Use of the 2D Navier-Stokes Equations to Explore Aerodynamic Properties of Trains

ME EN 6720 Semester Projects

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Motivation

- Want to determine the optimal design of trains to mitigate drag

Incompressible Flow Equations

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{1}{\rho}\frac{\partial P}{\partial x} = \nu\nabla^2 u \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{1}{\rho}\frac{\partial P}{\partial y} = \nu\nabla^2 v \tag{2}$$

with the continuity equation being

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

Algorithm

Algorithm 1 Solving for the Primitive Variables

while $\left|\left|P^{(n+1)} - P^{(n)}\right|\right|_2 > tolerance$ **do**

Impose Boundary Conditions

$\vec{u}^{(t)} = \vec{u}^{(n)} + \Delta t \left(-\vec{A}^{(n)} + \vec{D}^{(n)}\right)$

$\nabla^2 P^{(n+1)} = \frac{1}{\Delta t} \vec{\nabla} \cdot \vec{u}^{(t)}$

$\vec{u}^{(n+1)} = \vec{u}^{(t)} - \Delta t \nabla P^{(n+1)}$

$t = t + \Delta t$

end while

Boundary Conditions (Domain)

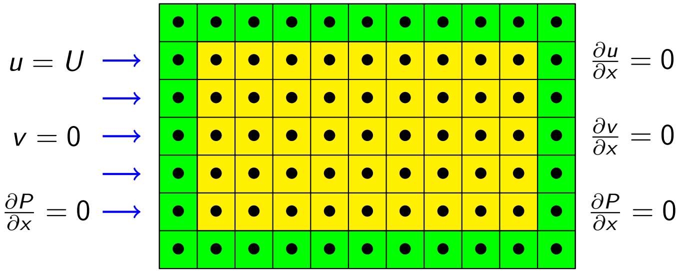

Figure : Grid used in the computation. Green nodes denote the "ghost nodes," while the yellow nodes cover the normal domain.

Boundary Conditions (Train)

We also needed to impose boundary conditions on the train to satisfy the *no-slip condition*. The no-slip condition requires that viscous fluids will have no velocity relative to a solid boundary. Therefore, to impose these conditions, we set both u and v to be 0 on the interior of the train cars, and the following on their boundaries:

- Left and Right Faces: $\frac{\partial u}{\partial x} = 0$ and $v = 0$
- Top and Bottom Faces: $\frac{\partial v}{\partial y} = 0$ and $u = 0$

Grid Type

The obvious choice of a grid is a collocated grid which stores $\{P, u, v\}$ at each point in the grid. However, this causes problems with pressure–velocity coupling and the occurrence of oscillations in the pressure. Instead, the use of the staggered grid was chosen to overcome these irregularities. A diagram for a single node of this staggered grid can be seen in Figure 2.

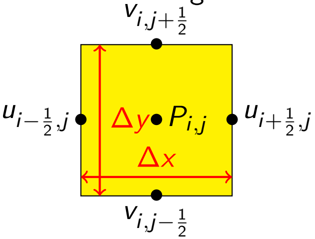

Figure : Staggered Grid: Single Cell

Verification of Simulation (cont.)

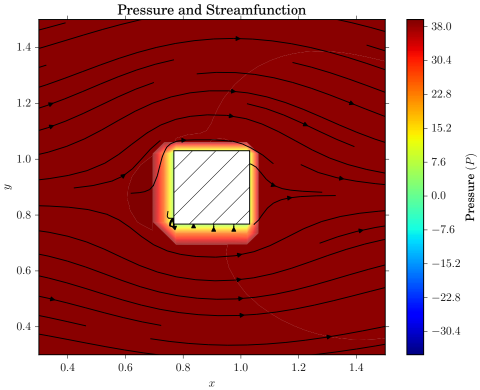

Figure : Typical plot for the pressure around a single block with $U = 10$

Verification of Simulation

For simple problems, we can check to make sure that the code performs correctly. In doing so, we can derive an analytical solution for a simplified problem and test it for this case. We can do so using the *Reynold's Transport Theorem* which is stated as

$$\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{\partial}{\partial t}\int_{C_v} \rho b \mathrm{d}V + \int_{C_s} \rho b \vec{u} \cdot \mathrm{d}\vec{s} \tag{4}$$

where B is any property of the local system. For the pressure, this equation turns into

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \sum \vec{F} = \cancel{\frac{\partial}{\partial t}\int_{C_v} \rho \vec{u} \mathrm{d}V} + \int_{C_s} \rho \vec{u} \left(\vec{u} \cdot \mathrm{d}\vec{A}\right) \tag{5}$$

Verification of Simulation (cont.)

with the first term going to zero due to conservation of flux. The forces that the train car are going to experience are the pressure (F_p) and the drag force (F_d). F_p is defined as

$$F_p = -\int p \mathrm{d}\vec{A} \tag{6}$$

$$F_L = \oint p \cdot \hat{n} \, \mathrm{d}A \tag{7}$$

Verification of Simulation (cont.)

where we can rearrange Equation (5) to solve for F_d, giving

$$F_d = \int p \, \mathrm{d}\vec{A} - \oint p \cdot \hat{n} \, \mathrm{d}A + \int_{C_s} \rho \vec{u} \left(\vec{u} \cdot \mathrm{d}\vec{A} \right) \tag{8}$$

Verification of Simulation (cont.)

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Exact:

$$F_d = \frac{1}{2} \rho u^2 C_D A \tag{9}$$

Verification of Simulation (cont.)

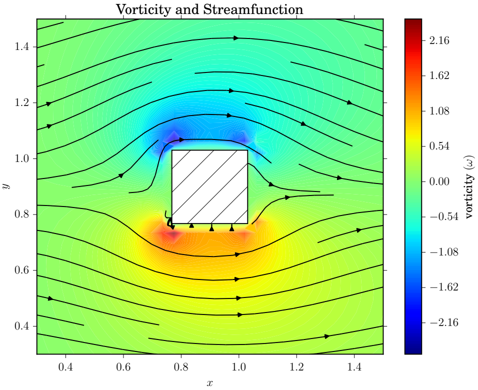
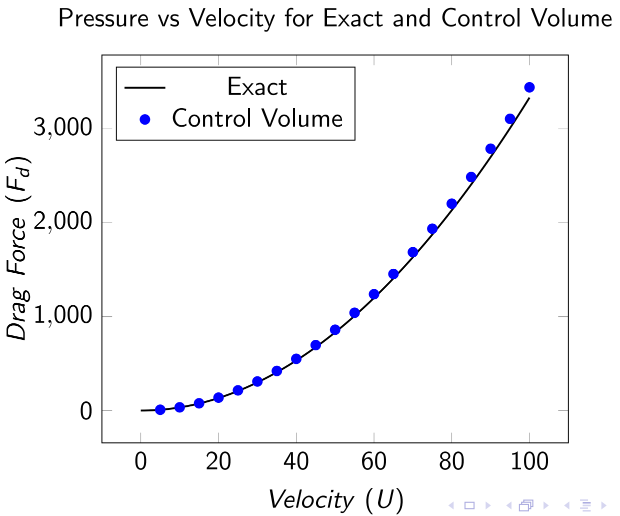
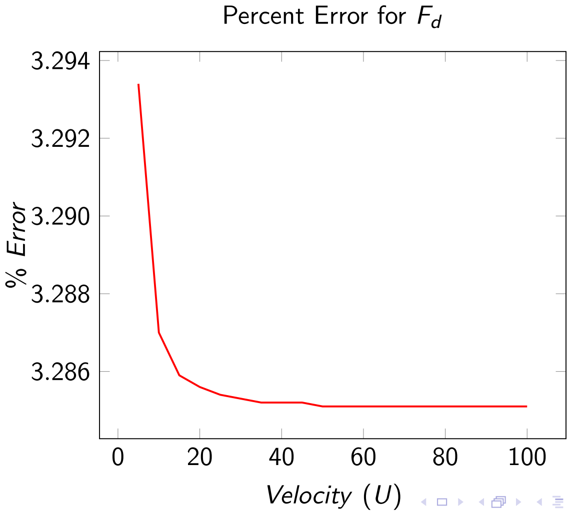


Figure : Typical plot of vorticity and streamfunction around a single block with $U = 10$

Verification of Simulation (cont.)



Verification of Simulation (cont.)



Spacing Between Cars

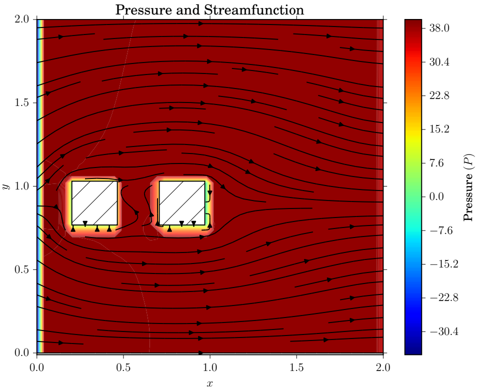


Figure : Pressure on the train for $U = 10$ and $separation/width = 11/15$

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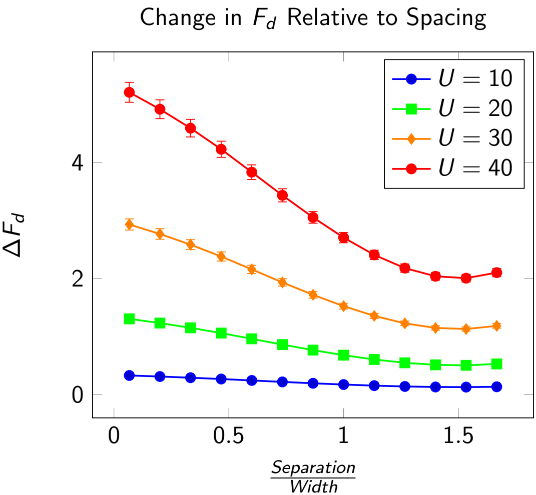

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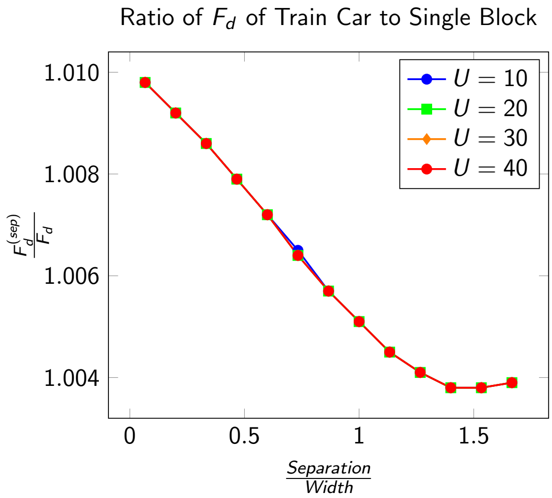

Ground Clearance

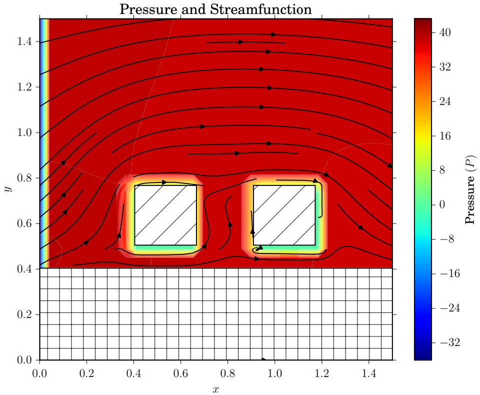

Figure : Pressure on train for $U = 10$ and $clearance/width = 1/3$

Ground Clearance

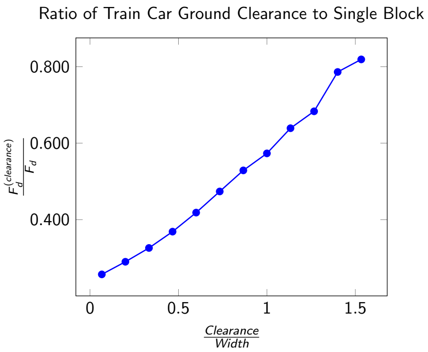

Figure : Ratio of F_d experienced on the second train car to that of a single block with $separation/width = 11/15$

Conclusion

- Ground clearance is dominant factor
- Spacing between cars is essentially negligable

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- Spacing between cars is essentially negligable
- Optimal Train:
  - Low Diameter Wheels
  - Close spacing since F_d is negligable (1% increase)
  - Conserve materials