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CS325
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Homework Assignment 6

NOTE: ALL CODE IS INCLUDED AT THE BOTTOM OF THIS FILE

1. <u>Shortest Paths using LP</u>: *(7 points)* Shortest paths can be cast as an LP using distances dv from the source s to a particular vertex v as variables.

- **We can compute the shortest path from s to t in a weighted directed graph by solving.**

  max dt
  subject to
  ds = 0
  dv – du ≤ w(u,v) for all (u,v)$\in$E

- **We can compute the single-source by changing the objective function to**

  $\max \sum_{v \in V} dv$

Use linear programming to answer the questions below. Submit a copy of the LP code and output.

a) **Find the distance of the shortest path from G to C in the graph below.**

To compute the length of the shortest path from G to C in a weighted directed graph we can use:

Maximize d_c
Subject To $d_g = 0$
$d_v - d_u \leq l_{u \to v}$ for every edge $u \to v$

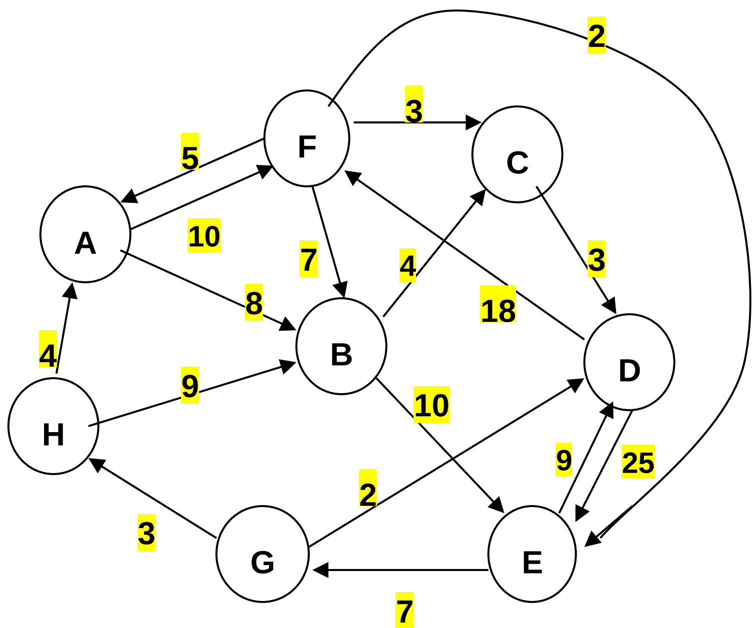
I used LINDO to carry out the calculation (see attached code and output) and found that:

Shortest Path from G to C = 16

b) **Find the distances of the shortest paths from G to all other vertices.**

Using the same method as in a, the shortest paths to all remaining points are:

G → A = 7
G → B = 12
G → C = 16
G → D = 2
G → E = 19
G → F = 17
G → H = 3



2. **<u>Product Mix</u>: (*7 points)* Acme Industries produces four types of men's ties using three types of material. Your job is to determine how many of each type of tie to make each month. The goal is to maximize profit, profit per tie = selling price - labor cost – material cost. Labor cost is $0.75 per tie for all four types of ties. The material requirements and costs are given below.**

Material	Cost per yard	Yards available per month
Silk	$20	1,000
Polyester	$6	2,000
Cotton	$9	1,250

Product Information	Type of Tie			
	Silk = s	Poly = p	Blend1 = b	Blend2 = c
Selling Price per tie	$6.70	$3.55	$4.31	$4.81
Monthly Minimum units	6,000	10,000	13,000	6,000
Monthly Maximum units	7,000	14,000	16,000	8,500

Material Information in yards	Type of Tie			
	Silk	Polyester	Blend 1 (50/50)	Blend 2 (30/70)
Silk	0.125	0	0	0
Polyester	0	0.08	0.05	0.03
Cotton	0	0	0.05	0.07

type	selling price	labor	material	profit per tie
silk s	6.7	0.75	2.5	3.45
poly p	3.55	0.75	0.48	2.32
blend1 b	4.31	0.75	0.75	2.81
blend2 c	4.81	0.75	0.81	3.25

Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. What are the optimal numbers of ties of each type to maximize profit? Include a copy of the code and output.

I used LINDO to solve the following (see attached code and output):

Maximize	$3.45s + 2.32p + 2.81b + 3.25c$	
Subject To	$0.125s \le 1000$	(1000 yards of Silk available each month)
	$0.08s + 0.05b + 0.03c \le 2000$	(2000 yards of Polyester available each month)
	$0.05b + 0.07c \le 1250$	(1250 yards of Cotton available each month)
	$s \le 7000$	(Silk has a monthly max of 7000 units)
	$s \ge 6000$	(Silk has a monthly min of 6000 units)
	$p \le 14000$	(Polyester has a monthly max of 14000 units)
	$p \ge 10000$	(Polyester has a monthly min of 10000 units)
	$b \le 16000$	(Cotton blend 1 has a monthly max of 16000 units)
	$b \ge 13000$	(Cotton blend 1 has a monthly min of 13000 units)
	$c \le 8500$	(Cotton Blend 2 has a monthly max of 8500 units)
	$c \ge 6000$	(Cotton Blend 2 has a monthly min of 6000 units)

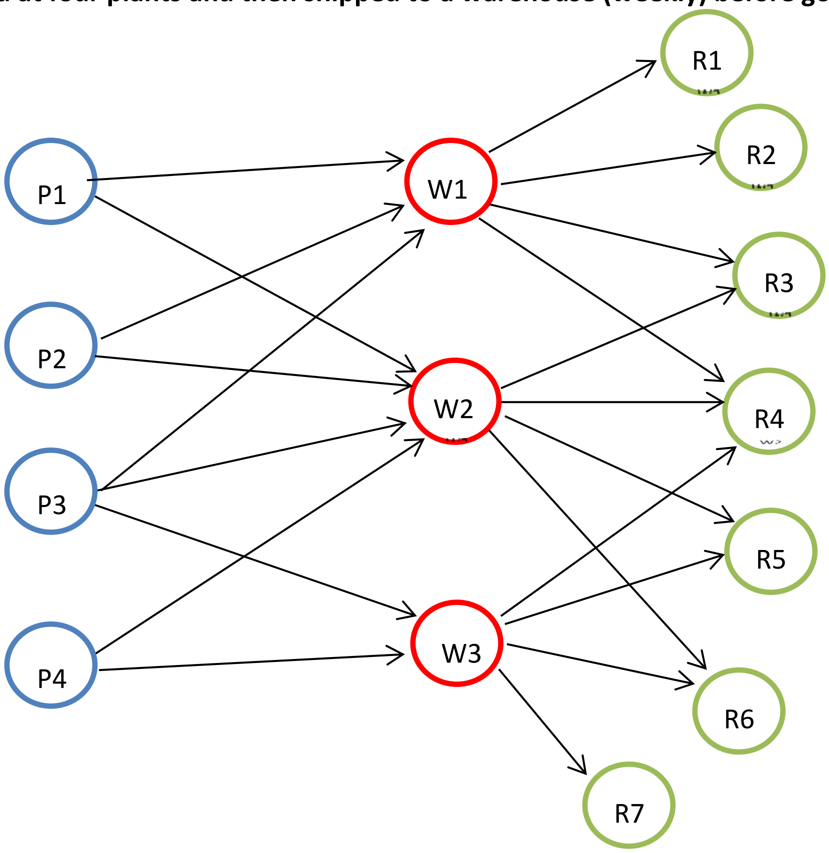
Solution (Optimal # of Ties to Maximize Profit):

Type	# to Make	Value
Silk	7000	$24,150
Polyester	13625	$31,610
Blend 1	13100	$36,811
Blend 2	8500	$27,625
	TOTAL:	**$120,196**

3. Transshipment Model *(7 points)*

This is an extension of the transportation model. There are now intermediate transshipment points added between the sources (plants) and destinations (retailers). Items being shipped from a Plant (p_i) must be shipped to a Warehouse (w_j) before being shipped to the Retailer (r_k). Each Plant will have an associated supply (s_i) and each Retailer will have a demand (d_k). The number of plants is n, number of warehouses is q and the number of retailers is m. The edges (i,j) from plant (p_i)to warehouse (w_j) have costs associated denoted cp(i,j). The edges (j,k) from a warehouse (w_j)to a retailer (r_k) have costs associated denoted cw(j,k).

The graph below shows the transshipment map for a manufacturer of refrigerators. Refrigerators are produced at four plants and then shipped to a warehouse (weekly) before going to the retailer.



Below are the costs of shipping from a plant to a warehouse and then a warehouse to a retailer. If it is impossible to ship between the two locations an X is placed in the table.

cost	W1	W2	W3
P1	$10	$15	X
P2	$11	$8	X
P3	$13	$8	$9
P4	X	$14	$8

cost	R1	R2	R3	R4	R5	R6	R7
W1	$5	$6	$7	$10	X	X	X

W2	X	X	$12	$8	$10	$14	X
W3	X	X	X	$14	$12	$12	$6

The tables below give the capacity of each plant (supply) and the demand for each retailer (per week).

	P1	P2	P3	P4
Supply	150	450	250	150

	R1	R2	R3	R4	R5	R6	R7
Demand	100	150	100	200	200	150	100

Your goal is to determine the number of refrigerators to be shipped plants to warehouses and then warehouses to retailers to minimize the cost. Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. What are the optimal shipping routes and minimum cost.
Include a copy of the code and output.

I used LINDO to solve the following (see attached code and output):

Minimize 10P1W1 + 15P1W2 + 11P2W1 + 8P2W2 + 13P3W1 + 8P3W2 + 9P3W3 + 14P4W2 + 8P4W3 + 5W1R1 + 6W1R2 + 7W1R3 + 10W1R4 + 12W2R3 + 8W2R4 + 10W2R5 + 14W2R6 + 14W3R4 + 12W3R5 + 12W3R6 + 6W3R7

Subject To

P1W1 + P2W1 + P3W1 - W1R1 - W1R2 - W1R3 - W1R4 ≥ 0
P1W2 + P2W2 + P3W2 + P4W2 - W2R3 - W2R4 - W2R5 - W2R6 ≥ 0
P3W3 + P4W3 - W3R4 - W3R5 - W3R6 - W3R7 ≥ 0
P1W1 + P1W2 ≤ 150
P2W1 + P2W2 ≤ 450
P3W1 + P3W2 + P3W3 ≤ 250
P4W2 + P4W3 ≤ 150
W1R1 ≥ 100
W1R2 ≥ 150
W1R3 + W2R3 ≥ 100
W1R4 + W2R4 + W3R4 ≥ 200
W2R5 + W3R5 ≥ 200
W2R6 + W3R6 ≥ 150
W3R7 ≥ 100

The first 3 constraints assure that the refrigerators shipped from each plant are ≥ 0. The next three constraints assure that the refrigerators shipped from each plant are less than or equal to the available supply. The final seven constraints assure that the demands are met.

Optimal Shipping Routes:

150 from P1 → W1
200 from P2 → W1
250 from P2 → W2
150 from P3 → W2
100 from P3 → W3
150 from P4 → W3
100 from W1 → R1
150 from W1 → R2
100 from W1 → R3
200 from W2 → R4
200 from W2 → R5
150 from W3 → R6
100 from W3 → R7

Total Minimum Cost: $17,100.00

4: <u>A Mixture Problem</u> *(9 points)*
Veronica the owner of Very Veggie Vegeria is creating a new healthy salad that is low in calories but meets certain nutritional requirements. A salad is any combination of the following ingredients: Tomato, Lettuce, Spinach, Carrot, Smoked Tofu, Sunflower Seeds, Chickpeas, Oil

Each salad must contain:

- **At least 15 grams of protein**
- **At least 2 and at most 8 grams of fat**
- **At least 4 grams of carbohydrates**
- **At most 200 milligrams of sodium**
- **At least 40% leafy greens by mass.**

The nutritional contents of these ingredients (per 100 grams) and cost are

Ingredient	Energy (Cal)	Protein (grams)	Fat (grams)	Carbohydrate (grams)	Sodium (mg)	Cost (100g)
Tomato	21	0.85	0.33	4.64	9.00	$1.00
Lettuce	16	1.62	0.20	2.37	28.00	$0.75
Spinach	40	2.86	0.39	3.63	65.00	$0.50
Carrot	41	0.93	0.24	9.58	69.00	$0.50
Sunflower Seeds	585	23.4	48.7	15.00	3.80	$0.45
Smoked Tofu	120	16.00	5.00	3.00	120.00	$2.15
Chickpeas	164	9.00	2.6	27.0	78.00	$0.95
Oil	884	0	100.00	0	0	$2.00

Part A: **Determine the combination of ingredients that minimizes calories but meets all nutritional requirements. Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. What is the cost of the low-calorie salad?**

I used LINDO to solve the following (see attached code and output):

Minimize	$21T + 16L + 40S + 41C + 585SS + 120ST + 164CP + 884O$	
Subject To	$0.85T + 1.62L + 2.86S + 0.93C + 23.4SS + 16ST + 9CP \geq 15$	(at least 15g of protein)
	$0.33T + 0.20L + 0.39S + 0.24C + 48.7SS + 5ST + 2.6CP + 100O \geq 2$	(at least 2g of fat)
	$0.33T + 0.20L + 0.39S + 0.24C + 48.7SS + 5ST + 2.6CP + 100O \leq 8$	(at most 8g of fat)
	$4.64T + 2.37L + 3.63S + 9.58C + 15SS + 3ST + 27CP >= 4$	(at least 4g of carbs)
	$9T + 28L + 65S + 69C + 3.8SS + 120ST + 78CP \leq 200$	(at most 200mg of sodium)
	$0.6L + 0.6S - 0.4T - 0.4C - 0.4SS - 0.4ST -0.4CP - 0.40O >= 0$	(40% leafy greens by mass)

Optimal Solution:

58.584g of Lettuce
87.822g of Smoked Tofu

Total Calories: 114.7541
Total Cost: $2.33

Part B: Veronica realizes that it is also important to minimize the cost associated with the new salad. Unfortunately, some of the ingredients can be expensive. Determine the combination of ingredients that minimizes cost. Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. How many calories are in the low-cost salad?

I used LINDO to solve the following (see attached code and output):

Minimize	$1T + 0.75L + 0.5S + 0.5C + 0.45SS + 2.15ST + 0.95CP + 2O$	
Subject To	$0.85T + 1.62L + 2.86S + 0.93C + 23.4SS + 16ST + 9CP \geq 15$	(at least 15g of protein)
	$0.33T + 0.20L + 0.39S + 0.24C + 48.7SS + 5ST + 2.6CP + 100O \geq 2$	(at least 2g of fat)
	$0.33T + 0.20L + 0.39S + 0.24C + 48.7SS + 5ST + 2.6CP + 100O \leq 8$	(at most 8g of fat)
	$4.64T + 2.37L + 3.63S + 9.58C + 15SS + 3ST + 27CP >= 4$	(at least 4g of carbs)
	$9T + 28L + 65S + 69C + 3.8SS + 120ST + 78CP \leq 200$	(at most 200mg of sodium)
	$0.6L + 0.6S - 0.4T - 0.4C - 0.4SS - 0.4ST -0.4CP - 0.40O >= 0$	(40% leafy greens by mass)

Optimal Solution:

83.2298g of Spinach
9.6083g of Sunflower Seeds
115.2364g of Chick Peas

Total Cost: $1.55
Total Calories: 278.488171

LINDO Code and Output for each problem:

Problem 1 Part A Code

```
max dc
st
dg = 0
dg - de <= 7
dd - dg <= 2
dh - dg <= 3
da - dh <= 4
db - dh <= 9
de - db <= 10
db - da <= 8
dd - de <= 9
de - dd <= 25
dd - dc <= 3
df - dd <= 18
dc - db <= 4
dc - df <= 3
db - df <= 7
df - da <= 10
da - df <= 5
de - df <= 2
end
```

Problem 1 Part A Output

```
LP OPTIMUM FOUND AT STEP      0

      OBJECTIVE FUNCTION VALUE

      1)      16.00000

 VARIABLE        VALUE          REDUCED COST
      DC       16.000000         0.000000
      DG        0.000000         0.000000
      DE        0.000000         0.000000
      DD        0.000000         0.000000
      DH        3.000000         0.000000
      DA        4.000000         0.000000
      DB       12.000000         0.000000
      DF       13.000000         0.000000


     ROW   SLACK OR SURPLUS     DUAL PRICES
```

2)	0.000000	1.000000
3)	7.000000	0.000000
4)	2.000000	0.000000
5)	0.000000	1.000000
6)	3.000000	0.000000
7)	0.000000	1.000000
8)	22.000000	0.000000
9)	0.000000	0.000000
10)	9.000000	0.000000
11)	25.000000	0.000000
12)	19.000000	0.000000
13)	5.000000	0.000000
14)	0.000000	1.000000
15)	0.000000	0.000000
16)	8.000000	0.000000
17)	1.000000	0.000000
18)	14.000000	0.000000
19)	15.000000	0.000000

NO. ITERATIONS= 0

Problem 1 Part B Code

```
max da + db + dc + dd + de + df + dh
st
dg = 0
dg - de <= 7
dd - dg <= 2
dh - dg <= 3
da - dh <= 4
db - dh <= 9
de - db <= 10
db - da <= 8
dd - de <= 9
de - dd <= 25
dd - dc <= 3
df - dd <= 18
dc - db <= 4
dc - df <= 3
db - df <= 7
df - da <= 10
da - df <= 5
de - df <= 2
end
```

Problem 1 Part B Output

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) 76.00000

VARIABLE	VALUE	REDUCED COST
DA	7.000000	0.000000
DB	12.000000	0.000000
DC	16.000000	0.000000
DD	2.000000	0.000000
DE	19.000000	0.000000
DF	17.000000	0.000000
DH	3.000000	0.000000
DG	0.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	7.000000
3)	26.000000	0.000000
4)	0.000000	1.000000
5)	0.000000	6.000000
6)	0.000000	3.000000
7)	0.000000	2.000000
8)	3.000000	0.000000
9)	3.000000	0.000000
10)	26.000000	0.000000
11)	8.000000	0.000000
12)	17.000000	0.000000
13)	3.000000	0.000000
14)	0.000000	1.000000
15)	4.000000	0.000000
16)	12.000000	0.000000
17)	0.000000	2.000000
18)	15.000000	0.000000
19)	0.000000	1.000000

NO. ITERATIONS= 2

Problem 2 Code

```
max 3.45s + 2.32p + 2.81b + 3.25c
st
0.125s <= 1000
0.08p + 0.05b + 0.03c <= 2000
0.05b + 0.07c <= 1250
s <= 7000
s >= 6000
p <= 14000
p >= 10000
b <= 16000
b >= 13000
c <= 8500
c >= 6000
end
```

Problem 2 Output

```
LP OPTIMUM FOUND AT STEP      2

      OBJECTIVE FUNCTION VALUE

      1)      120196.0

 VARIABLE        VALUE          REDUCED COST
        S      7000.000000         0.000000
        P     13625.000000         0.000000
        B     13100.000000         0.000000
        C      8500.000000         0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)       125.000000         0.000000
       3)         0.000000        29.000000
       4)         0.000000        27.200001
       5)         0.000000         3.450000
       6)      1000.000000         0.000000
       7)       375.000000         0.000000
       8)      3625.000000         0.000000
       9)      2900.000000         0.000000
      10)       100.000000         0.000000
      11)         0.000000         0.476000
      12)      2500.000000         0.000000

 NO. ITERATIONS=       2
```

Problem 3 Code

```
min 10P1W1 + 15P1W2 + 11P2W1 + 8P2W2 + 13P3W1 + 8P3W2 + 9P3W3 + 14P4W2 + 8P4W3 + 5W1R1
+ 6W1R2 + 7W1R3 + 10W1R4 + 12W2R3 + 8W2R4 + 10W2R5 + 14W2R6 + 14W3R4 + 12W3R5 + 12W3R6
+ 6W3R7
st
P1W1 + P2W1 + P3W1 - W1R1 - W1R2 - W1R3 - W1R4 >= 0
P1W2 + P2W2 + P3W2 + P4W2 - W2R3 - W2R4 - W2R5 - W2R6 >= 0
P3W3 + P4W3 - W3R4 - W3R5 - W3R6 - W3R7 >= 0
P1W1 + P1W2 <= 150
P2W1 + P2W2 <= 450
P3W1 + P3W2 + P3W3 <= 250
P4W2 + P4W3 <= 150
W1R1 >= 100
W1R2 >= 150
W1R3 + W2R3 >= 100
W1R4 + W2R4 + W3R4 >= 200
W2R5 + W3R5 >= 200
W2R6 + W3R6 >= 150
W3R7 >= 100
end
```

Problem 3 Output

```
LP OPTIMUM FOUND AT STEP     13

      OBJECTIVE FUNCTION VALUE

      1)      17100.00

 VARIABLE        VALUE          REDUCED COST
     P1W1       150.000000         0.000000
     P1W2         0.000000         8.000000
     P2W1       200.000000         0.000000
     P2W2       250.000000         0.000000
     P3W1         0.000000         2.000000
     P3W2       150.000000         0.000000
     P3W3       100.000000         0.000000
     P4W2         0.000000         7.000000
     P4W3       150.000000         0.000000
     W1R1       100.000000         0.000000
     W1R2       150.000000         0.000000
     W1R3       100.000000         0.000000
     W1R4         0.000000         5.000000
     W2R3         0.000000         2.000000
     W2R4       200.000000         0.000000
     W2R5       200.000000         0.000000
     W2R6         0.000000         1.000000
```

W3R4	0.000000	7.000000
W3R5	0.000000	3.000000
W3R6	150.000000	0.000000
W3R7	100.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	-11.000000
3)	0.000000	-8.000000
4)	0.000000	-9.000000
5)	0.000000	1.000000
6)	0.000000	0.000000
7)	0.000000	0.000000
8)	0.000000	1.000000
9)	0.000000	-16.000000
10)	0.000000	-17.000000
11)	0.000000	-18.000000
12)	0.000000	-16.000000
13)	0.000000	-18.000000
14)	0.000000	-21.000000
15)	0.000000	-15.000000

NO. ITERATIONS= 13

Problem 4 Part A Code

```
min 21T + 16L + 40S + 41C + 585SS + 120ST + 164CP + 884O
st
0.85T + 1.62L + 2.86S + 0.93C + 23.4SS + 16ST + 9CP >= 15
0.33T + 0.20L + 0.39S + 0.24C + 48.7SS + 5ST + 2.6CP + 100O >= 2
0.33T + 0.20L + 0.39S + 0.24C + 48.7SS + 5ST + 2.6CP + 100O <= 8
4.64T + 2.37L + 3.63S + 9.58C + 15SS + 3ST + 27CP >= 4
9T + 28L + 65S + 69C + 3.8SS + 120ST + 78CP <= 200
0.6L + 0.6S - 0.4T - 0.4C - 0.4SS - 0.4ST -0.4CP - 0.40O >= 0
End
```

Problem 4 Part A Output

```
LP OPTIMUM FOUND AT STEP      5

      OBJECTIVE FUNCTION VALUE

      1)      114.7541

  VARIABLE        VALUE          REDUCED COST
         T        0.000000        16.901640
         L        0.585480         0.000000
         S        0.000000        14.513662
         C        0.000000        36.289616
        SS        0.000000       408.387970
        ST        0.878220         0.000000
        CP        0.000000        97.551910
         O        0.000000       886.404358


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)        0.000000        -7.650273
        3)        2.508197         0.000000
        4)        3.491803         0.000000
        5)        0.022248         0.000000
        6)       78.220139         0.000000
        7)        0.000000        -6.010929

 NO. ITERATIONS=       5
```

Problem 4 Part B Code

```
min 1T + 0.75L + 0.5S + 0.5C + 0.45SS + 2.15ST + 0.95CP + 2O
st
0.85T + 1.62L + 2.86S + 0.93C + 23.4SS + 16ST + 9CP >= 15
0.33T + 0.20L + 0.39S + 0.24C + 48.7SS + 5ST + 2.6CP + 100O >= 2
0.33T + 0.20L + 0.39S + 0.24C + 48.7SS + 5ST + 2.6CP + 100O <= 8
4.64T + 2.37L + 3.63S + 9.58C + 15SS + 3ST + 27CP >= 4
9T + 28L + 65S + 69C + 3.8SS + 120ST + 78CP <= 200
0.6L + 0.6S - 0.4T - 0.4C - 0.4SS - 0.4ST -0.4CP - 0.40O >= 0
End
```

Problem 4 Part B Output

LP OPTIMUM FOUND AT STEP 3

OBJECTIVE FUNCTION VALUE

1) 1.554133

VARIABLE	VALUE	REDUCED COST
T	0.000000	1.002081
L	0.000000	0.402912
S	0.832298	0.000000
C	0.000000	0.486914
SS	0.096083	0.000000
ST	0.000000	0.405609
CP	1.152364	0.000000
O	0.000000	7.281258

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	-0.131261
3)	6.000000	0.000000
4)	0.000000	0.051847
5)	31.576324	0.000000
6)	55.651089	0.000000
7)	0.000000	-0.241358

NO. ITERATIONS= 3