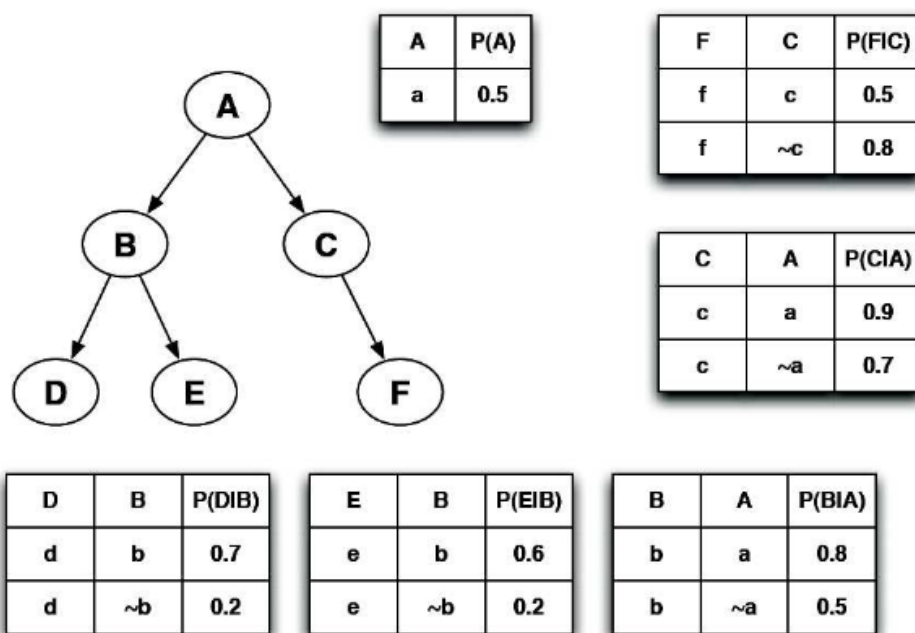


Please use the  $\text{\LaTeX}$  template to produce your writeups. See the Homework Assignments page on the class website for details. Hand in at: <https://webhandin.eng.utah.edu/index.php>.

## 1 Variable Elimination

Perform variable elimination on the network below to figure out  $P(C)$  given  $F = f$  and  $D = \sim d$ . Show your work and list all of the intermediate factors formed. Assume we eliminate the variables in the following order: F, E, D, B, A.



To eliminate the variables, we can sum over them all individually. This gives the equation

$$P(C) = P(f|C) \sum_A P(C|A)P(A) \sum_B P(B|A)P(\sim d|B) \sum_E P(E|B)$$

In using this, we can calculate the values for  $c$  and  $\sim c$ , giving

$$\begin{aligned} P(c, \sim d, f) &= 0.5(0.9 \cdot 0.5 + 0.7 \cdot 0.5)(0.8 \cdot 0.3 + 0.5 \cdot 0.8)(0.8) \\ &= 0.2048 \\ P(\sim c, \sim d, f) &= 0.8(0.1 \cdot 0.5 + 0.3 \cdot 0.5)(0.8 \cdot 0.3 + 0.5 \cdot 0.8)(0.8) \\ &= 0.0819 \end{aligned}$$

Now, solving for  $P(c | \sim d, f)$  and  $P(\sim c | \sim d, f)$  gives

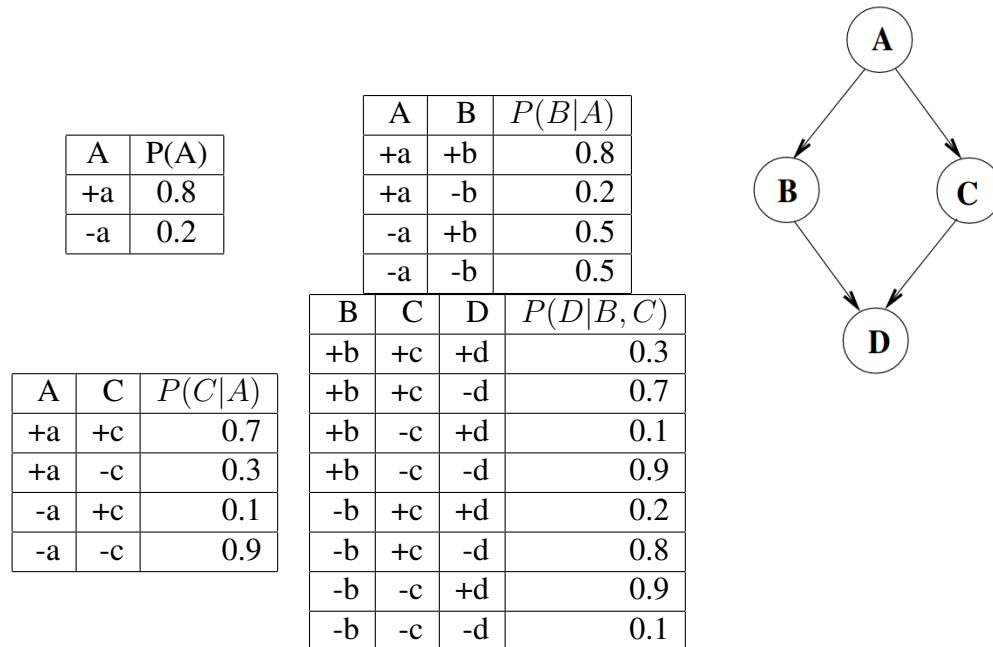
$$\begin{aligned}
 P(c | \sim d, f) &= \frac{P(c, f, \sim d)}{\sum_C P(c, f, \sim d)} \\
 &= \frac{0.2048}{0.2048 + 0.0819} = 0.7143 \\
 P(\sim c | \sim d, f) &= \frac{P(\sim c, f, \sim d)}{\sum_C P(c, f, \sim d)} \\
 &= \frac{0.08192}{0.2048 + 0.0819} = 0.2857
 \end{aligned}$$

## 2 Sampling

Consider the Bayes net below with corresponding CPTs.

1. Generate 2 samples using the following random numbers. The order for the random numbers is ABCD.

0.31	0.58	0.04	0.94	0.67	0.49	0.37	0.42	...
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2. Given the samples below, answer the subsequent queries.

+a -b -c -d  
 -a +b +c -d  
 +a -b +c -d  
 +a +b -c -d  
 +a -b +c +d  
 -a -b -c +d  
 -a -b -c -d  
 +a +b +c -d  
 -a +b -c -d  
 +a +b -c +d

- (a)  $P(+d) = 3/10$
- (b)  $P(+a, -b) = 2/10$
- (c)  $P(-a, -b, -c, -d) = 1/10$

(d)  $P(-c | -d) = 4/7$

(e)  $P(+d | -a, -b) = 1/2$

3. Consider the query  $P(-d | -a, -b)$ . Using the same random numbers as before, generate samples and their weights using likelihood weighting.

0.31	0.58	0.04	0.94	0.67	0.49	0.37	0.42	...
------	------	------	------	------	------	------	------	-----

4. Given the weighted samples below, answer the subsequent queries.

+a	-b	-c	-d	0.3
-a	+b	+c	-d	0.4
+a	-b	+c	-d	0.1
+a	+b	-c	-d	0.3
+a	-b	+c	+d	0.4
-a	-b	-c	+d	0.1
-a	-b	-c	-d	0.2
+a	+b	+c	-d	0.5
-a	+b	-c	-d	0.7
+a	+b	-c	+d	0.8

(a)  $P(+d)$

(b)  $P(+a, -b)$

(c)  $P(-a, -b, -c, -d)$

(d)  $P(-c | -d)$

(e)  $P(+d | -a, -b)$