Multiple Units

- Each type of good has multiple instances $\mathbf{Q} = (q_1, q_2, .., q_m)$
- Represent the bundle by a vector: $\mathbf{S}_i = (s_i^1, s_i^2, .., s_i^m)$
- define an ordering $\prec$ to compare bids
- For example: $(\mathbf{S}_j, b_j) \prec (\mathbf{S}_i, b_i)$ iff

$$\frac{b_j}{\sum_{k=1}^{m} s_j^k} > \frac{b_i}{\sum_{k=1}^{m} s_i^k}$$

- Examine bids in order and grant a bid if its request can be met

Critical Value

x_i = 1 iff ($\mathbf{S}_i$, b_i) is a winning bid

- $\boldsymbol{B} = \{(\mathbf{S}_i, b_i)\}_{i=1,..n}$, $\mathbf{Q} = (q_1, q_2, .., q_m)$
- $\mathbf{X}_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$ is the auction result without ($\mathbf{S}_i$, b_i)
- Critical value for b_i is a value c_i such that when $b_i > c_i$

$$\sum_{(\mathbf{S}_j, b_j) \prec (\mathbf{S}_i, b_i)} (x_j \cdot s_j^k) \le q_k - s_i^k \text{ for all } s_i^k > 0$$

and when $b_i < c_i$

$$\sum_{(\mathbf{S}_j, b_j) \prec (\mathbf{S}_i, b_i)} (x_j \cdot s_j^k) > q_k - s_i^k \text{ for some } s_i^k > 0$$

[ZG13]: Multi Units Extension to [LOS02]

- Five types of goods {A, B, C, D, E}. The system has (3, 2, 2, 2, 2)

bidder	P1	P2	P3	P4	P5
bid (b_i)	\$63	\$54	\$93	\$70	\$28
Bundle (S_i)	(1,0,2,1,0)	(2,1,1,0,0)	(0,1,0,1,2)	(0,0,0,2,1)	(1,0,1,0,0)
$b_i / \Sigma s_i$	15.75	13.5	23.25	23.33	9.33
v_c	93.33	0	93.33	69.75	0