Computational Fluid Dynamics (ME EN 6720)
Homework 3, Spring 2014
Due Tuesday Mar. 18^{th}

Problem 3.1

Consider the flow in a lid driven cavity illustrated in the figure below. The cavity is square and flow in the cavity is driven by an upper plate moving at a constant velocity of U. The Reynolds number for the flow is $Re = Uw/\nu = 100$. Solve for the steady state flow in the cavity using the vorticity-streamfunction method outlined in Lecture 12 (also see Ferziger pg. 181 and Tannehill et al, pg. 650). Construct a second order finite difference solution for the problem (for both ω and Ψ) and solve it using the following schemes:

1. Jacobi iterations
2. Gauss-Seidel
3. Gauss-Seidel with Successive Over-Relaxations (SOR)
4. Gauss-Seidel with a 1-level nested Multigrid method

For **extra credit** use a solver package (Matlab® or one of the packages listed on the class website)

1. Preconditioned Conjugate gradient method (PCG)
2. Generalized Minimal Residual Method (GMRES)

Use a uniform mesh to solve your problem with $N_x = N_y = 50$ points in the horizontal x and vertical y directions respectively. Start the iterations with $u(x, y) = v(x, y) = 0$ (you can also explore using $u(x, y) = U$ and $v(x, y) = 0$). For a fixed convergence criteria (on the fine grid for multigrid methods), which scheme converges fastest? How do all the schemes compare to each other (measured by performance)? What is the optimum value of the relaxation coefficient for the SOR scheme? Present your results by plotting the streamlines for each scheme overlaid on a filled contour (or pseudo color) plot of the vorticity in the cavity. Also include a table comparing the number of iterations and total execution times for each method along with a short discussion. If you do the extra credit, with the PCG and GMRES methods, report at least the minimum total time.