CS 6300 **HW3: Expectimax and Probability** **Due February 7, 2017**

Please use the LaTeX template to produce your writeups. See the Homework Assignments page on the class website for details. Hand in at: `https://webhandin.eng.utah.edu/index.php`.

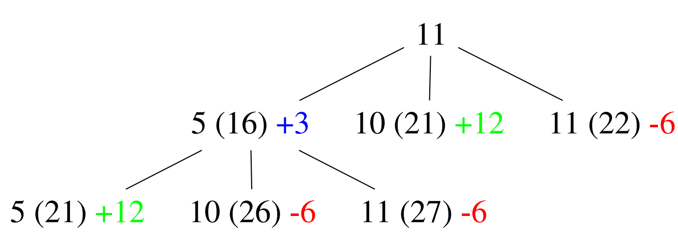
1 Expectimax

Consider this modified form of blackjack in Utah's only casino: this CS 6300 Artificial Intelligence class! By playing this game, you can add (or subtract) points to your course grade.

- There are three card values in an infinite deck: 5, 10, and 11. They are equally probable.
- There are two actions available: hit (draw a card), or stay (game ends).
- The total value of cards must be less than or equal to 21. If the value is more than 21, the player is bust and the game ends.
- The player must stay if the value is 21.
- The payoff schedule is:

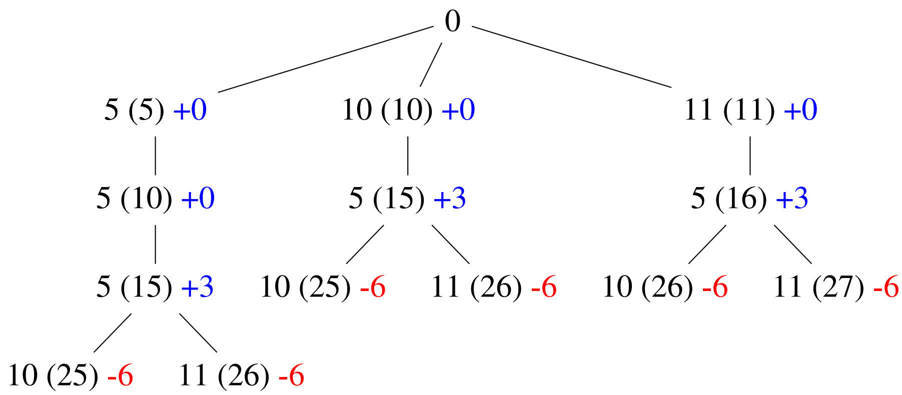
Hand value	Payoff
0-14	0
15-19	3
20	9
21	12
Bust	-6

1. Suppose the player's first card is an 11. Draw the expectimax tree using a drawing program. Show chance, max, and terminal nodes, and work out their values. What is the optimal strategy?



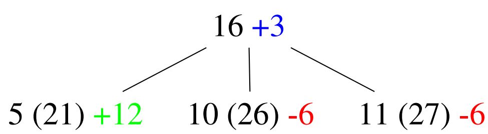
The optimal strategy would be to only draw one card and then stay. It has a 1/3 chance of going bust, but 2/3 chance of some payout. The average "winnings" of the 2nd level is zero after drawing 5. Stoping after the first gives an average winning of 11/2.

2. Unfortunately, you are playing with an unscrupulous dealer (one of the TAs) who after a fair first card then gives you the worst possible card thereafter. Draw the expectimax tree, and work out values for the nodes. What is the optimal strategy?



The optimal play strategy is to top at the second dealt card if given a 10 or an 11, but to go one card after after the second 5. This guarenttees a win of +3.

3. Unforunately, interest in AI blackjack is decreasing, and the instructor asks for your help in reworking the game by increasing the payoff for a value of 21 (currently $x = 12$). Suppose a player has a hand value of 16. What should the smallest value of x be (an integer) so that it is advantageous to hit rather than stay?



The average in the above figure is 0 gain from "hitting" with a 16 in hand. Therefore, the smallest that it would have to be is +15. This will have an average of +1 overall in playing.

2 Probability

Marijuana legalization has been in the news, and one of the states is having a gubernatorial election. The Libertarian candidate (random variable L) is more likely to legalize marijuana (random variable M) than the other candidates, but legalization may happen if any candidate is elected. The probabilities are modeled below.

	$+l$	$-l$
$P(L)$	0.1	0.9

Libertarian governor elected

	$P(+m\|L)$	$P(-m\|L)$
$+l$	0.667	0.333
$-l$	0.25	0.75

Marijuana legalized

1. What is $P(+m)$?

$$\begin{aligned} P(+m) &= P(+m|+l)P(+l) + P(+m|-l)P(-l) \\ &= 2/3 \cdot 1/10 + 1/4 \cdot 9/10 = 7/24 \\ P(-m) &= 1 - P(+m) = 17/24 \end{aligned}$$

2. What is $P(+l|+m)$?

$$\begin{aligned} P(+l|+m) &= \frac{P(+m|+l)P(+l)}{P(+m|+l)P(+m)} \\ &= \frac{2/3 \cdot 1/10}{7/24} = 8/35 \end{aligned}$$

	$P(+l\|M)$	$P(-l\|M)$
$+m$	8/35	27/35
$-m$	12/85	73/85

3. Fill in the joint distribution table below.

L	M	$P(L,M)$
$+l$	$+m$	1/15
$+l$	$-m$	1/30
$-l$	$+m$	9/40
$-l$	$-m$	27/40

4. More information is provided with new random variables B (balanced budget) and A (workplace absenteeism).

	$P(+b \mid M)$	$P(-b \mid M)$
$+m$	0.4	0.6
$-m$	0.2	0.8

Balanced Budget

	$P(+a \mid M)$	$P(-a \mid M)$
$+m$	0.75	0.25
$-m$	0.5	0.5

Absenteeism

Fill in the joint distribution table below.

L	M	B	A	$P(L, M, B, A)$
$+l$	$+m$	$+b$	$+a$	1/50
$+l$	$+m$	$+b$	$-a$	1/150
$+l$	$+m$	$-b$	$+a$	3/100
$+l$	$+m$	$-b$	$-a$	1/100
$+l$	$-m$	$+b$	$+a$	1/100
$+l$	$-m$	$+b$	$-a$	1/100
$+l$	$-m$	$-b$	$+a$	1/25
$+l$	$-m$	$-b$	$-a$	1/25

L	M	B	A	$P(L, M, B, A)$
$-l$	$+m$	$+b$	$+a$	27/400
$-l$	$+m$	$+b$	$-a$	9/400
$-l$	$+m$	$-b$	$+a$	81/800
$-l$	$+m$	$-b$	$-a$	27/800
$-l$	$-m$	$+b$	$+a$	73/1200
$-l$	$-m$	$+b$	$-a$	73/1200
$-l$	$-m$	$-b$	$+a$	73/300
$-l$	$-m$	$-b$	$-a$	73/300

The work for the first one is provided, while the others are left out. This is because the work is similar using Bayes' Nets.

$$P(L, M, B, A) = P(L|B, A, M)P(B|A, M)P(A|M)P(M)$$

Where we can reduce this to the dependent variables, giving

$$P(+l, +m, +b, +a) = P(+l| + m)P(+b| + m)P(+a| + m)P(+m)$$

Which is possible as the given conditional probabilities are only dependent upon M, and no the other conditions. Plugging in the values gives

$$P(+l, +m, +b, +a) = \left(\frac{8}{35}\right) \cdot \left(\frac{4}{10}\right) \cdot \left(\frac{3}{4}\right) \cdot \left(\frac{7}{24}\right) = \frac{1}{50}$$

5. Compute the following.

   (a) $P(+b| + m) = 4/10$ (directly from conditional probability)
   (b) $P(+b| + m, +l) = 4/10$ (from conditional probability since $+l$ is independent)
   (c) $P(+b) = \sum_{L,M,+b,A} = \frac{1}{50} + \frac{1}{150} + \frac{1}{100} + \frac{1}{100} + \frac{27}{400} + \frac{9}{400} + \frac{73}{1200} + \frac{73}{1200} = \frac{31}{120}$
   (d) $P(+a| + b) = \frac{P(+b,+a)}{P(+b)} = \frac{\sum_{G,M} P(G,M,+b,+a)}{31/120} = \left(\frac{1}{50} + \frac{1}{100} + \frac{27}{400} + \frac{73}{1200}\right) \cdot \frac{120}{31} = \frac{19}{31}$