Homework 2 Solutions

Abhinay Duppelly

1 Warmup: Boolean Functions

1.1

Three functions would be $y_1 = x_4$, $y_2 = x_2 \wedge x_3 \wedge x_4$ and $y_3 = x_4 \wedge \neg x_1$

1.2

y_1 would make 0 mistakes
y_2 would make 2 mistakes
y_3 would also make 2 mistakes

1.3

Yes, the data is linearly separable.
The linear threshold function would be : $\text{sgn}(x_4 - 0.5)$.

2 Mistake-bound learning

2.1

1. The number of functions in C_n is equal to the number of inputs in the input space, which is 2^n.

2. The halving algorithm will make at most one mistake. At any given iteration in the halving algorithm the algorithm will predict 0 as the label for f_z because the majority of the functions in C_n will predict 0.
   If the halving algorithm makes a mistake , all the functions that disagree are removed leaving only one true function
   It has to be noted that this is a classic counter example for the mistake bound of the halving algorithm

3. Yes. Halving is a mistake bound algorithm for this class.

2.2

It is given that instead of one perfect expert, we have M perfect experts in our pool. Suppose it makes n mistakes.Finally,we will have the final set of concepts

C_n with M elements. C_n will be created when a majority of the functions in C_{n-1} are incorrect.

$$
\begin{aligned}
M = |C_n| &< \frac{1}{2} \cdot |C_{n-1}| \\
&< \frac{1}{2} \cdot \frac{1}{2} |C_{n-2}| \\
&\quad . \\
&\quad . \\
&< \frac{1}{2^n} \cdot |C_0| \\
&< \frac{1}{2^n} \cdot |C|
\end{aligned}
$$

So

$$\frac{|C|}{M} = \frac{N}{M} < 2^n$$

Thus halving will make at-most $log_2 \frac{N}{M}$ number of mistakes. Or in other words the mistake bound is $\mathcal{O}(log_2 \frac{N}{M})$