log, a first difference, or a log-difference (the first differenced of logged values).

We use first differences to convert a non-stationary but linear series into a stationary one.

Logs are used primarily for interest rate and interest rate spread variables. This has the benefit of (1) forcing projections to be positive, which is economically desirable for most interest rates, and (2) weighting small changes in rates more when interest rates are low than when rates are high.

A log-differencing transformation is used when (1) the series is non-stationary and exponential, or when (2) compounded downsides need to be symmetrically equal to upsides (e.g., a non-logged series will weight a $+100\%$ gain more than a -50% loss even though $1 * 200\% * 50\% = 1$, whereas a logged series will weight them equally). The latter rationale can be primarily applied for variables representing assets or asset classes. Log-differencing is also used when the variance of a series is non-constant and increasing even after a first-difference.

Following transformation, we have our final list of input variables. These can be found in `macro-model/model-output/final-data.xlsx`.

- The `hist_base_m` worksheet contains untransformed monthly-frequency variables.
- The `hist_base_q` worksheet contains untransformed quarterly-frequency variables.
- The `hist_base` worksheet contains untransformed data for all variables, with monthly-frequency variables aggregated to quarterly frequency.

[9] e.g., "Operation Twist" increasing curvature of the rate premium but not level

- The `hist_pca_m` worksheet contains monthly-frequency variables transformed following the specifications given in the `pca_form` column of the `variables.xlsx` worksheet.
- The `hist_output` worksheet contains data for all variables, with monthly-frequency variabels aggregated to quarterly frequency, transformed following the specifications given in the `output_form` column of the `variables.xlsx` worksheet.

And so on for the other worksheets. The data in the `*_m` and `*_q` worksheets will later be used in the nowcasting model while the remaining transformations are for use in the forecasting model.

Principal Components Analysis

We first estimate the factors by taking a principal components analysis of the monthly-frequency variables. First, the variables are scaled and centered. We denote this standardized matrix of data as X, with $i = 1, .., N$ indexing each monthly covariate., and $t = 1, ..., T, ..., \tau$ as the number of monthly time series. T refers to the last month where all variables have available data while τ refers to the last month where any variable has available data. We use PCA to extract the time series of factors from 1 to T, then later project the time series up to τ using the Kalman Filter and Kalman Smoother.

Let F denote the $T \times R$ matrix of factors, where $r = 1, ..., R$ refers to the number of factors. We want the F and the coefficient matrix Λ which minimizes the least squares problem,

$$SSE = \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} (x_{it} - \lambda_i F_t)^2. \tag{1}$$

Stock and Watson (2002) show that the estimated solutions $\hat{F}$ and $\hat{\Lambda}$ are given by

$$\hat{F} = \frac{1}{N} X' \times \hat{\Lambda} \tag{2}$$

and setting Λ to the eigenvectors of $X'X$..

From this process we extract $R = N$ factors. It has been shown that a relatively large number of macroeconomic series can be explained by only a few common factors (Bernanke et al. 2005, Stock and Watson 2011), so to prevent overfitting and overparametrization, we select an $R \leq 10$.

To select the number of factors we will use, R, we utilize the scree chart below as well as the Bai-Ng (2002) factor model information criteria, restricted to a maximum of 10 factors.

$$IC = log(S\hat{S}E) + R \left(\frac{N+T}{NT} \right) log \left(\frac{NT}{N+T} \right) \tag{3}$$

$$\text{where } S\hat{S}E \text{ is the minimized value of } SSE. \tag{4}$$

We run a grid search over $R = 1, ..., N$ and select the R which gives gives the lowest IC.

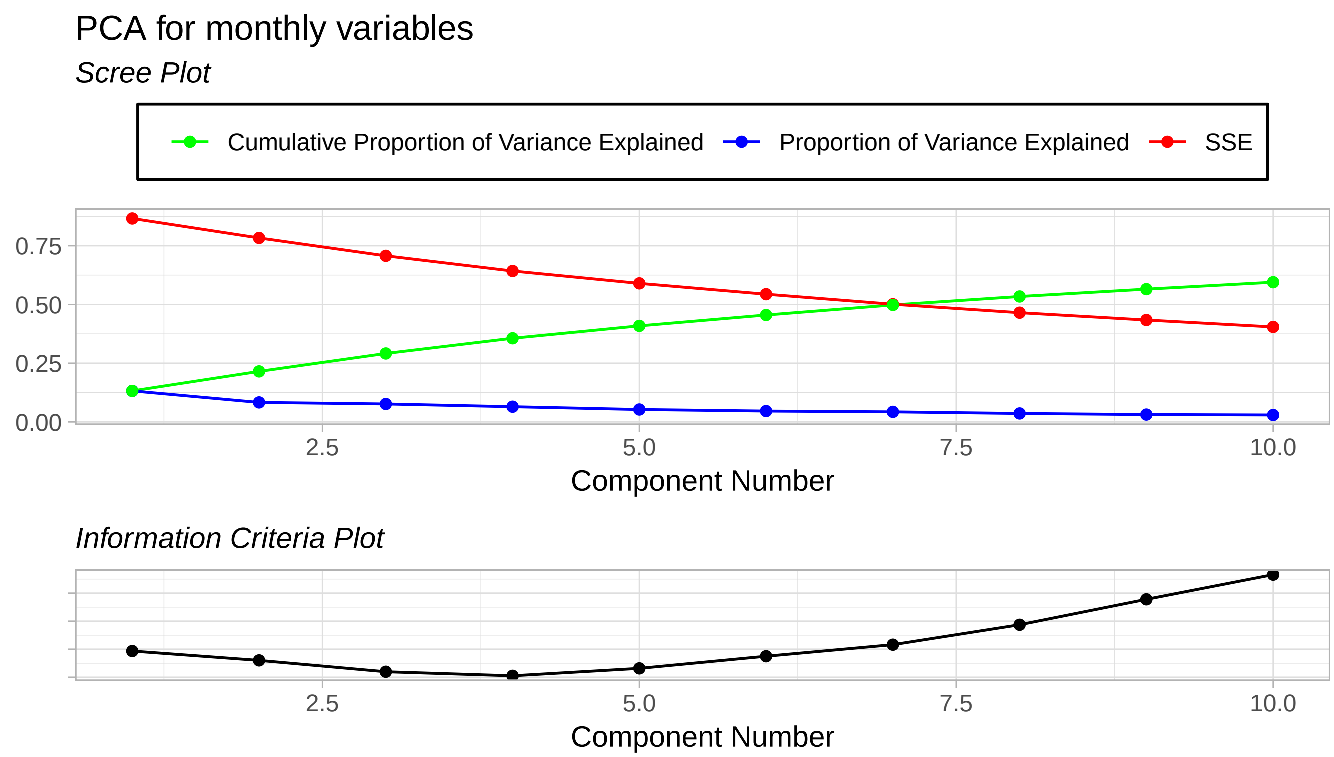

Figure 10: Principal Components Chart

Optimal R	4

The graph below shows the time series of the extracted principal components.

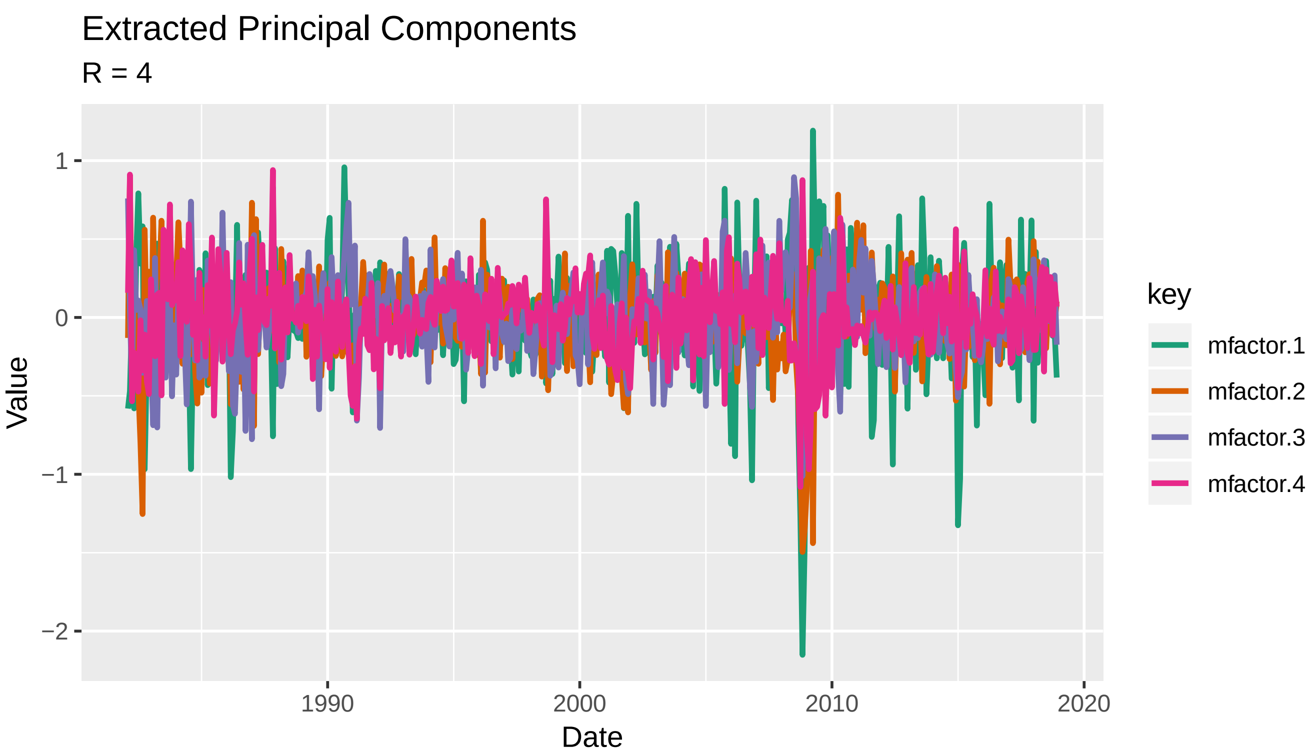

Figure 11: Principal Components Chart

3.2 Mean Modeling

Variable Partitioning

We begin by partitioning each model variable into several different groups. Variables are modeled either in the factor-augmented vector autoregression (FAVAR), a structural econometric model (SEM), or as individual AR-X equations. The FAVAR is used for forecasting variables which are difficult to model with standard economic theory but which can be causally interpreted. A variable is modeled through the SEM if there is strong and empirically proven economic theory which underlies it. Lastly, a variable is modeled with an AR-X equation if it does not fit in the above categories, or if the historical time series for a variable starts at a later point than the start point of the FAVAR time series.

A flowchart of the variable partitioning process is shown below.

The resulting forecasting model used for each variable can be found in the “forecast˙method” column of *variables.xlsx*. Note that some variables are marked as “IDENTITY” or “IDENTITY.SEM”. These are variables which are not estimated but instead calculated as identity equations of estimated variables (e.g., $net\_exports = exports - imports$).

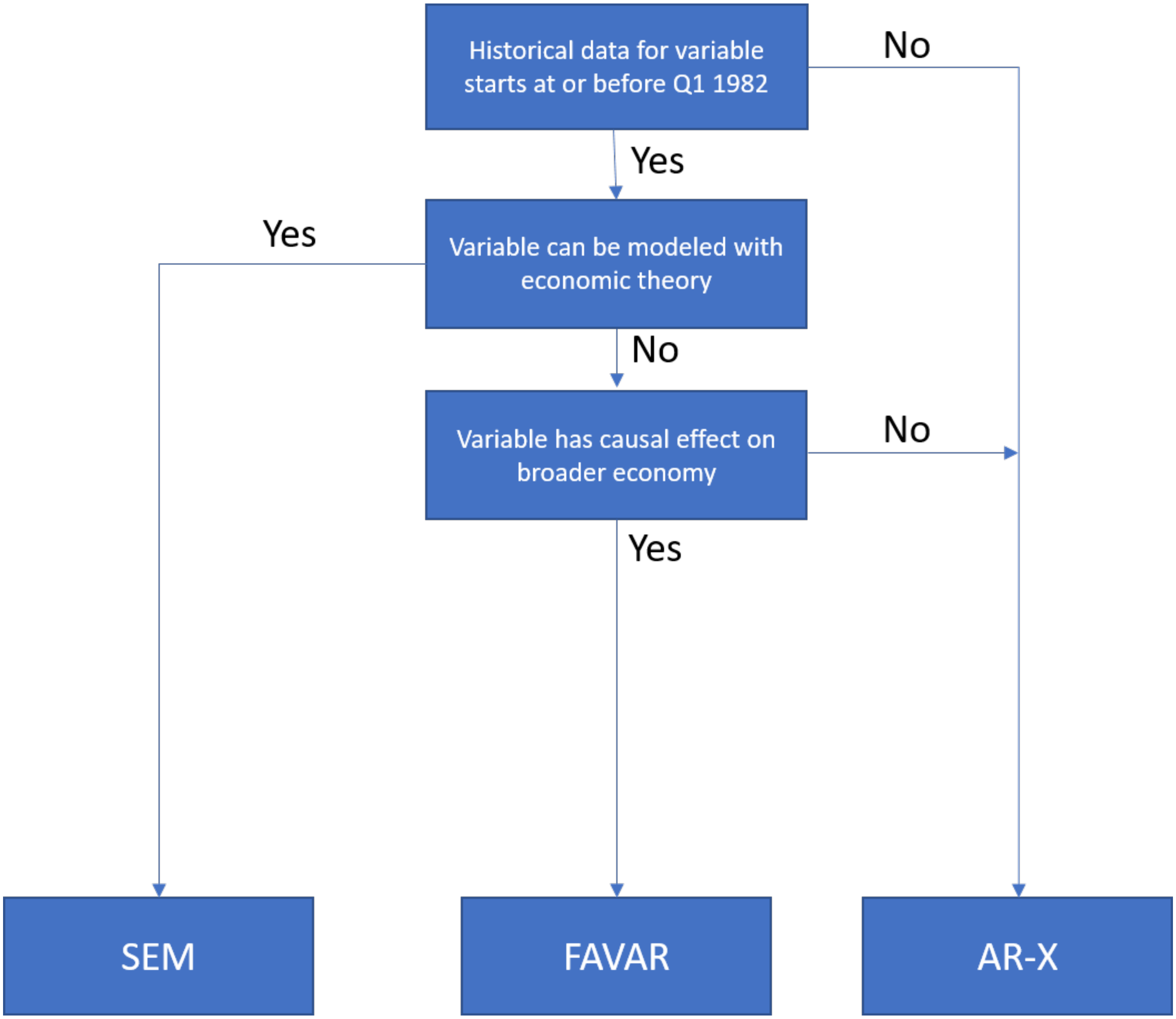

Figure 12: Variable Selection Flowchart

FAVAR Estimation

Because the dimensionality of a VAR increases by N for each one additional covariate, a VAR can only handle a limited set of information before running into degrees-of-freedom issues. This frequently leads to omitted variable bias in forecast results. Stock and Watson (2002), Bernanke and Boivin (2003), and Bernanke et al. (2005) show that this bias can be improved by extracting principal components from larger macroeconomic datasets, and that forecasts based on factors outperform univariate ARIMAs and VARs. These factors are included as covariates in the VAR.

To estimate the FAVAR, we begin by taking all quarterly, stationary-transformed data series which have the "FAVAR" forecast method in *variables.xlsx*, as well as the factors extracted from the principal components analysis performed earlier. These datasets are merged by date; any date where any of the variables or factors is missing an observation is thrown out. Our final date range includes all quarters between and inclusive of **1982-03-31** and **2018-09-30**.

We use the notation from Stock and Watson (2003), as well as our previous notation that R refers to the number of factors from extracted principal components. L here denotes the number of autoregressive lags in the VAR.

$$\begin{bmatrix} F_t \\ X_t \end{bmatrix} = \begin{bmatrix} \Lambda(L) \end{bmatrix} \begin{bmatrix} F_{t-1} \\ X_{t-1} \end{bmatrix} + U_t,$$

$$\begin{aligned} \text{where } \Lambda(L) \text{ is an } (NL+RL)\times(NL+RL) \text{ matrix of lag polynominal coefficients,} \\ X_t \text{ is an } NL\times 1 \text{ matrix of variables to be modeled in the FAVAR,} \\ \text{and } F_t \text{ is an } RL\times 1 \text{ matrix of factors extracted earlier.} \end{aligned}$$

This equation can be estimated efficiently by linewise estimation of each variable through OLS (Lutkepohl 2005, p.71).

We consider several different parameterizations of the equation above: allowing for the existence of a constant term in the model or not, and allowing for the autoregressive lag term to be any number between 1 and 4. This allows for 8 total combinations; for each, we calculate the coefficient estimates $\Lambda(L)$, we test the stability of the model, and we calculate the Schwarz information criteria.

Equation estimation uses linewise OLS as explained above. Stability of the FAVAR process is calculated by checking the modulus of the eigenvalues of a transformed coefficient matrix as described in Lutkepohl 2005, p. 15.

The Schwarz information criteria is given by

$$\begin{aligned} SC(L) = log(det(\Sigma(L))) + \frac{log(T)}{T} \times L(N+R)^2, \\ \Sigma(L) = \sum_{1}^{T} U_t' U_t \text{where } T \text{ is the number of observations,} \\ \text{and } \Sigma(L) \text{ is the estimated variance matrix of the residuals.} \end{aligned}$$

We use the Schwarz criteria here due to its stronger consistency properties over other common information criteria (Lutkepohl 2005). Given our 8 test models, we choose the combination which gives the lowest Schwarz value of all stable models. The table below shows the results.

Table 1:

	L	has_constant	stable	schwarz
1	1	TRUE	TRUE	-40.9848192629943
2	2	TRUE	TRUE	-26.6211542031906
3	3	TRUE	TRUE	-14.7094084262216
4	4	TRUE	TRUE	-22.15359054338
5	1	FALSE	TRUE	-40.2828125695518
6	2	FALSE	TRUE	-25.7623358359236
7	3	FALSE	TRUE	-13.9053272758286
8	4	FALSE	TRUE	-19.7289355526487

We select a lag order of **1** and **include** a constant in the model.

Following this, we then perform qualitative checks to verify that the in-time fits of the FAVAR variables are reasonable.

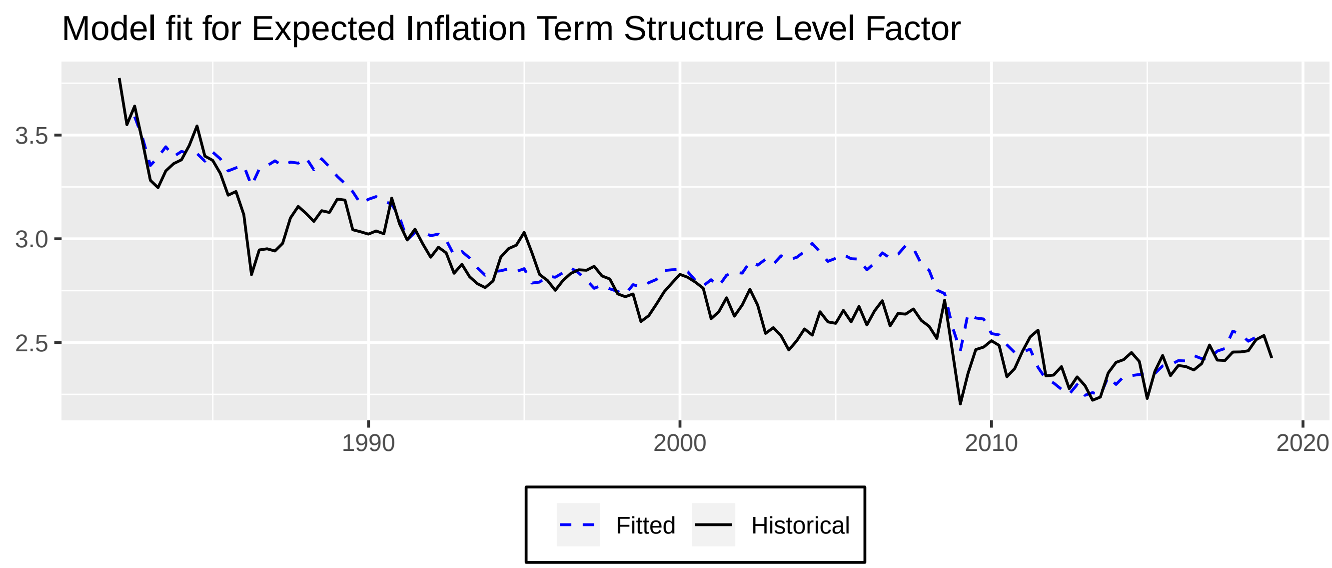

Figure 13: An example FAVAR fit

We create impulse response charts and qualitatively check them for intuitiveness. For example, the chart below shows the effect of a unit residual shock to the U.S. dollar index on the SP 500. Here the residual of the U.S. dollar almost certainly represents largely the strength or weakness of the U.S. economy relative to foreign economies, as those foreign economies are unmodeled in CHIMPS. Thus a positive residual likely indicates relative strength of the U.S. economy to foreign ones, which typically results in fund flows toward dollar-denominated assets, as in the impulse response chart shown below.

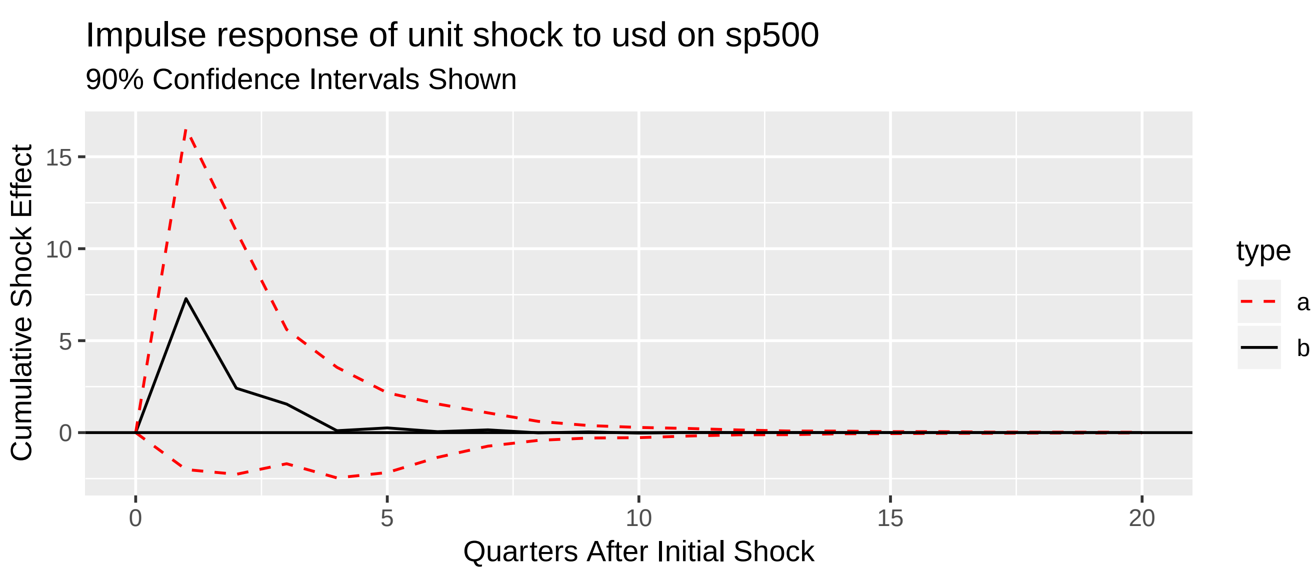

Figure 14: An example FAVAR impulse response chart

4 References

Bai, Jushan and Serena Ng (2002), "Determining the Number of Factors in Approximate Factor Models", *Econometrica*, 70:1, 191-221.

Bai, Jushan Serena Ng (2008), "Forecasting economic time series using targeted predictors", *Journal of Econometrics*, 146:2, 304-317.

Bai, Jushan, Kunpeng Li, Lina Lu (2014), "Estimation and Inference of FAVAR Models", *Journal of Business & Economic Statistics*, 34:4, 620-641.

Banbura, Marta, Gerhard Runstler (2011), "A look into the factor model black box: Publication lags and the role of hard and soft data in forecasting GDP", *International Journal of Forecasting*, 26:2, 333-346.

Bernanke, Ben, Jean Boivin, Piotr Eliasz (2005), "Measuring the Effects of Monetary Policy: A Factor-Augmented Vector Autoregressive (FAVAR) Approach", *Quarterly Journal of Economics*, 120:1, 387-422.

Bernanke, Ben (2015), "The Taylor Rule: A benchmark for monetary policy?", *Brookings Institute, The Hutchins Center on Fiscal and Monetary Policy*

Bok, Brandyn, Daniele Caratelli, Domenico Giannone, Argia Sbordone, Andrea Tambalotti

Carlstrom, Charles and Timothy Fuerst (2014), "Adding Double Inertia to Taylor Rules to Improve Accuracy", *Federal Reserve Bank of Cleveland Economic Commentary*, 2014:8.

Cao, Yifan, Adam Shapiro (2013), "Why Do Measures of Inflation Disagree?", *Federal Reserve Bank of San Francisco Economic Letters*, 2013:37.

Christensen, Jens, Francis Diebolid, Glenn Rudebusch (2010), "The Affine Arbitrage-Free Class of Nelson-Siegel Term Structure Models", *Journal of Econometrics*, 164:1, 4-20.

De Pooter, Michiel, Francesco Ravazzolo, Dick J.C. van Diyk (2010), "Term Structure Forecasting Using Macro Factors and Forecast Combination", *Board of Governors of the Federal Reserve System International Finance Discussion Papers*, 993.

Diebold, Francis, Canlin Li (2006), "Forecasting the term structure of government bond yields", *Journal of Econometrics*, 130, 337-364.

Diebold, Francis, Canlin Li, Vivian Yue (2008), "Global yield curve dynamics and interactions: A dynamic Nelson-Siegel approach", *Journal of Econometrics*, 146, 351-363.

Doz, Cathering, Domenico Giannone, Lucrezia Reichlin (2011), "A two-step estimator for large approximate dynamic factor models based on Kalman filtering".

ECB (2008), "Short-Term Forecasts of Economic Activity in the Euro Area", *ECB Monthly Bulletin*, April 2008.

Engle, Robert (2002), "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models", *Journal of Business & Economic Statistics*, 20:3, 339-350.

Gali, Jordi, Mark Gertler (1999), "Inflation dynamics: A structural econometric analysis", *Journal of Monetary Economics*, 44:2, 195-222.

Giannone, Domenico, Lucrezia Reichlin, David Small (2008), "Nowcasting: The real-time informational content of macroeconomic data", *Journal of Monetary Economics*, 55, 665-676.

Fernández-Villaverde, Jesús, Juan Rubio-Ramírez (2010), "Macroeconomics and Volatility: Data, Models, and Estimation", *NBER Working Papers*, 16618.

Foroni, Claudi, Massimilano Marcellino (2013), "A survey of econometric methods for mixed frequency data", *Norges Bank*, Working Paper 2013:06.

Goncalves, Silvia, Lutz Kilian (2004), "Bootstrapping autoregressions with conditional heteroskedasticity of unknown form", *Journal of Econometrics*, 123:1, 89-120.

Hamilton, James (1994). "Time Series Analysis", *Princeton University Press*, ISBN 0691042896.

Hamilton, James (2008). "Macroeconomics and ARCH", *NBER Working Papers*, 14151.

Haubrich, Joseph, George Pennachi, Peter Ritchken (2011), "Inflation Expectations, Real Rates, and Risk Premia: Evidence from Inflation Swaps", *Cleveland Fed Working Papers*, 11-07.

Justiniano, Alejandro, Giorgio Primiceri (2008), "The Time-Varying Volatility of Macroeconomic Fluctuations", *American Economic Review*, 98:3, 604-641.

Karlsson, Sune (2013), "Forecasting with Bayesian Vector Autoregression", *Handbook of Economic Forecasting*, 2:B, 791-897.

Laubach, Thomas and John Williams, "Measuring the Natural Rate of Interest", *The Review of Economics and Statistics*, 85:4, 1063-1070.

Lutkepohl, Helmut (2007), "New Introduction to Multiple Time Series Analysis", *Springer Publishing Company*, ISBN 3540262393.

Monech, Emanuel (2008), "Forecasting the yield curve in a data-rich environment: A no-arbitrage factor-augmented VAR approach", *Journal of Econometrics*, 146, 26-43.

Nelson, Charles, Andrew Siegel (1987), "Parsimonious Modeling of Yield Curves", *The Journal of Business*, 60:4, 473-489.

Stock, James and Mark Watson (2002), "Forecasting Using Principal Components From a Large Number of Predictors", *Journal of the American Statistical Association*, 97:460, 1167-1179.

Powell, Jerome (2018), "Monetary Policy in a Changing Economy".

Rudebusch, Glenn (2007), "Monetary Policy Inertia and Recent Fed Actions", *Federal Reserve Bank of San Francisco Economic Letters*, 2007:3.

Stock, James, Mark Watson (2002), "Has the Busienss Cycle Changed and Why", *NBER Working Papers*, 9127.

Stock, James, Mark Watson (2011), "Dynamic Factor Models", *Oxford Handbook on Economic Forecasting*, Oxford: Oxford University Press.

Taylor, John (1993), "Discretion versus policy rules in practice", *Carnegie-Rochester Conference Series on Public Policy*, 39, 195-214.

Taylor, John (1999), "A Historical Analysis of Monetary Policy Rules", *NBER Chapters in Monetary Policy Rules*, 319-348.

Woodford, Michael (1999), "Optimal Monetary Policy Inertia", *NBER Working Papers*, 7261.

Yellen, Janet (2017), "The Economic Outlook and the Conduct of Monetary Policy".

A Math

A.1 Log-Linearization

The multivariate first-order Taylor approximation of a function

$$z = f(x^1, \dots, x^n)$$

around the points $(\overline{x^1}, \dots, \overline{x^n})$ is given by

$$z \approx f(\overline{x^1}, \dots, \overline{x^n}) + \sum_{i=1}^{n} \frac{\partial}{\partial x^i} f(\overline{x^1}, \dots, \overline{x^n})(x^i - \overline{x^i}).$$

Now suppose we have an identity equation,

.

We wish to instead rewrite this function in terms of the stationary variables $dlog(y_t), dlog(x_t^1), \dots, dlog(x_t^n)$. Taking the logs of both sides,

).

We take a first-order Taylor rule approximation around the points $(\overline{x_{t-1}^1}, \dots, \overline{x_{t-1}^n})$, yielding

$$log(y_t) \approx log(x_{t-1}^1 + \dots + x_{t-1}^n) + \frac{1}{x_{t-1}^1 + \dots + x_{t-1}^n} \sum_{i=1}^{n} x_t^i - x_{t-1}^i$$