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THE HAMMER PRODUCTION PROBLEM

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LOGISTICS PROJECT

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1 Introduction

In the dynamic landscape of manufacturing, the efficient production and strategic pricing of goods are pivotal factors that define the success of any company. In this context, the company specializes in the production of three different types of hammers - designated as type A, type B and type C. The production process lies in the acquisition of iron, which may be purchased from a supplier who sells it at 4 Euro/Kg, till a maximum of 4800 Kg. The quantity of iron required to craft a single unit of each hammer type, the unit manufacturing cost per type, and the anticipated profit associated with the sale of each hammer type are reported in Table 1.

	Iron Kg/for hammer	Profit	Cost of manufacturing
Type A	1	27	11
Type B	1	21	6
Type C	2	33	5

Table 1: Input data

The objective of the company is to establish how many hammers of each type to produce in such a way to maximize the revenue for the company (given by total profit minus costs), knowing that the three types of hammers need to be produced by satisfying the following additional constraints:

- The number of hammers of type A must be at least the double of the number of hammers of type B;
- The number of hammers of type A must be not higher than the number of hammers of type C.

2 Model 1

Given the input data and the aim of the company to establish the number of hammers of each type to be produced, this can be formulated as an Integer Linear Programming (ILP) problem because they must take on integer values. It is not feasible to produce a fractional part of a hammer; production quantities need to be whole numbers.

2.1 Input data and decision variables

The set of the types of hammers, namely type A, type B and type C, is denoted with I. Therefore, the decision variables are:

- x_i is the number of hammers of type i to be produced, $\forall i \in I$.

The rest of the input data can be defined as follows:

- c is the fixed cost of the iron per Kg;
- a is the iron available to order from the supplier;
- q_i is the quantity of iron necessary to produce hammer of type i, $\forall i \in I$;
- p_i is the profit for hammer of type i, $\forall i \in I$;
- m_i is the cost of manufacturing to produce hammer of type i, $\forall i \in I$.

2.2 Model formulation

The aim of this model would be to maximize the revenue of the company (given by the total profit minus the costs). The objective function described could then be expressed as the following mathematical function:

$$max \sum_{i \in I} (p_i - q_i * c - m_i) * x_i$$

The maximization is subject to the maximum quantity deliverable by the supplier. The sum of the quantity of iron multiplied by the number of hammers, for every i, must be smaller than or equal to 4800 Kg:

$$\sum_{i \in I} q_i * x_i \leq a$$

The following are the additional constraints the company must satisfy:

$$x_{typeA} \geq 2 * x_{typeB}$$

$$x_{typeA} \leq x_{typeC}$$

Finally there are the non-negativity and integrality constraints:

$$x_i \geq 0, \forall i \in I$$

$$x_i \; integer, \forall i \in I$$

2.3 Resolution of the model and results

The problem has been implemented by means of the modelling language AMPL, and has been solved with the optimization solver CPLEX. The optimal solution for Model 1 returns a total revenue of 51428€ with 1372 hammers of type A, 684 hammers of type B and 1372 hammers of type C.

3 Model 2

The company then decides to allocate an extra budget of 800 Euros to invest in iron from another supplier, who sells it at 7 Euros/Kg. The rest of the data and constraints remain the same.

3.1 Input data and decision variables

The set of the types of hammers, namely type A, type B and type C, is denoted with I as before, whereas the set of the suppliers, namely supplier 1 and supplier 2, is denoted with J. Therefore, the decision variables are:

- x_{ij} is the number of hammers of type i to be produced by iron supplier j, $\forall i \in I$, $\forall j \in J$.

The rest of the input data can be defined as follows:

- c_j is the fixed cost of the iron per Kg by iron supplier j, $\forall j \in J$;
- a is the iron available to order from the supplier 1;
- e is the extra budget for supplier 2;
- q_i is the quantity of iron necessary to produce hammer of type i, $\forall i \in I$;
- p_i is the profit for hammer of type i, $\forall i \in I$;
- m_i is the cost of manufacturing to produce hammer of type i, $\forall i \in I$.

3.2 Model formulation

The aim of this model would still be to maximize the revenue of the company (given by the total profit minus the costs). The objective function described could then be expressed as the following mathematical function:

$$max \sum_{i \in I} \sum_{j \in J} (p_i - q_i * c_j - m_i) * x_{ij}$$

The maximization is subject to the maximum quantity deliverable by supplier 1. The sum of the quantity of iron multiplied by the number of hammers, for every i, must be smaller than or equal to 4800 Kg:

$$\sum_{i \in I} q_i * x_{i\ supplier1} \leq a$$

For the second supplier the maximum extra budget of 800 Euros must be taken into account:

$$c_{supplier2} * \sum_{i \in I} q_i * x_{i\ supplier2} \leq e$$

The following are the additional constraints the company must satisfy:

$$\sum_{j \in J} x_{typeAj} \geq 2 * \sum_{j \in J} x_{typeBj}$$

$$\sum_{j \in J} x_{typeAj} \leq \sum_{j \in J} x_{typeCj}$$

Finally there are the non-negativity and integrality constraints:

$$x_{ij} \geq 0, \forall i \in I, \forall j \in J$$

$$x_{ij}\ integer, \forall i \in I, \forall j \in J$$

In the case in which the company decides that the extra budget not allocated will be considered as revenue, and added to the objective function, the updated mathematical formulation would be the following:

$$max \sum_{i \in I} \sum_{j \in J} (p_i - q_i * c_j - m_i) * x_{ij} + (e - c_{supplier2} * \sum_{i \in I} q_i * x_{i\ supplier2})$$

3.3 Resolution of the model and results

The problem has been implemented by means of the modelling language AMPL, and has been solved with the optimization solver CPLEX. The optimal solution for Model 2 returns a total revenue of 52308€ with 1404 hammers of type A (supplier 1: 1404; supplier 2: 0), 702 hammers of type B (supplier 1: 702; supplier 2: 0) and 1404 hammers of type C (supplier 1: 1347; supplier 2: 57). For the case in which the remaining extra budget is added to the revenue, the number of hammers remains the same, however the total revenue now is 52310€ .

4 Comparison of the two models

The difference between the second model with respect to the first one comes from the additional supplier introduced, which brings to the introduction of the variable x_{ij}, where the set J of suppliers is taken into account, differently from before. This leads to a greater computational complexity of the model.