A Graph Theoretical Approach for Creating Building Floor Plans

Krishnendra Shekhawat[1(✉)], Pinki[1], and José P. Duarte[2]

[1] Department of Mathematics, BITS Pilani, Pilani Campus, Pilani, India
krishnendra.iitd@gmail.com
[2] SCDC, School of Architecture and Landscape Architecture, The Pennsylvania State University, University Park, PA 16802, USA

Abstract. Existing floor planning algorithms are mostly limited to rectangular room geometries. This restriction is a significant reason why they are not used much in design practice. To address this issue, we propose an algorithm (based on graph theoretic tools) that generates rectangular and, if required, orthogonal floor plans while satisfying the given adjacency requirements. If a floor plan does not exist for the given adjacency requirements, we introduce circulations within a floor plan to have a required floor plan.

Keywords: Adjacency · Algorithm · Graph theory · Rectangular floor plan · Orthogonal floor plan

1 Introduction

A *floor plan* (FP) is a polygon, the plan boundary, divided by straight lines into component polygons called *rooms*. The edges forming the perimeter of each room are termed *walls*. Two rooms in a floor plan are *adjacent* if they share a wall or a section of wall; it is not sufficient for them to touch at a point only.

A *rectangular floor plan* (RFP) is a FP in which plan's boundary and each room are rectangles. Any RFP with n rooms is represented by RFP(n). An *orthogonal floor plan* (OFP) has a rectangular plan boundary with the walls of each room parallel to the sides of the plan boundary, i.e., an OFP may have some non-rectangular rooms, such as L-shaped, T-shaped, etc. A FP with non-rectangular and rectilinear plan boundary is called *non-rectangular floor plan* (NRFP). For an illustration, refer to Figs. 1A, B and C demonstrating a RFP, an OFP and a NRFP respectively.

An essential task in the initial stages of most architectural design processes is the construction of planar floor plans, that are composed of non-overlapping rooms divided from each other by walls while satisfying given constraints. In this paper, we aim to construct a floor plan for any given adjacency graph, where an *adjacency graph* provides specific neighborhood between the given rooms. Constructing a floor plan involves the following sub-problems:

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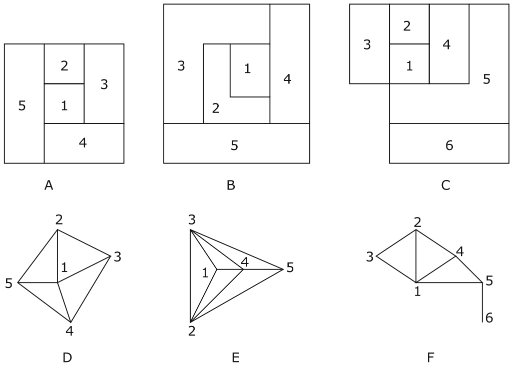

Fig. 1. Illustrating different concepts used in the paper

1. Checking the existence of a floor plan
   There may or may not exist a floor plan for the given adjacency graph. If it exists, then there may or may not exists a RFP. If it does not exist, then there may or may not exists an OFP. Hence, for constructing a floor plan, we first need to derive some conditions for checking the existence of a floor plan for the given adjacency graph.
2. Identifying the position of rooms
   Rooms need to be positioned in a way that each room should only be surrounded by its neighbors. Also, we need to consider their adjacencies with the exterior, if asked by the user.
3. Identifying the shape of rooms
   We need to look for the shape of a room that would make it adjacent to its neighbors. Initially, we prefer a room to be rectangular, otherwise we look for a shape that is made up of more than one rectangles, for example, L-shape is made up of 2 rectangles while U-shape is made up of 3 rectangles.
4. Identifying the shape of the layout
   We need to place all rooms inside a polygon while satisfying the above-discussed constraints. Here, we prefer to have a rectangular boundary, if possible.
5. The size of rooms and the layout
   At this stage, we are not considering the dimensional constraints, i.e., we will construct a floor plan corresponding to a given adjacency graph only.

1.1 Comparison with Existing Work

In the past, many researchers have presented graph theoretical techniques for the generation of a FP while satisfying given adjacency requirements. A brief literature review is as follows:

This approach was first presented by Levin [1] where a method was proposed for converting an adjacency graph into a FP. Then in 1970, Cousin [2] talked about the construction of a RFP for a given adjacency graph. In both the approaches, the problem of realizing the adjacency structure as a FP was not presented very clearly. In the same year, Grason [3] proposed a dual graph representation of a planar graph for generating a RFP. In this direction, Steadman [4] exhaustively generates all topologically distinct RFP (illustrating all possibilities up to six component rooms). These packings were produced by hand. In 1975, Sauda [5] designed a computer algorithm for generating all topologically distinct RFP having 8 rooms. In 1977, Lynes [6] showed that "all rooms of a FP may have windows if and only if the adjacency graph is outer-planar[1]." In 1980, Baybars and Eastman [7] demonstrated a systematic procedure for obtaining NRFP from a given underlying maximal planar graph (MPG)[2]. In 1982, Roth et al. [8] presented the construction of a dimensioned RFP from a given adjacency graph. In this method, the given graph is first split into two sub-graphs by a colouring technique; each of these graphs is then converted into a dimensioned graph; at the end, a set of alternative plans were derived where the size of FP is determined using the PERT algorithm [9]. In 1985, Robinson and Janjic [10] showed that, for a given maximal outer-planar graph, if areas are specified for rooms, then any convex polygon with the correct area can be divided into convex rooms to satisfy both area and adjacency requirements. In 1987, Bhasker and Sahni [11] gave a linear time algorithm for the existence of a RFP corresponding to a properly triangulated planar graph (PTPG)[3]. In 1988, Rinsma [12] provided conditions for the existence of a RFP and an OFP for a given tree[4]. In 1990, Rinsma et al. [13] demonstrated the automated generation of an OFP corresponding to a given MPG. In 1993, Yeap and Sarrafzadeh [14] showed that every planar triangulated graph (PTG) has a floor-plan using 1-, 2-, and 3-rectangle modules. In 1995, Giffin et al. [15] gave a linear time algorithm for constructing an OFP where rooms are at worst topologically equivalent T-shapes. In 1999, He [16] presented a linear time algorithm that constructs a floor-plan for PTG using only 1- and 2-rectangle modules. In 2000, Recuero et al. [17] presented a heuristic method for checking the existence of a RFP for a given graph. In 2003, Liao et al. [18] gave a linear time algorithm for constructing an OFP for any n-vertex PTG, which is based upon the concept of orderly spanning trees.

As a recent work, in 2011, Jokar and Sangchooli [19] introduced face area concept for constructing an OFP corresponding to a particular class of MPG. In the

[1] An undirected graph is an *outer-planar graph* if it can be drawn in the plane without crossings in such a way that all of the vertices belong to the unbounded face of the drawing.

[2] A planar graph G is *maximal* if no edges can be added to G without losing planarity.

[3] A *properly triangulated planar graph* (PTPG), G, is a connected planar graph that satisfies the following properties:

i. Every face (except the exterior) is a triangle (i.e., bounded by three edges),
ii. All internal vertices have degree ≥ 4,
iii. All cycles that are not faces have length ≥ 4.

[4] Any connected graph without cycles is a tree.

same year, Zhang and Sadasivam [20] studied adjacency-preserving transformations from MPG to PTPG. In 2012, Regateiro et al. [21] proposed an approach for architectural layout design problems, which is based on topological algebras and constraint satisfaction techniques. In 2014, Shekhawat [22] proposed the enumeration of a particular class of RFP, i.e., the RFP having $3n - 7$ edges in their dual graphs. In 2017, Shekhawat et al. [23] gave an algorithm to generate a NRFP corresponding to the given weighted adjacency matrix. In 2018, Slusarczyk [24] introduced the notion of hierarchical layout graph (HL-graph) and present a theoretical framework for extending local graph requirements to global requirements on HL-graphs. Recently, Shekhawat [25] presented an algorithm for enumerating all distinct maximal RFP.

It can be seen that most of the work done related to the existence and construction of a FP falls into any one of these categories:

i. Construction of a RFP corresponding to PTPG [8,11,22,26,27],
ii. Construction of an OFP corresponding to maximal outer planar (MOP) graphs [6,10,12] and MPG [13–16,18,19],
iii. Construction of a NRFP corresponding to MPG [7] or corresponding to weighted adjacency matrix [23].

Hence, most of the work related to the generation of a FP is restricted to a particular class of adjacency graphs. And to the best of our knowledge, there does not exist an algorithm for constructing a FP corresponding to adjacency graphs, other than PTG and MPG. In this work, we aim to provide a FP for any given adjacency graph, i.e., this work is not restricted to any particular class of adjacency graphs.

In 2006, Steadman [28] showed that rectangular packings offer the best flexibility of dimensioning and this is why most of the buildings are rectangular. Hence, in this paper, we first aim to have a RFP for a given adjacency graph. If RFP does not exist, then we look for an OFP. If both of them do not exist, then we insert some circulations within the floor plan.

Notations:
FP: floor plan/s,
$\mathrm{FP_G}$: floor plan graph,
RFP: rectangular FP,
RFP(n): RFP with n rooms,
MFP and $\mathrm{MFP_G}$: maximal FP and maximal $\mathrm{FP_G}$ respectively,
OFP: orthogonal FP,
G_n: $n-$vertex simple connected adjacency graph,
n: the number of rooms in a RFP or the number of vertices in an adjacency graph,
v_i: i^{th} vertex of an adjacency graph,
R_i: i^{th} room of a FP.

2 An Algorithm for Constructing Floor Plans

In this section, our aim is to construct a FP for a given adjacency graph G_n. We prefer to derive a RFP for G_n, otherwise we proceed for an OFP. If a floor plan does not exists, then we introduce some circulations within a FP. To proceed further, we introduce the following new terminologies.

Definition 1. An adjacency graph for which a FP exists is called *floor plan graph*, abbreviated as $\mathrm{FP_G}$.

For example, the adjacency graph in Fig. 1D is a $\mathrm{FP_G}$. In particular, it is a $\mathrm{RFP_G}$ because of the presence of a RFP in Fig. 1A while it is interesting to verify that the adjacency graph in Fig. 1E is an $\mathrm{OFP_G}$ but not a $\mathrm{RFP_G}$ (it is not possible to construct a RFP for the adjacency graph in Fig. 1E).

Definition 2. A $\mathrm{FP_G}$ is called *maximal* $\mathrm{FP_G}$, abbreviated as $\mathrm{MFP_G}$, if adding any new edge to it results in a graph that is not a $\mathrm{FP_G}$. A FP corresponding to a $\mathrm{MFP_G}$ is called *maximal floor plan*, abbreviated as MFP.

Similarly, maximal rectangular floor plan and maximal orthogonal floor plan are abbreviated as MRFP and MOFP respectively.

It is easy to verify that the FP in Fig. 1A is a MRFP (it is not possible to make two non-adjacent rooms adjacent in Fig. 1A while maintaining other adjacencies and rectangularity). Similarly, the FP in Fig. 1B is a MOFP (we cannot make two non-adjacent rooms adjacent while maintaining other adjacencies).

In 2018, Shekhawat [25] presented the enumeration of all distinct MRFP. Furthermore, it has been proved that if an adjacency graph G_n is not a sub-graph of any of the $\mathrm{MRFP_G}(n)$, then there does not exist a $\mathrm{RFP}(n)$ for G_n. It leads us to the following cases:

1. G_n is not a sub-graph of a $\mathrm{MRFP_G}(n)$
   In this case, there does not exist a RFP for G_n but there may or may not exist an OFP for G_n. If it exists, we construct an OFP for G_n. If it does not exist, we construct a FP with circulations.
2. G_n is a sub-graph of a $\mathrm{MRFP_G}(n)$
   In this case, there may or may not exist a RFP for G_n. If it exists, we construct a RFP otherwise we construct an OFP.

The steps for the construction of a FP are as follows (for a better understanding of all the steps involved, refer to the flow chart in Fig. 2):

1. Check if G_n is a sub-graph of any one of the $\mathrm{MRFP_G}(n)$
2. If G_n is a sub-graph of any one of the $\mathrm{MRFP_G}(n)$
   For example, G_6 in Fig. 3A is a sub-graph of $\mathrm{MRFP_G}(6)$ in Fig. 3B. The red edges in Fig. 3B are the extra connections, which are not a part of adjacency constraints. Similarly, G_6 in Fig. 4A is also a sub-graph of the $\mathrm{MRFP_G}(6)$.
   (a) Consider a $\mathrm{MRFP}(n)$ corresponding to the $\mathrm{MRFP_G}(n)$.
       For example, corresponding to the adjacency graphs G_6 in Figs. 3A and 4A, the required $\mathrm{MRFP}(6)$ are illustrated in Figs. 3C and 4B respectively.

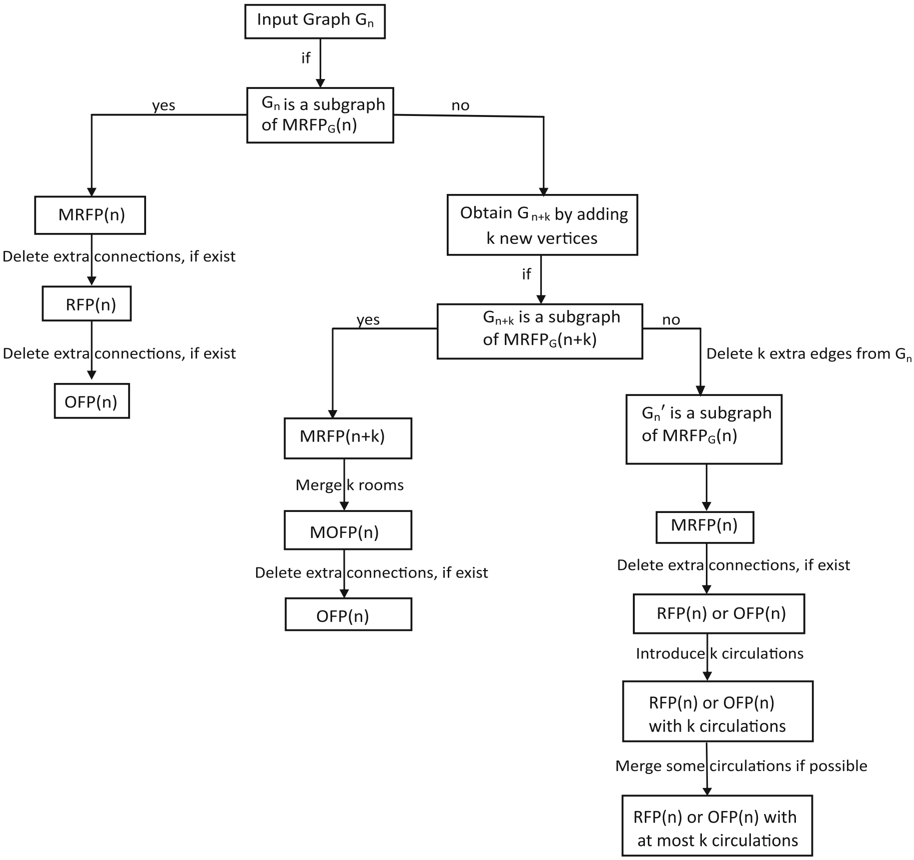

Fig. 2. Flow chart illustrating the different cases for constructing a floor plan corresponding to a given adjacency graph

(b) Reduce the obtained MRFP(n) into a required FP(n) by eliminating the extra connections.
One of the most common ways which has been used by many researchers to eliminate the extra connections is to introduce doors between adjacent rooms such that there are no doors between non-adjacent rooms. For example, refer to Fig. 3E with a RFP(6) from the given G_6, where the extra connections with no doors are shown in red. But, in this approach, we are compromising with the position of the rooms. As an example, G_6 in Fig. 3A clearly indicates that all rooms must be adjacent to exterior but RFP(6) in Fig. 3E have two rooms which are not adjacent to the exterior. Hence, the best possible approach is to adjust the shape and size of some rooms to have a required FP. For example, the reduction of MRFP(6) in Fig. 3C into a RFP(6) in Fig. 3F is shown in Figs. 3G to K. Further, the MRFP(6) in Figure cannot be reduced in a RFP, hence an OFP for G_6 in Fig. 4A has been shown in Fig. 4D.

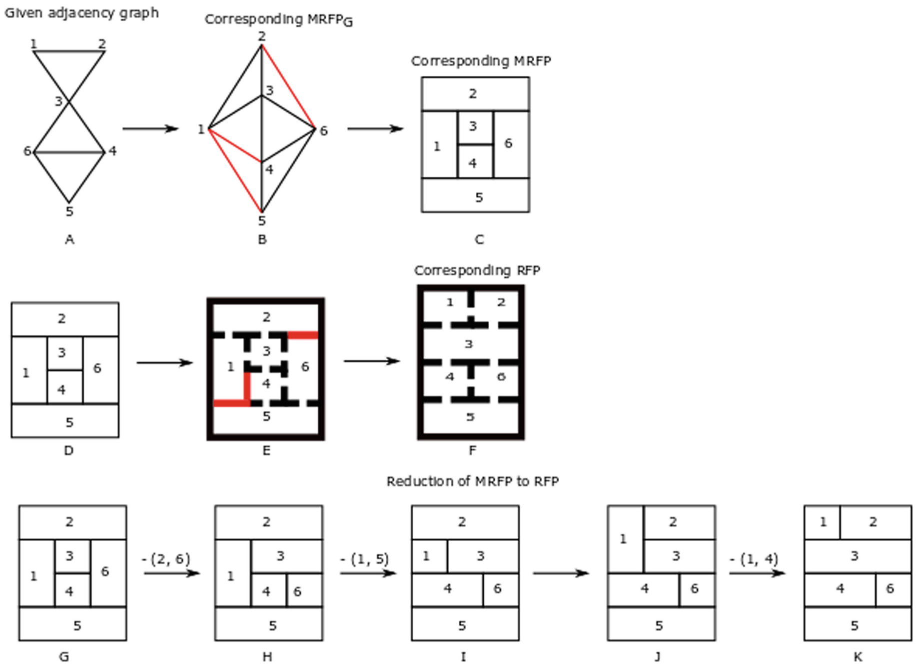

Fig. 3. Deriving a RFP from MRFP corresponding to a given adjacency graph (Color figure online)

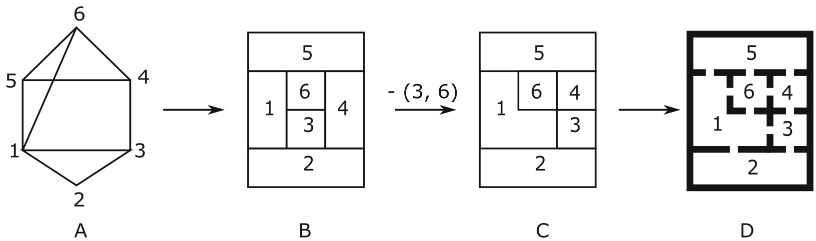

Fig. 4. Deriving an OFP from MRFP corresponding to a given adjacency graph

3. If G_n is not a sub-graph of any of the $\mathrm{MRFP_G}(n)$
   For example, G_8 in Fig. 5A is not a sub-graph of any of the $\mathrm{MRFP_G}(8)$ (for all $\mathrm{MRFP_G}(8)$, refer to [25]).
   (a) In this case, there does not exist a RFP corresponding to G_n. Hence, we need to construct an OFP for G_n.
   For example, G_8 in Fig. 5A is not a $\mathrm{RFP_G}(8)$.
   (b) Let k be the minimum number of edges of G_n whose deletion results in a graph G'_n such that G'_n is a sub-graph of at least one of the $\mathrm{MRFP_G}(n)$ (an algorithm for computing k is given in Sect. 3.1).

For example, for G_8 in Fig. 5A, we have $k = 2$, corresponding to the edges joining vertices (v_1, v_7) and (v_3, v_5).

(c) Subdivide each of the k edges, obtained in above step, into two edges, by adding a vertex to each of them, to have a new graph, say G_r with $r = n + k$.
For example, two vertices, v_9 and v_{10}, have been added to G_8 in Fig. 5A to derive graph G_{10}, illustrated in Fig. 5B.

(d) If G_r is a sub-graph of any one of the $\mathrm{MRFP_G}(r)$

i. Consider a MRFP(r) corresponding to the $\mathrm{MRFP_G}(r)$.
For example, G_{10} in Fig. 5B is a sub-graph of $\mathrm{MRFP_G}(10)$ in Fig. 5C and corresponding MRFP(10) is shown in Fig. 5D.

ii. Let e_{ij} denotes an edge with endpoints v_i and v_j and v_p is the added vertex on e_{ij}.
Corresponding to each new vertex v_p, $n < p \leq n + k$, either merge rooms R_i and R_p, or merge rooms R_p and R_j, to have a MOFP with k orthogonal rooms.
For example, corresponding to G_8 in Fig. 5A, a MOFP(8) is shown in Fig. 5E, that is obtained from MRFP(10) by merging (R_3, R_9) and (R_7, R_{10}).

iii. Reduce the obtained MOFP(n) into a required OFP(n) by eliminating the extra connections.
For example, an OFP(8) for G_8 in Fig. 5A is shown in Fig. 5H. The steps for deriving the OFP(8) in Fig. 5H from MOFP(8) in Fig. 5F are shown in Fig. 5I to L.

(e) If G_r is not a sub-graph of any one of the $\mathrm{MRFP_G}(r)$
For example, G_7 in Fig. 6B is not a subgraph of any of the $\mathrm{MRFP_G}(7)$.

i. Consider the graph G'_n (obtained by deleting k edges in Step 3b).
For example, G'_5 for G_5 in Fig. 6A is shown in Fig. 6C.

ii. Obtain a MRFP(n) for G'_n.
For example, a MRFP for G'_5 in Fig. 6C is illustrated in Fig. 6D.

iii. Eliminate extra connections, if exist in the obtained MRFP.
For example, there does not exist extra connections in the MRFP corresponding to G'_5 in Fig. 6C.

iv. Introduce k circulations corresponding to the deleted edges. If possible, insert circulations such that they are adjacent to exterior.
For example, two circulations have been introduced in Fig. 6E corresponding to the edges deleted from G_5 in Fig. 6A.

v. Merge circulations with the rooms, if possible.
For example, a circulation has been merged to room R_3 to have a required floor plan for G_5 in Fig. 6A, as shown in Fig. 6G.

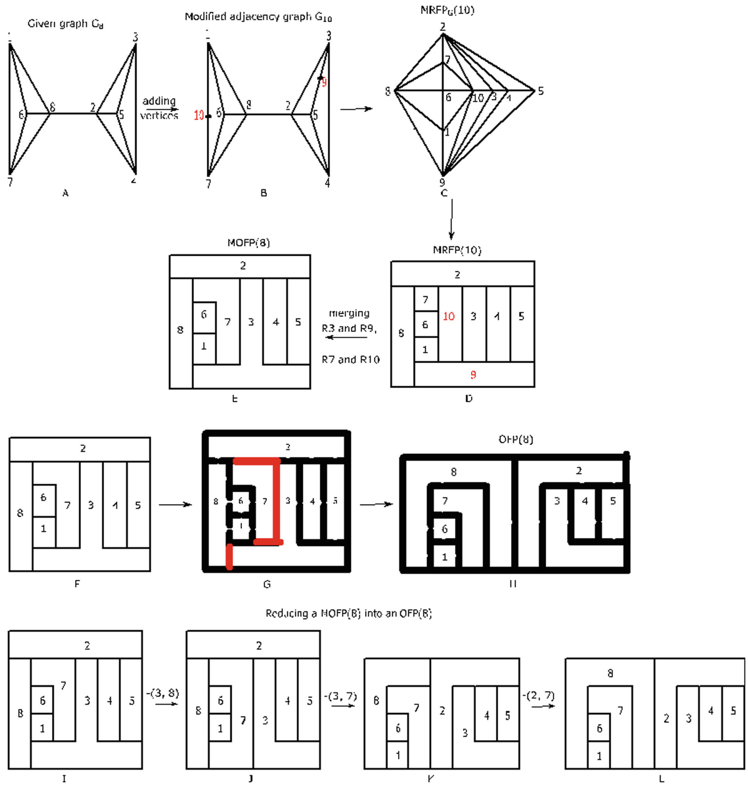

Fig. 5. Deriving an OFP from MOFP corresponding to a given adjacency graph

2.1 Computing Extra Edges

If a given adjacency graph G_n is not a $\mathrm{RFP_G}(n)$, we need to construct an OFP corresponding to it. To have an OFP, we require k, where k is the minimum number of edges of G_n whose deletion results in a graph G'_n such that G'_n is a sub-graph of at least one of the $\mathrm{MRFP_G}(n)$. The steps for computing k are as follows:

1. Compute power set of edge set of G_n and denote it as $P(E(G_n))$, where $E(G_n)$ is the edge set of G_n.
   Let the members of $P(E(G_n))$ are denoted as $S_1, S_2, \ldots, S_{2^m}$.
2. Order $S_1, S_2, \ldots, S_{2^m}$ in decreasing order on the basis of size of each set S_i.
   Let the members of $P(E(G_n))$ in the descending order be $S'_1, S'_2, \ldots, S'_{2^m}$.
3. Let $i = 2$.
4. Choose S'_i.
5. If $S'_i \subseteq E(MRFP_\mathrm{G}(n))$, then $k = m - n(S'_i)$ and edges corresponding to k are $E(G_n) - S'_i$.
6. If condition in Step 5 holds, stop the algorithm otherwise increase i by 1.
7. If $i \leq (2^m)$ go to Step 4 otherwise stop.

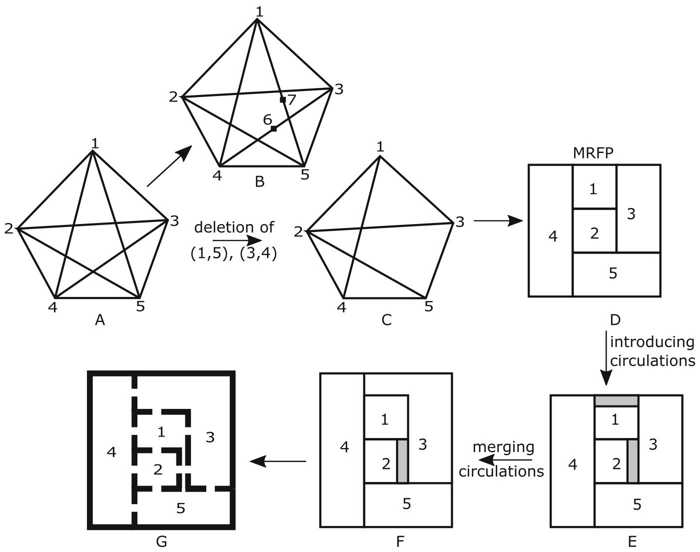

Fig. 6. Deriving a FP with circulations corresponding to a given adjacency graph

3 Conclusion and Future Work

In this paper, we first aim to check the existence of a floor plan corresponding to a given adjacency graph, and if it exists, we aim to construct it.

To the best of our knowledge, there does not exist a mathematical theory for checking the existence of a FP for a given adjacency graph, other than PTPG and MPG. In this paper, we are the first one to categorize the adjacency graphs on the basis of existence of a RFP, or OFP, or none of them. In addition, we propose a method to construct a FP for any given adjacency graph, where we prefer to have a RFP first, then an OFP. If both of them do not exist, then we go for a floor plan with circulations.

We know that in architectural design, a multitude of aspects with different nature need to be considered. In this paper, we are dealing with adjacency requirements in the strict sense only. In the future, we first aim to introduce dimensional constraints for constructing a floor plan. Then, we will cover other aspects, like (aesthetic) composition, style, functionality, access to light, etc.

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