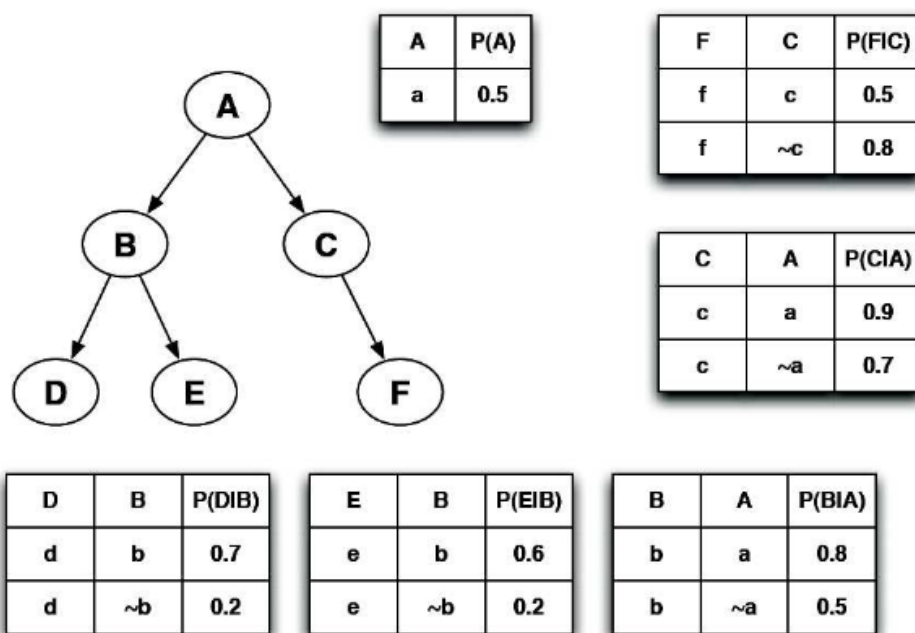


Please use the \LaTeX template to produce your writeups. See the Homework Assignments page on the class website for details. Hand in at: <https://webhandin.eng.utah.edu/index.php>.

1 Variable Elimination

Perform variable elimination on the network below to figure out $P(C)$ given $F = f$ and $D = \sim d$. Show your work and list all of the intermediate factors formed. Assume we eliminate the variables in the following order: F, E, D, B, A.



To eliminate the variables, we can sum over them all individually. This gives the equation

$$P(C) = P(f|C) \sum_A P(C|A)P(A) \sum_B P(B|A)P(\sim d|B) \sum_E P(E|B)$$

In using this, we can calculate the values for c and $\sim c$, giving

$$\begin{aligned} P(c, \sim d, f) &= 0.5(0.9 \cdot 0.5 + 0.7 \cdot 0.5)(0.8 \cdot 0.3 + 0.5 \cdot 0.8)(0.8) \\ &= 0.2048 \\ P(\sim c, \sim d, f) &= 0.8(0.1 \cdot 0.5 + 0.3 \cdot 0.5)(0.8 \cdot 0.3 + 0.5 \cdot 0.8)(0.8) \\ &= 0.0819 \end{aligned}$$

Now, solving for $P(c | \sim d, f)$ and $P(\sim c | \sim d, f)$ gives

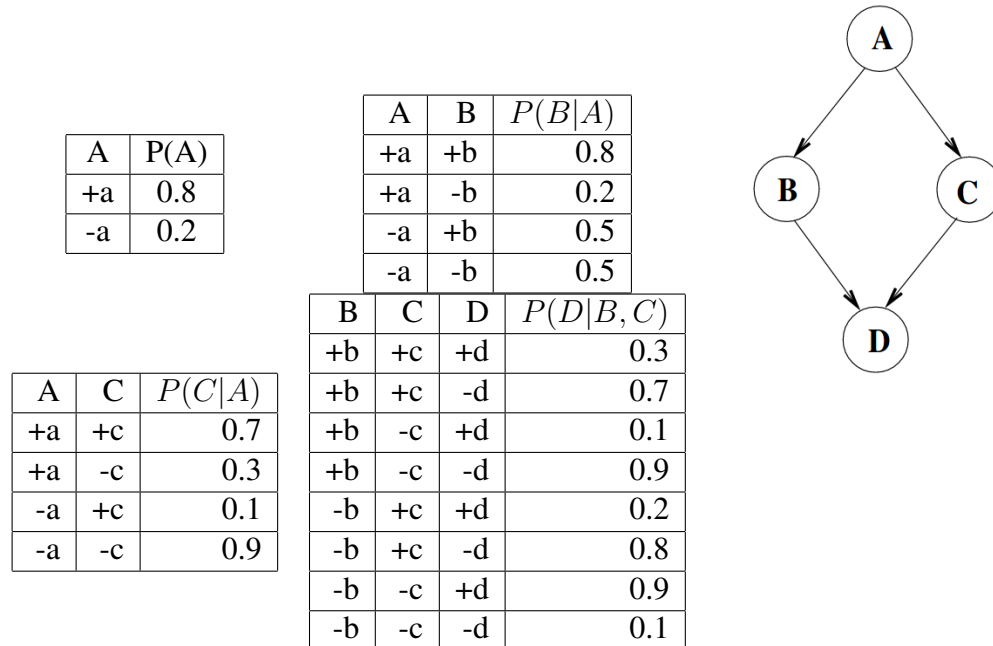
$$\begin{aligned} P(c | \sim d, f) &= \frac{P(c, f, \sim d)}{\sum_C P(c, f, \sim d)} \\ &= \frac{0.2048}{0.2048 + 0.0819} = 0.7143 \\ P(\sim c | \sim d, f) &= \frac{P(\sim c, f, \sim d)}{\sum_C P(c, f, \sim d)} \\ &= \frac{0.08192}{0.2048 + 0.0819} = 0.2857 \end{aligned}$$

2 Sampling

Consider the Bayes net below with corresponding CPTs.

1. Generate 2 samples using the following random numbers. The order for the random numbers is ABCD.

0.31	0.58	0.04	0.94	0.67	0.49	0.37	0.42	...
------	------	------	------	------	------	------	------	-----



Sample 1: +a, +b, +c, -d

Sample 2: +a, +b, +c, -d

The method used was the range was chosen from a “top down” approach. For example, $P(B|+a)$, the first range for +b was $0 \leq \text{rand}() < 0.8$, and -b was $0.8 \leq \text{rand}() < 1$.

2. Given the samples below, answer the subsequent queries.

```

+a  -b  -c  -d
-a  +b  +c  -d
+a  -b  +c  -d
+a  +b  -c  -d
+a  -b  +c  +d
-a  -b  -c  +d
-a  -b  -c  -d
+a  +b  +c  -d
-a  +b  -c  -d
+a  +b  -c  +d
  
```

- (a) $P(+d) = 3/10$
- (b) $P(+a, -b) = 2/10$
- (c) $P(-a, -b, -c, -d) = 1/10$
- (d) $P(-c | -d) = 4/7$
- (e) $P(+d | -a, -b) = 1/2$

3. Consider the query $P(-d | -a, -b)$. Using the same random numbers as before, generate samples and their weights using likelihood weighting.

0.31	0.58	0.04	0.94	0.67	0.49	0.37	0.42	...
------	------	------	------	------	------	------	------	-----

Sample 1: -a, -b, +c, -d, $w = 1.0 \times 0.2 \times 0.2 \times 0.8 = 0.032$

Sample 2: -a, -b, +c, -d, $w = 1.0 \times 0.2 \times 0.2 \times 0.8 = 0.032$

4. Given the weighted samples below, answer the subsequent queries.

+a	-b	-c	-d	0.3
-a	+b	+c	-d	0.4
+a	-b	+c	-d	0.1
+a	+b	-c	-d	0.3
+a	-b	+c	+d	0.4
-a	-b	-c	+d	0.1
-a	-b	-c	-d	0.2
+a	+b	+c	-d	0.5
-a	+b	-c	-d	0.7
+a	+b	-c	+d	0.8

- (a) $P(+d) = \frac{0.4+0.1+0.8}{0.3+0.4+0.1+0.3+0.4+0.1+0.2+0.5+0.7+0.8} = \frac{1.3}{3.8} = 0.342$
- (b) $P(+a, -b) = \frac{0.3+0.1+0.4}{3.8} = \frac{0.8}{3.8} = 0.211$
- (c) $P(-a, -b, -c, -d) = \frac{0.2}{3.8} = 0.053$
- (d) $P(-c | -d) = \frac{0.3+0.3+0.2+0.7}{0.3+0.4+0.1+0.3+0.2+0.5+0.7} = \frac{1.5}{2.5} = 0.6$
- (e) $P(+d | -a, -b) = \frac{0.1}{0.1+0.2} = \frac{0.1}{0.3} = 0.333$