

Irreducible angular momentum and spin eigenspaces on atomic subshells

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Reference:

Christian B. Mendl. *Efficient algorithm for many-electron angular momentum and spin diagonalization on atomic subshells*. [arXiv:1409.6860](#) (2014)

config	sym	L_z	S_z	Ψ
$\wedge^1 V_s$	2S	0	$\frac{1}{2}$	$ s\rangle$
$\wedge^2 V_s$	1S	0	0	$ s\bar{s}\rangle$
$\wedge^1 V_p$	$^2P^o$	1	$\frac{1}{2}$	$ p_1\rangle$
$\wedge^2 V_p$	1D	2	0	$ p_1\bar{p}_1\rangle$
	3P	1	1	$ p_1p_0\rangle$
	1S	0	0	$\frac{1}{\sqrt{3}}(- p_1\bar{p}_1\rangle + \bar{p}_1p_1\rangle + p_0\bar{p}_0\rangle)$
$\wedge^3 V_p$	$^2D^o$	2	$\frac{1}{2}$	$ p_1\bar{p}_1p_0\rangle$
	$^2P^o$	1	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}(p_1\bar{p}_1p_1\rangle + p_1p_0\bar{p}_0\rangle)$
	$^4S^o$	0	$\frac{3}{2}$	$ p_1p_0p_1\rangle$
$\wedge^4 V_p$	1D	2	0	$ p_1\bar{p}_1p_0\bar{p}_0\rangle$
	3P	1	1	$ p_1\bar{p}_1p_0p_1\rangle$
	1S	0	0	$\frac{1}{\sqrt{3}}(- p_1\bar{p}_1p_1\bar{p}_1\rangle - p_1p_0\bar{p}_0\bar{p}_1\rangle + \bar{p}_1p_0\bar{p}_0p_1\rangle)$
$\wedge^5 V_p$	$^2P^o$	1	$\frac{1}{2}$	$ p_1\bar{p}_1p_0\bar{p}_0p_1\rangle$
$\wedge^6 V_p$	1S	0	0	$ p_1\bar{p}_1p_0\bar{p}_0p_1\bar{p}_1\rangle$
$\wedge^1 V_d$	2D	2	$\frac{1}{2}$	$ d_2\rangle$
$\wedge^2 V_d$	1G	4	0	$ d_2\bar{d}_2\rangle$
	3F	3	1	$ d_2d_1\rangle$
	1D	2	0	$\frac{1}{\sqrt{7}}(-\sqrt{2} \cdot d_2\bar{d}_0\rangle + \sqrt{2} \cdot \bar{d}_2d_0\rangle + \sqrt{3} \cdot d_1\bar{d}_1\rangle)$
	3P	1	1	$\frac{1}{\sqrt{5}}(-\sqrt{2} \cdot d_2d_1\rangle + \sqrt{3} \cdot d_1d_0\rangle)$
	1S	0	0	$\frac{1}{\sqrt{5}}(d_2\bar{d}_2\rangle - \bar{d}_2d_2\rangle - d_1\bar{d}_1\rangle + \bar{d}_1d_1\rangle + d_0\bar{d}_0\rangle)$
$\wedge^3 V_d$	2H	5	$\frac{1}{2}$	$ d_2\bar{d}_2d_1\rangle$
	2G	4	$\frac{1}{2}$	$\frac{1}{\sqrt{5}}(\sqrt{2} \cdot d_2\bar{d}_2d_0\rangle + \sqrt{3} \cdot d_2d_1\bar{d}_1\rangle)$
	4F	3	$\frac{3}{2}$	$ d_2d_1d_0\rangle$
	2F	3	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}(\sqrt{6} \cdot d_2\bar{d}_2d_1\rangle - d_2d_1\bar{d}_0\rangle - d_2\bar{d}_1d_0\rangle + 2 \cdot \bar{d}_2d_1d_0\rangle)$
	2D	2	$\frac{1}{2}$	$\frac{1}{2}(- d_2\bar{d}_2d_2\rangle - d_2d_1\bar{d}_1\rangle + d_2\bar{d}_1d_1\rangle + d_2d_0\bar{d}_0\rangle)$
	2D	2	$\frac{1}{2}$	$\frac{1}{2\sqrt{21}}(5 \cdot d_2\bar{d}_2d_2\rangle - 3 \cdot d_2d_1\bar{d}_1\rangle - d_2\bar{d}_1d_1\rangle + 3 \cdot d_2d_0\bar{d}_0\rangle + 4 \cdot \bar{d}_2d_1d_1\rangle + 2\sqrt{6} \cdot d_1\bar{d}_1d_0\rangle)$

Table 1: Irreducible LS eigenspaces, showing states with maximal L_z, S_z only

config	sym	L_z	S_z	Ψ
	^4P	1	$\frac{3}{2}$	$\frac{1}{\sqrt{5}} (-\sqrt{3} \cdot d_2 d_1 d_{-2}\rangle + \sqrt{2} \cdot d_2 d_0 d_{-1}\rangle)$
	^2P	1	$\frac{1}{2}$	$\frac{1}{\sqrt{210}} (4\sqrt{3} \cdot d_2 d_1 \bar{d}_{-2}\rangle - 2\sqrt{3} \cdot d_2 \bar{d}_1 d_{-2}\rangle - 4\sqrt{2} \cdot d_2 d_0 \bar{d}_{-1}\rangle - \sqrt{2} \cdot d_2 \bar{d}_0 d_{-1}\rangle$ $- 2\sqrt{3} \cdot \bar{d}_2 d_1 d_{-2}\rangle + 5\sqrt{2} \cdot \bar{d}_2 d_0 d_{-1}\rangle + 3\sqrt{3} \cdot d_1 \bar{d}_1 d_{-1}\rangle + 3\sqrt{3} \cdot d_1 d_0 \bar{d}_0\rangle)$
$\wedge^4 V_d$	^1I	6	0	$ d_2 \bar{d}_2 d_1 \bar{d}_1\rangle$
	^3H	5	1	$ d_2 \bar{d}_2 d_1 d_0\rangle$
	^3G	4	1	$\frac{1}{\sqrt{5}} (\sqrt{2} \cdot d_2 \bar{d}_2 d_1 d_{-1}\rangle + \sqrt{3} \cdot d_2 d_1 \bar{d}_1 d_0\rangle)$
	^1G	4	0	$\frac{1}{\sqrt{3}} (- d_2 \bar{d}_2 d_1 \bar{d}_{-1}\rangle + d_2 \bar{d}_2 d_1 d_{-1}\rangle + d_2 \bar{d}_2 d_0 \bar{d}_0\rangle)$
	^1G	4	0	$\frac{1}{\sqrt{66}} (-\sqrt{2} \cdot d_2 \bar{d}_2 d_1 \bar{d}_{-1}\rangle + \sqrt{2} \cdot d_2 \bar{d}_2 d_1 d_{-1}\rangle - 2\sqrt{2} \cdot d_2 \bar{d}_2 d_0 \bar{d}_0\rangle$ $- 3\sqrt{3} \cdot d_2 d_1 \bar{d}_1 d_0\rangle + 3\sqrt{3} \cdot \bar{d}_2 d_1 \bar{d}_1 d_0\rangle)$
	^3F	3	1	$\frac{1}{\sqrt{3}} (d_2 \bar{d}_2 d_1 d_{-2}\rangle + d_2 d_1 \bar{d}_1 d_{-1}\rangle + d_2 d_1 d_0 \bar{d}_0\rangle)$
	^3F	3	1	$\frac{1}{2\sqrt{3}} (-2 \cdot d_2 \bar{d}_2 d_1 d_{-2}\rangle + \sqrt{6} \cdot d_2 \bar{d}_2 d_0 d_{-1}\rangle + d_2 d_1 \bar{d}_1 d_{-1}\rangle$ $+ d_2 d_1 d_0 \bar{d}_0\rangle)$
	^1F	3	0	$\frac{1}{2\sqrt{10}} (-2\sqrt{3} \cdot d_2 \bar{d}_2 d_1 \bar{d}_{-2}\rangle + 2\sqrt{3} \cdot d_2 \bar{d}_2 d_1 d_{-2}\rangle + \sqrt{2} \cdot d_2 \bar{d}_2 d_0 \bar{d}_{-1}\rangle$ $- \sqrt{2} \cdot d_2 \bar{d}_2 d_0 d_{-1}\rangle - \sqrt{3} \cdot d_2 d_1 \bar{d}_1 \bar{d}_{-1}\rangle - \sqrt{3} \cdot d_2 \bar{d}_1 d_0 \bar{d}_0\rangle$ $+ \sqrt{3} \cdot \bar{d}_2 d_1 \bar{d}_1 d_{-1}\rangle + \sqrt{3} \cdot \bar{d}_2 d_1 d_0 \bar{d}_0\rangle)$
	^5D	2	2	$ d_2 d_1 d_0 d_{-1}\rangle$
	^3D	2	1	$\frac{1}{2\sqrt{21}} (4 \cdot d_2 \bar{d}_2 d_0 d_{-2}\rangle + 2\sqrt{6} \cdot d_2 d_1 \bar{d}_1 d_{-2}\rangle - 3 \cdot d_2 d_1 d_0 \bar{d}_{-1}\rangle$ $- 3 \cdot d_2 d_1 \bar{d}_0 d_{-1}\rangle + d_2 \bar{d}_1 d_0 d_{-1}\rangle + 5 \cdot \bar{d}_2 d_1 d_0 d_{-1}\rangle)$
	^1D	2	0	$\frac{1}{\sqrt{42}} (-\sqrt{2} \cdot d_2 \bar{d}_2 d_0 \bar{d}_{-2}\rangle + \sqrt{2} \cdot d_2 \bar{d}_2 d_0 d_{-2}\rangle + 2\sqrt{3} \cdot d_2 \bar{d}_2 d_{-1} \bar{d}_{-1}\rangle$ $- \sqrt{3} \cdot d_2 d_1 \bar{d}_1 d_{-2}\rangle - \sqrt{2} \cdot d_2 \bar{d}_1 d_0 \bar{d}_{-1}\rangle + \sqrt{2} \cdot d_2 \bar{d}_1 d_0 d_{-1}\rangle$ $+ \sqrt{3} \cdot \bar{d}_2 d_1 \bar{d}_1 d_{-2}\rangle + \sqrt{2} \cdot \bar{d}_2 d_1 d_0 \bar{d}_{-1}\rangle - \sqrt{2} \cdot \bar{d}_2 d_1 d_0 d_{-1}\rangle$ $+ 2\sqrt{3} \cdot d_1 \bar{d}_1 d_0 \bar{d}_0\rangle)$
	^1D	2	0	$\frac{1}{\sqrt{21}} (\sqrt{2} \cdot d_2 \bar{d}_2 d_0 \bar{d}_{-2}\rangle - \sqrt{2} \cdot d_2 \bar{d}_2 d_0 d_{-2}\rangle + \sqrt{3} \cdot d_2 d_1 \bar{d}_1 \bar{d}_{-2}\rangle$ $- \sqrt{2} \cdot d_2 d_1 \bar{d}_0 \bar{d}_{-1}\rangle + \sqrt{2} \cdot d_2 \bar{d}_1 d_0 \bar{d}_{-1}\rangle - \sqrt{3} \cdot \bar{d}_2 d_1 \bar{d}_1 d_{-2}\rangle$ $+ \sqrt{2} \cdot \bar{d}_2 d_1 d_0 \bar{d}_{-1}\rangle - \sqrt{2} \cdot \bar{d}_2 d_1 d_0 d_{-1}\rangle + \sqrt{3} \cdot d_1 \bar{d}_1 d_0 \bar{d}_0\rangle)$
	^3P	1	1	$\frac{1}{5} (2\sqrt{2} \cdot d_2 \bar{d}_2 d_1 d_{-2}\rangle + \sqrt{3} \cdot d_2 d_1 d_0 \bar{d}_{-2}\rangle - \sqrt{3} \cdot d_2 d_1 \bar{d}_0 d_{-2}\rangle$ $- \sqrt{2} \cdot d_2 d_1 d_{-1} \bar{d}_{-1}\rangle - \sqrt{3} \cdot d_2 \bar{d}_1 d_0 d_{-2}\rangle + \sqrt{3} \cdot \bar{d}_2 d_1 d_0 d_{-2}\rangle$ $+ \sqrt{3} \cdot d_1 \bar{d}_1 d_0 d_{-1}\rangle)$

Table 2: Irreducible LS_3 eigenspaces (continued)

config	sym	L_z	S_z	Ψ
	3P	1	1	$\frac{1}{5\sqrt{14}} (3\sqrt{2} \cdot d_2\bar{d}_2d_1d_2\rangle - 6\sqrt{3} \cdot d_2d_1d_0\bar{d}_2\rangle + \sqrt{3} \cdot d_2d_1\bar{d}_0d_2\rangle$ $+ 6\sqrt{2} \cdot d_2d_1d_1\bar{d}_1\rangle + \sqrt{3} \cdot d_2\bar{d}_1d_0d_2\rangle + 5\sqrt{2} \cdot d_2d_0\bar{d}_0d_1\rangle$ $+ 4\sqrt{3} \cdot \bar{d}_2d_1d_0d_2\rangle + 4\sqrt{3} \cdot d_1\bar{d}_1d_0d_1\rangle)$
	1S	0	0	$\frac{1}{5\sqrt{3}} (4 \cdot d_2\bar{d}_2d_2\bar{d}_2\rangle - 2 \cdot d_2\bar{d}_1d_1\bar{d}_2\rangle + 2 \cdot d_2\bar{d}_1\bar{d}_1d_2\rangle$ $+ \sqrt{6} \cdot d_2\bar{d}_0d_1\bar{d}_1\rangle + 2 \cdot d_2\bar{d}_1d_1\bar{d}_2\rangle - 2 \cdot d_2\bar{d}_1\bar{d}_1d_2\rangle$ $- \sqrt{6} \cdot \bar{d}_2d_0d_1\bar{d}_1\rangle + \sqrt{6} \cdot d_1\bar{d}_1d_0\bar{d}_2\rangle - \sqrt{6} \cdot d_1\bar{d}_1d_0d_2\rangle$ $+ d_1\bar{d}_1d_1\bar{d}_1\rangle + 3 \cdot d_1d_0\bar{d}_0d_1\rangle - 3 \cdot \bar{d}_1d_0\bar{d}_0d_1\rangle)$
	1S	0	0	$\frac{1}{5\sqrt{7}} (d_2\bar{d}_2d_2\bar{d}_2\rangle + 5 \cdot d_2d_1\bar{d}_1d_2\rangle - 3 \cdot d_2\bar{d}_1d_1\bar{d}_2\rangle$ $- 2 \cdot d_2\bar{d}_1\bar{d}_1d_2\rangle - 5 \cdot d_2d_0\bar{d}_0d_2\rangle - \sqrt{6} \cdot d_2\bar{d}_0d_1\bar{d}_1\rangle$ $- 2 \cdot \bar{d}_2d_1d_1\bar{d}_2\rangle - 3 \cdot \bar{d}_2d_1\bar{d}_1d_2\rangle + 5 \cdot \bar{d}_2\bar{d}_1d_1d_2\rangle$ $+ 5 \cdot \bar{d}_2d_0\bar{d}_0d_2\rangle + \sqrt{6} \cdot \bar{d}_2d_0d_1\bar{d}_1\rangle - \sqrt{6} \cdot d_1\bar{d}_1d_0\bar{d}_2\rangle$ $+ \sqrt{6} \cdot d_1\bar{d}_1d_0d_2\rangle + 4 \cdot d_1\bar{d}_1d_1\bar{d}_1\rangle + 2 \cdot d_1d_0\bar{d}_0d_1\rangle$ $- 2 \cdot \bar{d}_1d_0\bar{d}_0d_1\rangle)$
$\wedge^5 V_d$	2I	6	$\frac{1}{2}$	$ d_2\bar{d}_2d_1\bar{d}_1d_0\rangle$
	2H	5	$\frac{1}{2}$	$\frac{1}{\sqrt{2}} (d_2\bar{d}_2d_1\bar{d}_1d_1\rangle + d_2\bar{d}_2d_1d_0\bar{d}_0\rangle)$
	4G	4	$\frac{3}{2}$	$ d_2\bar{d}_2d_1d_0d_1\rangle$
	2G	4	$\frac{1}{2}$	$\frac{1}{\sqrt{39}} (3\sqrt{3} \cdot d_2\bar{d}_2d_1\bar{d}_1d_2\rangle - \sqrt{2} \cdot d_2\bar{d}_2d_1d_0\bar{d}_1\rangle - \sqrt{2} \cdot d_2\bar{d}_2d_1\bar{d}_0d_1\rangle$ $+ 2\sqrt{2} \cdot d_2\bar{d}_2\bar{d}_1d_0d_1\rangle)$
	2G	4	$\frac{1}{2}$	$\frac{1}{\sqrt{715}} (2\sqrt{3} \cdot d_2\bar{d}_2d_1\bar{d}_1d_2\rangle + 8\sqrt{2} \cdot d_2\bar{d}_2d_1d_0\bar{d}_1\rangle - 5\sqrt{2} \cdot d_2\bar{d}_2d_1\bar{d}_0d_1\rangle$ $- 3\sqrt{2} \cdot d_2\bar{d}_2\bar{d}_1d_0d_1\rangle + 13\sqrt{3} \cdot d_2d_1\bar{d}_1d_0\bar{d}_0\rangle)$
	4F	3	$\frac{3}{2}$	$\frac{1}{\sqrt{2}} (d_2\bar{d}_2d_1d_0d_2\rangle + d_2d_1\bar{d}_1d_0d_1\rangle)$
	2F	3	$\frac{1}{2}$	$\frac{1}{6} (-4 \cdot d_2\bar{d}_2d_1d_0\bar{d}_2\rangle + 2 \cdot d_2\bar{d}_2d_1\bar{d}_0d_2\rangle + \sqrt{6} \cdot d_2\bar{d}_2d_1d_1\bar{d}_1\rangle$ $+ 2 \cdot d_2\bar{d}_2\bar{d}_1d_0d_2\rangle - d_2d_1\bar{d}_1d_0\bar{d}_1\rangle - d_2d_1\bar{d}_1d_0d_1\rangle$ $+ 2 \cdot \bar{d}_2d_1\bar{d}_1d_0d_1\rangle)$
	2F	3	$\frac{1}{2}$	$\frac{1}{6\sqrt{5}} (2 \cdot d_2\bar{d}_2d_1d_0\bar{d}_2\rangle - d_2\bar{d}_2d_1\bar{d}_0d_2\rangle - 2\sqrt{6} \cdot d_2\bar{d}_2d_1d_1\bar{d}_1\rangle$ $- d_2\bar{d}_2\bar{d}_1d_0d_2\rangle - 3\sqrt{6} \cdot d_2\bar{d}_2d_0\bar{d}_0d_1\rangle - 4 \cdot d_2d_1\bar{d}_1d_0\bar{d}_1\rangle$ $- 4 \cdot d_2d_1\bar{d}_1d_0d_1\rangle + 8 \cdot \bar{d}_2d_1\bar{d}_1d_0d_1\rangle)$

Table 3: Irreducible LS_4 eigenspaces (continued)

config	sym	L_z	S_z	Ψ
	4D	2	$\frac{3}{2}$	$\frac{1}{\sqrt{7}} (\sqrt{2} \cdot d_2 \bar{d}_2 d_1 d_1 d_2\rangle + \sqrt{3} \cdot d_2 d_1 \bar{d}_1 d_0 d_2\rangle + \sqrt{2} \cdot d_2 d_1 d_0 \bar{d}_0 d_1\rangle)$
	2D	2	$\frac{1}{2}$	$\frac{1}{2\sqrt{6}} (-\sqrt{2} \cdot d_2 \bar{d}_2 d_1 d_1 \bar{d}_2\rangle - \sqrt{2} \cdot d_2 \bar{d}_2 d_1 \bar{d}_1 d_2\rangle + 2\sqrt{2} \cdot d_2 \bar{d}_2 d_1 d_1 d_2\rangle$ $+ \sqrt{2} \cdot d_2 \bar{d}_2 d_0 \bar{d}_0 d_2\rangle - \sqrt{3} \cdot d_2 d_1 \bar{d}_1 d_0 \bar{d}_2\rangle - \sqrt{2} \cdot d_2 d_1 d_0 \bar{d}_0 d_1\rangle$ $+ \sqrt{3} \cdot \bar{d}_2 d_1 \bar{d}_1 d_0 d_2\rangle + \sqrt{2} \cdot \bar{d}_2 d_1 d_0 \bar{d}_0 d_1\rangle)$
	2D	2	$\frac{1}{2}$	$\frac{1}{2\sqrt{210}} (3\sqrt{2} \cdot d_2 \bar{d}_2 d_1 d_1 \bar{d}_2\rangle - \sqrt{2} \cdot d_2 \bar{d}_2 d_1 \bar{d}_1 d_2\rangle - 2\sqrt{2} \cdot d_2 \bar{d}_2 d_1 d_1 d_2\rangle$ $- 7\sqrt{2} \cdot d_2 \bar{d}_2 d_0 \bar{d}_0 d_2\rangle - 8\sqrt{3} \cdot d_2 \bar{d}_2 d_0 d_1 \bar{d}_1\rangle + 3\sqrt{3} \cdot d_2 d_1 \bar{d}_1 d_0 \bar{d}_2\rangle$ $- 8\sqrt{3} \cdot d_2 d_1 \bar{d}_1 d_0 d_2\rangle - 8\sqrt{2} \cdot d_2 d_1 \bar{d}_1 d_1 \bar{d}_1\rangle - 5\sqrt{2} \cdot d_2 d_1 d_0 \bar{d}_0 d_1\rangle$ $+ 5\sqrt{3} \cdot \bar{d}_2 d_1 \bar{d}_1 d_0 d_2\rangle + 5\sqrt{2} \cdot \bar{d}_2 d_1 d_0 \bar{d}_0 d_1\rangle)$
	2D	2	$\frac{1}{2}$	$\frac{1}{\sqrt{105}} (-2 \cdot d_2 \bar{d}_2 d_1 d_1 \bar{d}_2\rangle + 4 \cdot d_2 \bar{d}_2 d_1 \bar{d}_1 d_2\rangle - 2 \cdot d_2 \bar{d}_2 d_1 d_1 d_2\rangle$ $- 2 \cdot d_2 \bar{d}_2 d_0 \bar{d}_0 d_2\rangle + \sqrt{6} \cdot d_2 \bar{d}_2 d_0 d_1 \bar{d}_1\rangle - \sqrt{6} \cdot d_2 d_1 \bar{d}_1 d_0 \bar{d}_2\rangle$ $+ \sqrt{6} \cdot d_2 d_1 \bar{d}_1 d_0 d_2\rangle - 3 \cdot d_2 d_1 \bar{d}_1 d_1 \bar{d}_1\rangle - 5 \cdot d_2 d_1 d_0 \bar{d}_0 d_1\rangle$ $+ 5 \cdot d_2 \bar{d}_1 d_0 \bar{d}_0 d_1\rangle)$
	4P	1	$\frac{3}{2}$	$\frac{1}{\sqrt{10}} (\sqrt{2} \cdot d_2 \bar{d}_2 d_0 d_1 d_2\rangle + \sqrt{3} \cdot d_2 d_1 \bar{d}_1 d_1 d_2\rangle + \sqrt{3} \cdot d_2 d_1 d_0 \bar{d}_0 d_2\rangle$ $+ \sqrt{2} \cdot d_2 d_1 d_0 d_1 \bar{d}_1\rangle)$
	2P	1	$\frac{1}{2}$	$\frac{1}{2\sqrt{105}} (6\sqrt{3} \cdot d_2 \bar{d}_2 d_1 d_2 \bar{d}_2\rangle - 5\sqrt{2} \cdot d_2 \bar{d}_2 d_0 d_1 \bar{d}_2\rangle + 4\sqrt{2} \cdot d_2 \bar{d}_2 d_0 \bar{d}_1 d_2\rangle$ $+ \sqrt{2} \cdot d_2 \bar{d}_2 d_0 d_1 d_2\rangle + \sqrt{3} \cdot d_2 d_1 \bar{d}_1 d_1 \bar{d}_2\rangle - 2\sqrt{3} \cdot d_2 d_1 \bar{d}_1 d_1 d_2\rangle$ $+ \sqrt{3} \cdot d_2 d_1 d_0 \bar{d}_0 d_2\rangle + \sqrt{2} \cdot d_2 d_1 \bar{d}_0 d_1 \bar{d}_1\rangle - 2\sqrt{3} \cdot d_2 \bar{d}_1 d_0 \bar{d}_0 d_2\rangle$ $+ 4\sqrt{2} \cdot d_2 \bar{d}_1 d_0 d_1 \bar{d}_1\rangle + \sqrt{3} \cdot \bar{d}_2 d_1 \bar{d}_1 d_1 d_2\rangle + \sqrt{3} \cdot \bar{d}_2 d_1 d_0 \bar{d}_0 d_2\rangle$ $- 5\sqrt{2} \cdot \bar{d}_2 d_1 d_0 d_1 \bar{d}_1\rangle + 6\sqrt{3} \cdot d_1 \bar{d}_1 d_0 \bar{d}_0 d_1\rangle)$
	6S	0	$\frac{5}{2}$	$ d_2 d_1 d_0 d_1 d_2\rangle$
	2S	0	$\frac{1}{2}$	$\frac{1}{\sqrt{210}} (4 \cdot d_2 \bar{d}_2 d_0 d_2 \bar{d}_2\rangle + 2\sqrt{6} \cdot d_2 \bar{d}_2 d_1 \bar{d}_1 d_2\rangle + 2\sqrt{6} \cdot d_2 d_1 \bar{d}_1 d_2 \bar{d}_2\rangle$ $- 3 \cdot d_2 d_1 d_0 \bar{d}_1 d_2\rangle - 3 \cdot d_2 d_1 \bar{d}_0 d_1 \bar{d}_2\rangle + 3 \cdot d_2 d_1 \bar{d}_0 d_1 d_2\rangle$ $+ d_2 \bar{d}_1 d_0 d_1 \bar{d}_2\rangle - d_2 \bar{d}_1 d_0 \bar{d}_1 d_2\rangle + 3 \cdot d_2 \bar{d}_1 d_0 d_1 d_2\rangle$ $+ 2\sqrt{6} \cdot d_2 d_0 \bar{d}_0 d_1 \bar{d}_1\rangle + 5 \cdot \bar{d}_2 d_1 d_0 d_1 \bar{d}_2\rangle + \bar{d}_2 d_1 d_0 \bar{d}_1 d_2\rangle$ $- 3 \cdot \bar{d}_2 d_1 \bar{d}_0 d_1 d_2\rangle - 3 \cdot \bar{d}_2 d_1 d_0 d_1 d_2\rangle + 2\sqrt{6} \cdot d_1 \bar{d}_1 d_0 \bar{d}_0 d_2\rangle$ $+ 4 \cdot d_1 \bar{d}_1 d_0 d_1 \bar{d}_1\rangle)$

Table 4: Irreducible LS eigenspaces (continued)

config	sym	L_z	S_z	Ψ
$\wedge^6 V_d$	1I	6	0	$ d_2 \bar{d}_2 d_1 \bar{d}_1 d_0 \bar{d}_0\rangle$
	3H	5	1	$ d_2 \bar{d}_2 d_1 \bar{d}_1 d_0 d_1\rangle$
	3G	4	1	$\frac{1}{\sqrt{5}} (\sqrt{3} \cdot d_2 \bar{d}_2 d_1 \bar{d}_1 d_0 d_2\rangle + \sqrt{2} \cdot d_2 \bar{d}_2 d_1 d_0 \bar{d}_0 d_1\rangle)$
	1G	4	0	$\frac{1}{2\sqrt{2}} (-\sqrt{3} \cdot d_2 \bar{d}_2 d_1 \bar{d}_1 d_0 \bar{d}_2\rangle + \sqrt{3} \cdot d_2 \bar{d}_2 d_1 \bar{d}_1 d_0 d_2\rangle$ $+ \sqrt{2} \cdot d_2 \bar{d}_2 d_1 \bar{d}_1 d_1 \bar{d}_1\rangle)$
	1G	4	0	$\frac{1}{2\sqrt{22}} (-\sqrt{3} \cdot d_2 \bar{d}_2 d_1 \bar{d}_1 d_0 \bar{d}_2\rangle + \sqrt{3} \cdot d_2 \bar{d}_2 d_1 \bar{d}_1 d_0 d_2\rangle$ $- 3\sqrt{2} \cdot d_2 \bar{d}_2 d_1 \bar{d}_1 d_1 \bar{d}_1\rangle - 4\sqrt{2} \cdot d_2 \bar{d}_2 d_1 d_0 \bar{d}_0 d_1\rangle$ $+ 4\sqrt{2} \cdot d_2 \bar{d}_2 d_1 d_0 \bar{d}_0 d_1\rangle)$
	3F	3	1	$\frac{1}{2\sqrt{2}} (\sqrt{3} \cdot d_2 \bar{d}_2 d_1 \bar{d}_1 d_1 d_2\rangle + \sqrt{3} \cdot d_2 \bar{d}_2 d_1 d_0 \bar{d}_0 d_2\rangle$ $+ \sqrt{2} \cdot d_2 \bar{d}_2 d_1 d_0 d_1 \bar{d}_1\rangle)$
	3F	3	1	$\frac{1}{2\sqrt{6}} (d_2 \bar{d}_2 d_1 \bar{d}_1 d_1 d_2\rangle + d_2 \bar{d}_2 d_1 d_0 \bar{d}_0 d_2\rangle$ $- \sqrt{6} \cdot d_2 \bar{d}_2 d_1 d_0 d_1 \bar{d}_1\rangle + 4 \cdot d_2 d_1 \bar{d}_1 d_0 \bar{d}_0 d_1\rangle)$
	1F	3	0	$\frac{1}{2\sqrt{10}} (-\sqrt{3} \cdot d_2 \bar{d}_2 d_1 \bar{d}_1 d_1 \bar{d}_2\rangle + \sqrt{3} \cdot d_2 \bar{d}_2 d_1 \bar{d}_1 d_1 d_2\rangle$ $- \sqrt{3} \cdot d_2 \bar{d}_2 d_1 d_0 \bar{d}_0 d_2\rangle + \sqrt{2} \cdot d_2 \bar{d}_2 d_1 \bar{d}_0 d_1 \bar{d}_1\rangle$ $+ \sqrt{3} \cdot d_2 \bar{d}_2 d_1 d_0 \bar{d}_0 d_2\rangle - \sqrt{2} \cdot d_2 \bar{d}_2 d_1 d_0 d_1 \bar{d}_1\rangle$ $- 2\sqrt{3} \cdot d_2 d_1 \bar{d}_1 d_0 \bar{d}_0 d_1\rangle + 2\sqrt{3} \cdot d_2 d_1 \bar{d}_1 d_0 \bar{d}_0 d_1\rangle)$
	5D	2	2	$ d_2 \bar{d}_2 d_1 d_0 d_1 d_2\rangle$
	3D	2	1	$\frac{1}{2\sqrt{21}} (5 \cdot d_2 \bar{d}_2 d_1 d_0 d_1 \bar{d}_2\rangle + d_2 \bar{d}_2 d_1 d_0 \bar{d}_1 d_2\rangle$ $- 3 \cdot d_2 \bar{d}_2 d_1 \bar{d}_0 d_1 d_2\rangle - 3 \cdot d_2 \bar{d}_2 d_1 d_0 d_1 \bar{d}_2\rangle$ $+ 2\sqrt{6} \cdot d_2 d_1 \bar{d}_1 d_0 \bar{d}_0 d_2\rangle + 4 \cdot d_2 d_1 \bar{d}_1 d_0 d_1 \bar{d}_1\rangle)$
	1D	2	0	$\frac{1}{\sqrt{21}} (\sqrt{3} \cdot d_2 \bar{d}_2 d_1 \bar{d}_1 d_2 \bar{d}_2\rangle - \sqrt{2} \cdot d_2 \bar{d}_2 d_1 d_0 \bar{d}_1 d_2\rangle$ $+ \sqrt{2} \cdot d_2 \bar{d}_2 d_1 \bar{d}_0 d_1 d_2\rangle + \sqrt{2} \cdot d_2 \bar{d}_2 d_1 d_0 d_1 \bar{d}_2\rangle$ $- \sqrt{2} \cdot d_2 \bar{d}_2 d_1 \bar{d}_0 d_1 d_2\rangle - \sqrt{3} \cdot d_2 d_1 \bar{d}_1 d_0 \bar{d}_0 d_2\rangle$ $- \sqrt{2} \cdot d_2 d_1 \bar{d}_1 d_0 d_1 \bar{d}_1\rangle + \sqrt{3} \cdot d_2 d_1 \bar{d}_1 d_0 \bar{d}_0 d_2\rangle$ $+ \sqrt{2} \cdot d_2 d_1 \bar{d}_1 d_0 d_1 \bar{d}_1\rangle)$
	1D	2	0	$\frac{1}{\sqrt{42}} (-2\sqrt{3} \cdot d_2 \bar{d}_2 d_1 \bar{d}_1 d_2 \bar{d}_2\rangle + \sqrt{2} \cdot d_2 \bar{d}_2 d_1 \bar{d}_0 d_1 \bar{d}_2\rangle$ $- \sqrt{2} \cdot d_2 \bar{d}_2 d_1 d_0 \bar{d}_1 d_2\rangle - \sqrt{2} \cdot d_2 \bar{d}_2 d_1 d_0 d_1 \bar{d}_2\rangle$ $+ \sqrt{2} \cdot d_2 \bar{d}_2 d_1 d_0 \bar{d}_1 d_2\rangle - 2\sqrt{3} \cdot d_2 \bar{d}_2 d_0 \bar{d}_0 d_1 \bar{d}_1\rangle$ $- \sqrt{3} \cdot d_2 d_1 \bar{d}_1 d_0 \bar{d}_0 d_2\rangle - \sqrt{2} \cdot d_2 d_1 \bar{d}_1 d_0 d_1 \bar{d}_1\rangle$ $+ \sqrt{3} \cdot d_2 d_1 \bar{d}_1 d_0 \bar{d}_0 d_2\rangle + \sqrt{2} \cdot d_2 d_1 \bar{d}_1 d_0 d_1 \bar{d}_1\rangle)$

Table 5: Irreducible LS eigenspaces (continued)

config	sym	L_z	S_z	Ψ
	^3P	1	1	$\frac{1}{5\sqrt{2}} (2\sqrt{3} \cdot d_2\bar{d}_2d_1d_0d_{-2}\bar{d}_{-2}\rangle + \sqrt{2} \cdot d_2\bar{d}_2d_1d_{-1}\bar{d}_{-1}d_{-2}\rangle$ $+ 2\sqrt{3} \cdot d_2d_1\bar{d}_1d_0d_{-1}\bar{d}_{-2}\rangle - \sqrt{3} \cdot d_2d_1\bar{d}_1d_0\bar{d}_{-1}d_{-2}\rangle$ $- \sqrt{3} \cdot d_2d_1\bar{d}_1d_0d_{-1}d_{-2}\rangle + 3\sqrt{2} \cdot d_2d_1d_0\bar{d}_0d_{-1}\bar{d}_{-1}\rangle)$
	^3P	1	1	$\frac{1}{5\sqrt{7}} (-\sqrt{3} \cdot d_2\bar{d}_2d_1d_0d_{-2}\bar{d}_{-2}\rangle - 3\sqrt{2} \cdot d_2\bar{d}_2d_1d_{-1}\bar{d}_{-1}d_{-2}\rangle$ $- 5\sqrt{2} \cdot d_2\bar{d}_2d_0\bar{d}_0d_{-1}d_{-2}\rangle - \sqrt{3} \cdot d_2d_1\bar{d}_1d_0d_{-1}\bar{d}_{-2}\rangle$ $- 2\sqrt{3} \cdot d_2d_1\bar{d}_1d_0\bar{d}_{-1}d_{-2}\rangle - 2\sqrt{3} \cdot d_2d_1\bar{d}_1\bar{d}_0d_{-1}d_{-2}\rangle$ $+ \sqrt{2} \cdot d_2d_1d_0\bar{d}_0d_{-1}\bar{d}_{-1}\rangle + 5\sqrt{3} \cdot \bar{d}_2d_1\bar{d}_1d_0d_{-1}d_{-2}\rangle)$
	^1S	0	0	$\frac{1}{\sqrt{55}} (2 \cdot d_2\bar{d}_2d_1\bar{d}_{-1}d_{-2}\bar{d}_{-2}\rangle - 2 \cdot d_2\bar{d}_2\bar{d}_{-1}d_{-1}d_{-2}\bar{d}_{-2}\rangle$ $+ \sqrt{6} \cdot d_2\bar{d}_2d_0d_{-1}\bar{d}_{-1}d_{-2}\rangle - \sqrt{6} \cdot d_2\bar{d}_2d_0d_{-1}\bar{d}_{-1}d_{-2}\rangle$ $+ \sqrt{6} \cdot d_2d_1\bar{d}_1d_0d_{-2}\bar{d}_{-2}\rangle + d_2d_1\bar{d}_1d_{-1}\bar{d}_{-1}d_{-2}\rangle$ $+ d_2d_1d_0\bar{d}_0\bar{d}_{-1}d_{-2}\rangle + d_2\bar{d}_1d_0\bar{d}_0d_{-1}\bar{d}_{-2}\rangle$ $- 2 \cdot d_2\bar{d}_1d_0\bar{d}_0\bar{d}_{-1}d_{-2}\rangle - \sqrt{6} \cdot \bar{d}_2d_1\bar{d}_1d_0d_{-2}\bar{d}_{-2}\rangle$ $- \bar{d}_2d_1\bar{d}_1d_{-1}\bar{d}_{-1}d_{-2}\rangle - 2 \cdot \bar{d}_2d_1d_0\bar{d}_0d_{-1}\bar{d}_{-2}\rangle$ $+ \bar{d}_2d_1d_0\bar{d}_0\bar{d}_{-1}d_{-2}\rangle + \bar{d}_2\bar{d}_1d_0\bar{d}_0d_{-1}d_{-2}\rangle$ $- 3 \cdot d_1\bar{d}_1d_0\bar{d}_0d_{-1}\bar{d}_{-1}\rangle)$
	^1S	0	0	$\frac{1}{\sqrt{1155}} (9 \cdot d_2\bar{d}_2d_1\bar{d}_{-1}d_{-2}\bar{d}_{-2}\rangle - 9 \cdot d_2\bar{d}_2\bar{d}_{-1}d_{-1}d_{-2}\bar{d}_{-2}\rangle$ $- 11 \cdot d_2\bar{d}_2d_0\bar{d}_0d_{-2}\bar{d}_{-2}\rangle - \sqrt{6} \cdot d_2\bar{d}_2d_0d_{-1}\bar{d}_{-1}d_{-2}\rangle$ $+ \sqrt{6} \cdot d_2\bar{d}_2d_0d_{-1}\bar{d}_{-1}d_{-2}\rangle - \sqrt{6} \cdot d_2d_1\bar{d}_1d_0d_{-2}\bar{d}_{-2}\rangle$ $- 12 \cdot d_2d_1\bar{d}_1d_{-1}\bar{d}_{-1}d_{-2}\rangle - 12 \cdot d_2d_1d_0\bar{d}_0\bar{d}_{-1}d_{-2}\rangle$ $+ 10 \cdot d_2\bar{d}_1d_0\bar{d}_0d_{-1}\bar{d}_{-2}\rangle + 2 \cdot d_2\bar{d}_1d_0\bar{d}_0d_{-1}d_{-2}\rangle$ $+ \sqrt{6} \cdot \bar{d}_2d_1\bar{d}_1d_0d_{-2}\bar{d}_{-2}\rangle + 12 \cdot \bar{d}_2d_1\bar{d}_1d_{-1}\bar{d}_{-1}d_{-2}\rangle$ $+ 2 \cdot \bar{d}_2d_1d_0\bar{d}_0d_{-1}\bar{d}_{-2}\rangle + 10 \cdot \bar{d}_2d_1d_0\bar{d}_0d_{-1}d_{-2}\rangle$ $- 12 \cdot \bar{d}_2\bar{d}_1d_0\bar{d}_0d_{-1}d_{-2}\rangle - 8 \cdot d_1\bar{d}_1d_0\bar{d}_0d_{-1}\bar{d}_{-1}\rangle)$
$\wedge^7 V_d$	^2H	5	$\frac{1}{2}$	$ d_2\bar{d}_2d_1\bar{d}_1d_0\bar{d}_0d_{-1}\rangle$
	^2G	4	$\frac{1}{2}$	$\frac{1}{\sqrt{5}} (\sqrt{3} \cdot d_2\bar{d}_2d_1\bar{d}_1d_0\bar{d}_0d_{-2}\rangle + \sqrt{2} \cdot d_2\bar{d}_2d_1\bar{d}_1d_0d_{-1}\bar{d}_{-1}\rangle)$
	^4F	3	$\frac{3}{2}$	$ d_2\bar{d}_2d_1\bar{d}_1d_0d_{-1}d_{-2}\rangle$

Table 6: Irreducible LS_7 eigenspaces (continued)

config	sym	L_z	S_z	Ψ
	2F	3	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}} (2 \cdot d_2\bar{d}_2d_1\bar{d}_1d_0\bar{d}_1\bar{d}_2\rangle - d_2\bar{d}_2d_1\bar{d}_1d_0\bar{d}_1d_2\rangle - d_2\bar{d}_2d_1\bar{d}_1d_0\bar{d}_1d_2\rangle + \sqrt{6} \cdot d_2\bar{d}_2d_1d_0\bar{d}_0\bar{d}_1\bar{d}_1\rangle)$
	2D	2	$\frac{1}{2}$	$\frac{1}{\sqrt{15}} (\sqrt{3} \cdot d_2\bar{d}_2d_1\bar{d}_1d_0d_2\bar{d}_2\rangle + \sqrt{2} \cdot d_2\bar{d}_2d_1d_0\bar{d}_0\bar{d}_1\bar{d}_2\rangle - \sqrt{2} \cdot d_2\bar{d}_2d_1d_0\bar{d}_0\bar{d}_1d_2\rangle + 2\sqrt{2} \cdot d_2d_1\bar{d}_1d_0\bar{d}_0\bar{d}_1\bar{d}_1\rangle)$
	2D	2	$\frac{1}{2}$	$\frac{1}{\sqrt{70}} (-\sqrt{6} \cdot d_2\bar{d}_2d_1\bar{d}_1d_0d_2\bar{d}_2\rangle - 5 \cdot d_2\bar{d}_2d_1\bar{d}_1d_1\bar{d}_1\bar{d}_2\rangle - 2 \cdot d_2\bar{d}_2d_1d_0\bar{d}_0\bar{d}_1\bar{d}_2\rangle - 3 \cdot d_2\bar{d}_2d_1d_0\bar{d}_0\bar{d}_1d_2\rangle + 5 \cdot d_2\bar{d}_2\bar{d}_1d_0\bar{d}_0\bar{d}_1d_2\rangle + d_2d_1\bar{d}_1d_0\bar{d}_0\bar{d}_1\bar{d}_1\rangle)$
	4P	1	$\frac{3}{2}$	$\frac{1}{\sqrt{5}} (-\sqrt{2} \cdot d_2\bar{d}_2d_1d_0\bar{d}_1\bar{d}_1\bar{d}_2\rangle + \sqrt{3} \cdot d_2d_1\bar{d}_1d_0\bar{d}_0\bar{d}_1d_2\rangle)$
	2P	1	$\frac{1}{2}$	$\frac{1}{\sqrt{210}} (3\sqrt{3} \cdot d_2\bar{d}_2d_1\bar{d}_1d_1d_2\bar{d}_2\rangle + 3\sqrt{3} \cdot d_2\bar{d}_2d_1d_0\bar{d}_0d_2\bar{d}_2\rangle + 5\sqrt{2} \cdot d_2\bar{d}_2d_1d_0\bar{d}_1\bar{d}_1\bar{d}_2\rangle - \sqrt{2} \cdot d_2\bar{d}_2d_1\bar{d}_0\bar{d}_1\bar{d}_1\bar{d}_2\rangle - 4\sqrt{2} \cdot d_2\bar{d}_2\bar{d}_1d_0\bar{d}_1\bar{d}_1\bar{d}_2\rangle - 2\sqrt{3} \cdot d_2d_1\bar{d}_1d_0\bar{d}_0\bar{d}_1\bar{d}_2\rangle - 2\sqrt{3} \cdot d_2d_1\bar{d}_1d_0\bar{d}_0\bar{d}_1d_2\rangle + 4\sqrt{3} \cdot \bar{d}_2d_1\bar{d}_1d_0\bar{d}_0\bar{d}_1d_2\rangle)$
$\wedge^8 V_d$	1G	4	0	$ d_2\bar{d}_2d_1\bar{d}_1d_0\bar{d}_0\bar{d}_1\bar{d}_1\rangle$
	3F	3	1	$ d_2\bar{d}_2d_1\bar{d}_1d_0\bar{d}_0\bar{d}_1d_2\rangle$
	1D	2	0	$\frac{1}{\sqrt{7}} (-\sqrt{3} \cdot d_2\bar{d}_2d_1\bar{d}_1d_0\bar{d}_0d_2\bar{d}_2\rangle - \sqrt{2} \cdot d_2\bar{d}_2d_1\bar{d}_1d_0\bar{d}_1\bar{d}_1\bar{d}_2\rangle + \sqrt{2} \cdot d_2\bar{d}_2d_1\bar{d}_1d_0\bar{d}_1\bar{d}_1d_2\rangle)$
	3P	1	1	$\frac{1}{\sqrt{5}} (-\sqrt{3} \cdot d_2\bar{d}_2d_1\bar{d}_1d_0\bar{d}_1d_2\bar{d}_2\rangle + \sqrt{2} \cdot d_2\bar{d}_2d_1d_0\bar{d}_0\bar{d}_1\bar{d}_1\bar{d}_2\rangle)$
	1S	0	0	$\frac{1}{\sqrt{5}} (d_2\bar{d}_2d_1\bar{d}_1d_1\bar{d}_1d_2\bar{d}_2\rangle + d_2\bar{d}_2d_1d_0\bar{d}_0\bar{d}_1d_2\bar{d}_2\rangle - d_2\bar{d}_2d_1d_0\bar{d}_0\bar{d}_1d_2\bar{d}_2\rangle - d_2d_1\bar{d}_1d_0\bar{d}_0\bar{d}_1\bar{d}_1\bar{d}_2\rangle + \bar{d}_2d_1\bar{d}_1d_0\bar{d}_0\bar{d}_1\bar{d}_1d_2\rangle)$
$\wedge^9 V_d$	2D	2	$\frac{1}{2}$	$ d_2\bar{d}_2d_1\bar{d}_1d_0\bar{d}_0\bar{d}_1\bar{d}_1d_2\rangle$
$\wedge^{10} V_d$	1S	0	0	$ d_2\bar{d}_2d_1\bar{d}_1d_0\bar{d}_0\bar{d}_1\bar{d}_1d_2\bar{d}_2\rangle$
$\wedge^1 V_f$	$^2F^\circ$	3	$\frac{1}{2}$	$ f_3\rangle$
$\wedge^2 V_f$	1I	6	0	$ f_3\bar{f}_3\rangle$
	3H	5	1	$ f_3f_2\rangle$
	1G	4	0	$\frac{1}{\sqrt{11}} (-\sqrt{3} \cdot f_3\bar{f}_1\rangle + \sqrt{3} \cdot \bar{f}_3f_1\rangle + \sqrt{5} \cdot f_2\bar{f}_2\rangle)$

Table 7: Irreducible LS eigenspaces (continued)

config	sym	L_z	S_z	Ψ
	3F	3	1	$\frac{1}{\sqrt{3}} (- f_3 f_0\rangle + \sqrt{2} \cdot f_2 f_1\rangle)$
	1D	2	0	$\frac{1}{\sqrt{42}} (\sqrt{5} \cdot f_3 \bar{f}_1\rangle - \sqrt{5} \cdot \bar{f}_3 f_1\rangle - \sqrt{10} \cdot f_2 \bar{f}_0\rangle + \sqrt{10} \cdot \bar{f}_2 f_0\rangle + 2\sqrt{3} \cdot f_1 \bar{f}_1\rangle)$
	3P	1	1	$\frac{1}{\sqrt{14}} (\sqrt{3} \cdot f_3 f_2\rangle - \sqrt{5} \cdot f_2 f_1\rangle + \sqrt{6} \cdot f_1 f_0\rangle)$
	1S	0	0	$\frac{1}{\sqrt{7}} (- f_3 \bar{f}_3\rangle + \bar{f}_3 f_3\rangle + f_2 \bar{f}_2\rangle - \bar{f}_2 f_2\rangle - f_1 \bar{f}_1\rangle + \bar{f}_1 f_1\rangle + f_0 \bar{f}_0\rangle)$
$\wedge^3 V_f$	$^2K^\circ$	8	$\frac{1}{2}$	$ f_3 \bar{f}_3 f_2\rangle$
	$^2J^\circ$	7	$\frac{1}{2}$	$\frac{1}{2\sqrt{2}} (\sqrt{3} \cdot f_3 \bar{f}_3 f_1\rangle + \sqrt{5} \cdot f_3 f_2 \bar{f}_2\rangle)$
	$^4I^\circ$	6	$\frac{3}{2}$	$ f_3 f_2 f_1\rangle$
	$^2I^\circ$	6	$\frac{1}{2}$	$\frac{1}{\sqrt{21}} (3 \cdot f_3 \bar{f}_3 f_0\rangle - \sqrt{2} \cdot f_3 f_2 \bar{f}_1\rangle - \sqrt{2} \cdot f_3 \bar{f}_2 f_1\rangle + 2\sqrt{2} \cdot \bar{f}_3 f_2 f_1\rangle)$
	$^2H^\circ$	5	$\frac{1}{2}$	$\frac{1}{\sqrt{6}} (\sqrt{2} \cdot f_3 \bar{f}_3 f_1\rangle - f_3 \bar{f}_2 f_0\rangle + \bar{f}_3 f_2 f_0\rangle + \sqrt{2} \cdot f_2 \bar{f}_2 f_1\rangle)$
	$^2H^\circ$	5	$\frac{1}{2}$	$\frac{1}{\sqrt{273}} (-\sqrt{5} \cdot f_3 \bar{f}_3 f_1\rangle - 3\sqrt{10} \cdot f_3 f_2 \bar{f}_0\rangle + 2\sqrt{10} \cdot f_3 \bar{f}_2 f_0\rangle + 6\sqrt{3} \cdot f_3 f_1 \bar{f}_1\rangle + \sqrt{10} \cdot \bar{f}_3 f_2 f_0\rangle + 2\sqrt{5} \cdot f_2 \bar{f}_2 f_1\rangle)$
	$^4G^\circ$	4	$\frac{3}{2}$	$\frac{1}{\sqrt{11}} (-\sqrt{5} \cdot f_3 f_2 f_1\rangle + \sqrt{6} \cdot f_3 f_1 f_0\rangle)$
	$^2G^\circ$	4	$\frac{1}{2}$	$\frac{1}{7\sqrt{5}} (5\sqrt{3} \cdot f_3 \bar{f}_3 f_2\rangle + \sqrt{5} \cdot f_3 f_2 \bar{f}_1\rangle - 3\sqrt{5} \cdot f_3 \bar{f}_2 f_1\rangle - \sqrt{6} \cdot f_3 f_1 f_0\rangle + \sqrt{6} \cdot f_3 \bar{f}_1 f_0\rangle + 2\sqrt{5} \cdot \bar{f}_3 f_2 f_1\rangle + 2\sqrt{10} \cdot f_2 \bar{f}_2 f_0\rangle + 4\sqrt{3} \cdot f_2 f_1 \bar{f}_1\rangle)$
	$^2G^\circ$	4	$\frac{1}{2}$	$\frac{1}{7\sqrt{429}} (-18\sqrt{6} \cdot f_3 \bar{f}_3 f_2\rangle + 16\sqrt{10} \cdot f_3 f_2 \bar{f}_1\rangle + \sqrt{10} \cdot f_3 \bar{f}_2 f_1\rangle - 32\sqrt{3} \cdot f_3 f_1 \bar{f}_0\rangle - 17\sqrt{3} \cdot f_3 \bar{f}_1 f_0\rangle - 17\sqrt{10} \cdot \bar{f}_3 f_2 f_1\rangle + 49\sqrt{3} \cdot \bar{f}_3 f_1 f_0\rangle + 15\sqrt{5} \cdot f_2 \bar{f}_2 f_0\rangle + 15\sqrt{6} \cdot f_2 f_1 \bar{f}_1\rangle)$
	$^4F^\circ$	3	$\frac{3}{2}$	$\frac{1}{2} (f_3 f_2 f_2\rangle - f_3 f_1 f_1\rangle + \sqrt{2} \cdot f_2 f_1 f_0\rangle)$
	$^2F^\circ$	3	$\frac{1}{2}$	$\frac{1}{\sqrt{6}} (f_3 \bar{f}_3 f_3\rangle + f_3 f_2 \bar{f}_2\rangle - f_3 \bar{f}_2 f_2\rangle - f_3 f_1 \bar{f}_1\rangle + f_3 \bar{f}_1 f_1\rangle + f_3 f_0 \bar{f}_0\rangle)$
	$^2F^\circ$	3	$\frac{1}{2}$	$\frac{1}{2\sqrt{33}} (7 \cdot f_3 \bar{f}_3 f_3\rangle - 3 \cdot f_3 f_2 \bar{f}_2\rangle - 2 \cdot f_3 \bar{f}_2 f_2\rangle + 3 \cdot f_3 f_1 \bar{f}_1\rangle - f_3 \bar{f}_1 f_1\rangle - 2 \cdot f_3 f_0 \bar{f}_0\rangle + 5 \cdot \bar{f}_3 f_2 f_2\rangle - 2 \cdot \bar{f}_3 f_1 f_1\rangle + \sqrt{15} \cdot f_2 \bar{f}_2 f_1\rangle - \sqrt{2} \cdot f_2 f_1 f_0\rangle - \sqrt{2} \cdot f_2 \bar{f}_1 f_0\rangle + 2\sqrt{2} \cdot \bar{f}_2 f_1 f_0\rangle)$

Table 8: Irreducible LS eigenspaces (continued)

config	sym	L_z	S_z	Ψ
	$4D^\circ$	2	$\frac{3}{2}$	$\frac{1}{\sqrt{21}} (\sqrt{10} \cdot f_3 f_2 f_3\rangle - \sqrt{6} \cdot f_3 f_1 f_2\rangle + \sqrt{5} \cdot f_3 f_0 f_1\rangle)$
	$2D^\circ$	2	$\frac{1}{2}$	$\frac{1}{2\sqrt{42}} (2\sqrt{5} \cdot f_3 f_2 \bar{f}_3\rangle - \sqrt{5} \cdot f_3 \bar{f}_2 f_3\rangle - 2\sqrt{3} \cdot f_3 f_1 \bar{f}_2\rangle$ $-\sqrt{3} \cdot f_3 \bar{f}_1 f_2\rangle + \sqrt{10} \cdot f_3 f_0 \bar{f}_1\rangle + \sqrt{10} \cdot f_3 \bar{f}_0 f_1\rangle$ $-\sqrt{5} \cdot \bar{f}_3 f_2 f_3\rangle + 3\sqrt{3} \cdot \bar{f}_3 f_1 f_2\rangle - 2\sqrt{10} \cdot \bar{f}_3 f_0 f_1\rangle$ $+2\sqrt{5} \cdot f_2 \bar{f}_2 f_2\rangle - \sqrt{5} \cdot f_2 \bar{f}_1 f_1\rangle + \sqrt{5} \cdot \bar{f}_2 f_1 f_1\rangle$ $+ \sqrt{6} \cdot f_1 \bar{f}_1 f_0\rangle)$
	$2D^\circ$	2	$\frac{1}{2}$	$\frac{1}{6\sqrt{154}} (-14\sqrt{5} \cdot f_3 f_2 \bar{f}_3\rangle + 7\sqrt{5} \cdot f_3 \bar{f}_2 f_3\rangle + 14\sqrt{3} \cdot f_3 f_1 \bar{f}_2\rangle$ $-13\sqrt{3} \cdot f_3 \bar{f}_1 f_2\rangle - \sqrt{10} \cdot f_3 f_0 \bar{f}_1\rangle + 5\sqrt{10} \cdot f_3 \bar{f}_0 f_1\rangle$ $+7\sqrt{5} \cdot \bar{f}_3 f_2 f_3\rangle - \sqrt{3} \cdot \bar{f}_3 f_1 f_2\rangle - 4\sqrt{10} \cdot \bar{f}_3 f_0 f_1\rangle$ $+6\sqrt{5} \cdot f_2 \bar{f}_2 f_2\rangle - 12\sqrt{5} \cdot f_2 \bar{f}_1 f_1\rangle + 3\sqrt{5} \cdot f_2 f_1 \bar{f}_1\rangle$ $+12\sqrt{5} \cdot f_2 f_0 \bar{f}_0\rangle + 9\sqrt{5} \cdot \bar{f}_2 f_1 f_1\rangle + 9\sqrt{6} \cdot f_1 \bar{f}_1 f_0\rangle)$
	$2P^\circ$	1	$\frac{1}{2}$	$\frac{1}{2\sqrt{21}} (\sqrt{6} \cdot f_3 \bar{f}_1 f_3\rangle + \sqrt{3} \cdot f_3 f_0 \bar{f}_2\rangle - 2\sqrt{3} \cdot f_3 \bar{f}_0 f_2\rangle$ $-\sqrt{10} \cdot f_3 f_1 \bar{f}_1\rangle - \sqrt{6} \cdot \bar{f}_3 f_1 f_3\rangle + \sqrt{3} \cdot \bar{f}_3 f_0 f_2\rangle$ $-\sqrt{10} \cdot f_2 \bar{f}_2 f_3\rangle - \sqrt{6} \cdot f_2 f_1 \bar{f}_2\rangle + \sqrt{6} \cdot f_2 \bar{f}_1 f_2\rangle$ $+ \sqrt{5} \cdot f_2 f_0 \bar{f}_1\rangle - \sqrt{5} \cdot \bar{f}_2 f_0 f_1\rangle - \sqrt{6} \cdot f_1 \bar{f}_1 f_1\rangle$ $- \sqrt{6} \cdot f_1 f_0 \bar{f}_0\rangle)$
	$4S^\circ$	0	$\frac{3}{2}$	$\frac{1}{\sqrt{7}} (- f_3 f_0 f_3\rangle + \sqrt{2} \cdot f_3 f_1 f_2\rangle + \sqrt{2} \cdot f_2 f_1 f_3\rangle$ $- f_2 f_0 f_2\rangle + f_1 f_0 f_1\rangle)$

Table 9: Irreducible LS eigenspaces (continued)