Math Story

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0.1 TODO

Theorem List

- Pythagoras Theorem.
- Midpoint Theorem.
- Remainder Theorem.
- Fundamental Theorem of Arithmetic.
- Angle Bisector Theorem.
- Inscribed Angle Theorem.
- Ceva's Theorem.
- Bayes' Theorem.

Chapter 1

Foundation

1.1 The Axioms of Mathematics

1.1.1 Set Theory

The standard foundation of modern mathematics is set theory, or more precisely Zermelo-Fraenkel plus the Axiom of Choice (ZFC) set theory. [cite `https://en.wikipedia.org/wiki/Zermelo%E2%80%93Fraenkel_set_theory`]

Axiom 1.1.1 (Axiom of extensionality)**.** Two sets are equal if they have the same elements.

$$\forall x \forall y [\forall z (z \in x \iff z \in y) \Rightarrow x = y]$$

Story 1.1.2.

Axiom 1.1.3 (Axiom of regularity)**.** Every non-emtpy set x contains a member y such that x and y are disjoint sets.

$$\forall x [\exists a (a \in x) \Rightarrow \exists y (y \in x \wedge \neg \exists z (z \in y \wedge z \in x))]$$

Story 1.1.4.

Axiom 1.1.5 (Axiom schema of specification)**.** In general, the subset of a set z obeying a formula $\phi(x)$ with one free variable x may be written as $\{x \in z : \phi(x)\}$,

$$\forall z \forall w_1 \forall w_2 \dots \forall w_n \exists y \forall x [x \in y \iff ((x \in z) \wedge \phi)]$$

Story 1.1.6.

Axiom 1.1.7 (Axiom of pairing)**.** If x and y are sets, then there exists a set which contains x and y as elements

$$\forall x \forall y \exists z ((x \in z) \wedge (y \in z))$$

Story 1.1.8.

Axiom 1.1.9 (Axiom of union)**.** The axiom of union states that for any set of sets $\mathcal{F}$ there is a set A containing every element that is a member of some member of $\mathcal{F}$.

$$\forall \mathcal{F} \exists A \forall Y \forall x [(x \in Y \wedge Y \in \mathcal{F}) \Rightarrow x \in A]$$

Story 1.1.10.

Axiom 1.1.11 (Axiom schema of replacement)**.** Let ϕ be any formula in the language of ZFC whose free variables are among $x, y, A, w_1, \ldots, w_n$, so that in particular B is not free in ϕ. Then:

$$\forall A \forall w_1 \forall w_2 \ldots \forall w_n [\forall x(x \in A \Rightarrow \exists! y \phi) \Rightarrow \exists B \forall x(x \in A \Rightarrow \exists y(y \in B \wedge \phi))]$$

Story 1.1.12.

Axiom 1.1.13 (Axiom of infinity)**.**

$$\exists X[\emptyset \in X \wedge \forall y(y \in X \Rightarrow S(y) \in X)]$$

Story 1.1.14.

Axiom 1.1.15 (Axiom of power set)**.** For any set x , there is a set y that contains every subset of x:

$$\forall x \exists y \forall x[z \subseteq x \Rightarrow z \in y]$$

Story 1.1.16.

Axiom 1.1.17 (Well-ordering theorem)**.** For any set X, there is a binary relation R which well-orders X. This means R is a linear order on X such that every nonempty subset of X has a member which is minimal under R

$$\forall X \exists R(R \texttt{ well} - \texttt{orders } X)$$

Story 1.1.18.

1.1.2 Type Theory

1.1.3 Category Theory

Formalization inside topos, build on preshealfs $\mathbf{Sets}^{C^{op}}$

1.2 Numbers

Theorem 1.2.1.

Proof

Story