Computational Fluid Dynamics (ME EN 6720/CH EN 6355)
Homework 2, Spring 2014
Due Thursday Feb. 13th

Problem 2.1

Consider the Crank-Nicolson scheme for the 1-D heat equation ($\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$ for $\alpha > 0$):

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \alpha \frac{1}{2} \left(\frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{(\Delta x)^2} + \frac{u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}}{(\Delta x)^2} \right)$$

Part A) Show that the truncation error (T.E.) is $O[(\Delta t)^2, (\Delta x)^2]$. Is the scheme consistent?

Part B) Study the stability of the Crank-Nicholson scheme. Why is this scheme more popular than the simple implicit scheme?

Problem 2.2

Two infinite parallel plates are seperated by a distance $h = 4$ cm. The fluid between the plates has a kinematic viscosity of 0.000217 m^2/s[1] and density of 800 Kg/m^3. The upper plate is stationary and the lower plate is suddenly set in motion with a constant velocity of 40 m/s. A constant streamwise pressure gradient $dp/dx = 2000$ N/m^2/m is imposed within the domain at the instant motion starts.

Part A) Show that the governing equation for this problem is reduced from the Navier-Stokes equation and is given by

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho} \frac{\partial p}{\partial x}$$

Part B) Using a forward in time and centered in space explicit finite difference scheme, compute the velocity within the domain. Use a spatial grid size of $\Delta y = 0.05$ cm and a time step of your choice. Make sure you justify your time step choice. Plot the solution at time levels of 0.0, 0.2, 0.4, 0.6, 0.8 and 1.0 seconds.

Part C) Use the Crank-Nicolson implicit scheme to compute the velocity profiles and print them at the times specified in part B) for

- $\Delta t = 0.0005$ s
- $\Delta t = 0.00333$ s

Compute and compare the CPU time for each method.

Problem 2.3

A long, rectangular bar has dimensions of L_x by L_y, as shown in the figure. The bar is initially heated to a temperature of T_o. Subsequently, its surfaces are subjected to the constant temperature of $T1$, $T2$, $T3$ and $T4$ as depicted in the figure. The problem is governed by the 2D unsteady heat equation given by

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

The bar is composed of copper with constant thermal conductivity of 380 W/(m^{o}C) and a constant thermal diffusivity of 1.1234×10^{-4} m^2/s. The bar has dimensions of $L_x = 0.3$ m and $L_y = 0.4$ m.
Use the ADI (alternating direction implicit) scheme with timesteps of 0.1 s and 0.5 s and a computational grid with $N_x = 61$ and $N_y = 81$ grid points to compute the transient solution. The initial condition and boundary conditions are given by: $T_o = 0.0$ ^{o}C, $T_1 = 40.0$ ^{o}C, $T_2 = 0.0$ ^{o}C, $T_3 = 10.0$ ^{o}C and $T_4 = 0.0$ ^{o}C.
Print contour plots (of isotemperature lines) corresponding to $t = 5.0$ s, $t = 10.0$ s, $t = 20.0$ s, $t = 40.0$ s and steady state. Assume the simulation has reached steady-state when the total variation (TV) in the temperature from one time step to the next is less than 0.0001^{o}C where

$$TV = \frac{1}{N_x N_y} \sum_{j=2}^{N_y} \sum_{i=2}^{N_x} |T_{i,j}^{n+1} - T_{i,j}^{n}|$$

Discuss your results.