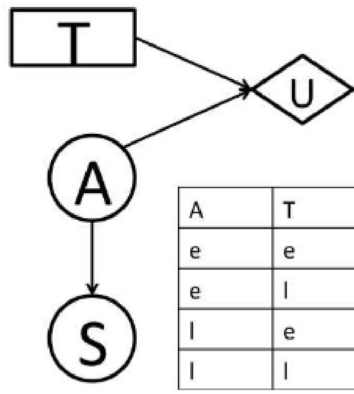
CS 6300 **HW9: VPI and HMMs** **Due April 18, 2017**

Please use the LaTeX template to produce your writeups. See the Homework Assignments page on the class website for details. Hand in at: `https://webhandin.eng.utah.edu/index.php`.

1 Decision Networks and VPI

A	P(A)
e	0.5
l	0.5



S	P(S)
e	
l	

A	S	P(S|A)
e	e	0.8
e	l	0.2
l	e	0.4
l	l	0.6

A	T	U(A,T)
e	e	600
e	l	0
l	e	300
l	l	600

S	A	P(A|S)
e	e	
e	l	
l	e	
l	l	

Your parents are visiting you for graduation. You are in charge of picking them up at the airport. Their arrival time (A) might be early (e) or late (l). You decide on a time (T) to go to the airport, also either early (e) or late (l). Your sister (S) is a noisy source of information about their arrival time. The probability values and utilities are shown in the tables above.

Compute $P(S), P(A|S)$ and compute the quantities below.

$$P(S=e) = 1.2 \Rightarrow \frac{1.2}{0.8+1.2} = 0.6$$
$$P(S=l) = 0.8 \Rightarrow \frac{0.8}{0.8+1.2} = 0.4$$

$$P(A=e|S=e) = \frac{P(S=e|A=e)P(A=e)}{P(s=e)} = \frac{0.8(0.5)}{0.6} = 2/3$$
$$P(A=l|S=e) = 1-2/3 = 1/3$$
$$P(A=l|S=l) = \frac{P(S=l|A=l)P(A=l)}{P(s=e)} = \frac{0.6(0.5)}{0.4} = 3/4$$
$$P(A=e|S=l) = 1-3/4 = 1/4$$

1. $EU(T=e) = 0.5(600) + 0.5(300) = 450$
2. $EU(T=l) = 0.5(0) + 0.5(600) = 300$
3. $MEU(\{\}) = \max_T EU(T,A) = 450$

4. Optimal action with no observations

   $T = e$

Now we consider the case where you decide to ask your sister for input.

1. $EU(T = e|S = e)$

   $= P(A = e|S = e)EU(T = e|A = e) + P(A = l|S = e)EU(T = e|A = l)$

   $= 2/3(600) + 1/3(300) = 500$

2. $EU(T = l|S = e)$

   $= P(A = e|S = e)EU(T = l|A = e) + P(A = l|S = e)EU(T = l|A = l)$

   $= 2/3(0) + 1/3(600) = 200$

3. $MEU(\{S = e\})$

   $= \max_T \sum_A P(A|S = e)EU(T|A) = 500$

4. Optimal action with observation $\{S = e\}$

   $T = e$

5. $EU(T = e|S = l)$

   $= P(A = l|S = l)EU(T = e, A = l) + P(A = e|S = l)EU(T = e, A = e)$

   $= 3/4(300) + 1/4(600) = 375$

6. $EU(T = l|S = l)$

   $= P(A = l|S = l)EU(T = l, A = l) + P(A = e|S = l)EU(T = l, A = e)$

   $= 3/4(600) + 1/4(0) = 450$

7. $MEU(\{S = l\})$

   $\max_T \sum_A P(A|S = l)EU(T) = 450$

8. Optimal action with observation $S = l$

   $T = l$

9. $VPI(S)$

   $= MEU(\{S = e\})P(S = e) + MEU(\{S = l\})P(S = l) - MEU(\{\})$

   $= 500(0.6) + 450(0.4) - 450 = 30$

2 Wherefore art thou Romeo?

Romeo and Juliet are two lovesick robots; they function best when each knows where the other is. Romeo has become lost, and is trying to figure out where he is so he can tell Juliet. Romeo is on the grid below, which also lists transition probabilities and properties of the sensors. At each step, Romeo senses, and then transitions to an adjacent room to get to the next time step. Romeo observed the following evidence while wandering in grief over his inability to tell Juliet where he is: 2 walls, 2 walls, 3 walls.

Forward Algorithm Compute the most likely location, given the evidence.

The forward algorithm is given by

$$P(x_{t+1}|e_{1:t+1}) = P(e_{1:t+1}|x_{t+1}) \sum_{x_t} P(x_{t+1}|x_t) P(x_t, e_{1:t})$$

We can use this for each time step to calculate the probability of each square. The *possible* squares for the robot to be in for $t = 1$ are $\{B\}$ and $\{D, G, F\}$, where B would be where number of walls were wrong and $\{D, G, F\}$ being a correct reading. From here, we can calculate the probability of each square for $t = 1$.

$$\begin{aligned}
P(B) &= 1/4[1(1/7) + 1(1/7)] = 1/14 \\
P(D) &= 3/4[1/3(1/7) + 1/2(1/7)] = 5/56 \\
P(G) &= 3/4[1/2(1/7) + 1/2(1/7)] = 3/28 \\
P(F) &= 3/4[1(1/7) + 1/2(1/7)] = 3/7
\end{aligned}$$

For $t = 2$ we get $\{B\}$ and $\{D, G, F\}$ again, except this time each room isn't equally likely, as we're constrained to being in the previous states for $t = 1$. This gives

$$\begin{aligned}
P(B) &= 1/4[1/2(5/56)] = 5/448 \\
P(D) &= 3/4[1/3(1/14) + 1/2(3/28)] = 13/224 \\
P(G) &= 3/4[1/2(5/56) + 1/2(3/7)] = 87/448 \\
P(F) &= 3/4[1/2(3/28)] = 9/224
\end{aligned}$$

Finally, for $t = 3$ we have the following sets $\{D, G, F\}$ and $\{A, C, E\}$, where again we're constrained from starting at $\{B, D, G, F\}$ for $t = 2$.

$$
\begin{aligned}
P(D) &= 1/4[1/3(5/448) + 1/2(87/448)] = 271/10752 \approx 0.0252 \\
P(G) &= 1/4[1/2(13/224) + 1/2(9/224)] = 11/896 \approx 0.0123 \\
P(F) &= 1/4[1/2(87/448)] = 87/3584 \approx 0.0243 \\
P(A) &= 3/4[1/3(5/448)] = 5/1792 \approx 0.0028 \\
P(B) &= 3/4[1/3(5/448)] = 5/1792 \approx 0.0028 \\
P(E) &= 3/4[1/2(9/224)] = 27/1792 \approx 0.0151
\end{aligned}
$$

The largest is $P(D)$ meaning the robot is most likely in square D

Viterbi Algorithm Compute the most likely sequence of steps he took in the maze for the above evidence.

The equation for the Viterbi Algorithm is similar to that of the Forward Algorithm except for a minor change. It can be seen below

$$P(x_{t+1}|e_{1:t+1}) = P(e_{1:t+1}|x_{t+1}) \max_{x_t} P(x_{t+1}|x_t)P(x_t, e_{1:t})$$

For $t = 1$ we get the same set of states, $\{B\}$ and $\{D, G, F\}$, giving

$$
\begin{aligned}
P(B) &= 1/4[1(1/7)] = 1/28 \\
P(D) &= 3/4[1/2(1/7)] = 3/56 \\
P(G) &= 3/4[1/2(1/7)] = 3/56 \\
P(F) &= 3/4[1(1/7)] = 3/28
\end{aligned}
$$

For $t = 2$ we get the same set of states, $\{B\}$ and $\{D, G, F\}$, giving

$$
\begin{aligned}
P(B) &= 1/4[1/2(3/56)] = 3/448 \\
P(D) &= 3/4[1/2(3/56)] = 9/448 \\
P(G) &= 3/4[1/2(3/28)] = 9/224 \\
P(F) &= 3/4[1/2(3/56)] = 9/448
\end{aligned}
$$

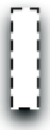
Finally, for $t = 3$ we have the following sets $\{D, G, F\}$ and $\{A, C, E\}$, where again we're constrained from starting at $\{B, D, G, F\}$ for $t = 2$.

$$
\begin{aligned}
P(D) &= 1/4[1/2(9/224)] = 9/1792 \approx 0.0050 \\
P(G) &= 1/4[1/2(9/448)] = 9/3584 \approx 0.0025 \\
P(F) &= 1/4[1/2(9/224)] = 9/1792 \approx 0.0050 \\
P(A) &= 3/4[1/3(3/448)] = 3/1792 \approx 0.0017 \\
P(C) &= 3/4[1/3(3/448)] = 3/1792 \approx 0.0017 \\
P(E) &= 3/4[1/2(9/448)] = 37/3584 \approx 0.0075
\end{aligned}
$$

With the highest being block E, meaning the robot has the highest probability of being in block E.

Probability of transitioning to any adjoining room is uniform (e.g., if you could go either north or south, but not east or west, you go north with probability 1/2 and south with probability 1/2).

The initial probabilities are uniform: the robot is in any square at time t=0 with probability 1/7.



The symbol to the left is a door; robots can only get to other rooms via doors.

Robots sense the number of walls in the current room without doors. Their sensors are noisy, however. With probability 3/4 the robot will sense the correct number of walls. With probability 1/4, the robot will sense one more than the actual number of walls. For example, if the robot is in square D, it will sense 2 with probability 3/4 and 3 with probability 1/4.