

A Semidefinite Programming Approach to Hybrid Localization using RSSI and TOA

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Abstract—In this paper, we exploit the semidefinite programming and the Cramer Rao lower bound techniques to study the hybrid fusion of RSSI and TOA. A semidefinite program is developed to fuse these two location-dependent parameters. The Cramer Rao lower bound expressions are also developed in order to assess theoretical performances of the RSSI and TOA fusion. In order to evaluate this localization scheme, Monte Carlo simulations are carried out in a generic environment using realistic parameters extracted from an ultra wide band measurement campaign. The semidefinite approach is compared to the weighted least-squares, the maximum likelihood, and the Cramer Rao lower bound for the different schemes (i.e. sole RSSI, sole TOA, and hybrid RSSI+TOA). The importance of the fusion of RSSI and TOA is highlighted while assessing the different factors influencing the positioning accuracy.

Index Terms—Localization, TOA, RSSI, Weighted least square, Semidefinite programming, Hybrid Data Fusion, Maximum likelihood, Cramer Rao Bound.

I. INTRODUCTION

Security, emergency, management, and entertainment are some, but not all, of fields where the location information is being more and more demanded. Nowadays, most of fields use location information in order to perform tasks and to offer valued services. Moreover, the location information becomes even requested by telecommunication networks themselves in order to enhance their functionalities and performances [1].

Wireless communications are, by any measure, the fastest growing segment of the communications industry. Today's wireless applications like cellular phone services or television broadcast are a part of the day to day life of many people. Wireless communication has evolved immensely from the time it was first implemented. The ease of setting up a wireless network, tetherless communication, and low cost of deployment are some of the key reasons for its popularity. Also, the reliability of wireless communication has improved significantly and is reflected in its application to a wide variety of civilian and military fields. The today's landscape of wireless communications is mainly characterized by the coexistence of different technologies (e.g. Bluetooth, WiFi, Zigbee, UWB, Cellular, WiMax, etc). The widespread implementation of these heterogeneous wireless networks make wireless localization a service that is available “anytime” and “anywhere” [2], [3].

A device's position is usually estimated by monitoring a location dependent parameter (LDP) such as received signal strength indicator (RSSI), time of arrival (TOA), time difference of arrival (TDOA), etc, from another device whose location is known [4]. The localization is done by computing

distances from these LDPs and then applying estimation techniques to find the device's position. Different techniques of estimation are defined such as least-squares, maximum likelihood, and convex optimization. The localization accuracy is mainly factor of the LDP measurement nature and accuracy, the wireless standard, and the estimation technique itself [5]. Since each location based service, before being proposed to users, require a minimal positioning accuracy, the localization system should choose the best standards, the best LDPs, and the best estimators, able to perform accurately the requested service.

The expansion, the heterogeneity, and the coexistence of wireless networks are the motivations make it possible for localization systems to implement novel techniques of localization. These techniques use more than one LDP type and we call them “Hybrid Localization Techniques”. This paper presents an application of semidefinite programming (SDP) on the fusion of TOA and RSSI. This technique is then compared to maximum likelihood (ML) and weighted least-squares (WLS) techniques. The enhancement brought by the fusion of RSSI and TOA is also evaluated and Cramer-Rao lower bounds (CRLB) are developed for both hybrid and non-hybrid schemes.

The rest of the document is organized as follows: In section II, the generic hybrid scenario is described. The SDP based estimator is developed in section III. The formulation of the Cramer Rao lower bounds is investigated in section IV. In section V, the performances of the proposed estimator are shown and the importance of the fusion of TOA with RSSI is highlighted. Finally, our concluding remarks are given in section VI.

II. SCENARIO AND ASSUMPTIONS

The assumed scenario here is a situation where the targeted mobile is connected to different anchors and is able to get different LDPs from these anchors. Let q be the total number of all anchors implied in the scenario. Without any loss of generality, assume that the targeted MS can get p RSSIs from anchors with indexes $k \in (1, \dots, p)$ and $q - p$ TOAs from anchors with indexes $k \in (p + 1, \dots, q)$. Fig. 1 depicts an example of hybrid scenario where $p = 4$ and $q = 8$. This plotted scenario will be used for simulations later in this paper. Let $\mathbf{X} = (x, y)$ and $\hat{\mathbf{X}} = (\hat{x}, \hat{y})$ be respectively the actual and estimated MS position and $\mathbf{X}_k = (x_k, y_k)$ the position of the k^{th} anchor.

For measurements, we consider Gaussian statistical models. For TOA, we assume that the measurement is centered on the

true value with a standard deviation σ_k for the k^{th} TOA:

$$c\tau_k \sim \mathcal{N}(d_k, \sigma_k^2) \quad (1)$$

where $d_k = \|\mathbf{X} - \mathbf{X}_k\|_2$ is the actual range between the targeted MS and the k^{th} anchor and c is the speed of light. RSSI is modeled using log-normal shadowing model. This model is given in (2) and represents the k^{th} received power P_k as a random variable centered on the mean received power with a standard deviation of shadowing σ_{shk} . n_p is the propagation exponent, d_0 is a reference distance taken equal to 1 meter, and P_0 is the received power at d_0 .

$$P_k \sim \mathcal{N}(P_0 - 10n_p \log_{10} \left(\frac{d_k}{d_0} \right), \sigma_{shk}^2) \quad (2)$$

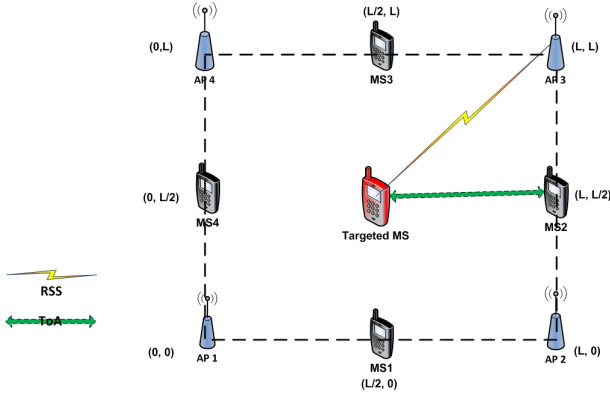


Fig. 1: Generic hybrid scenario.

III. FUSION OF TOA AND RSSI USING SDP

Assuming Gaussian models independence between considered LDP measurements, the likelihood functions are given respectively for RSSI and TOA by [6], [7]:

$$\begin{cases} f_{\text{RSSI}}(\mathbf{X}) = \prod_{k=1}^p \frac{1}{\sqrt{2\pi} S_k} e^{-\frac{(\ln d_k - M_k)^2}{2S_k^2}} \\ f_{\text{TOA}}(\mathbf{X}) = \prod_{k=p+1}^q \frac{1}{\sqrt{2\pi} \sigma_k} e^{-\frac{(c\tau_k - d_k)^2}{2\sigma_k^2}} \end{cases} \quad (3)$$

where S_k and M_k are defined for each k respectively by [6]:

$$S_k = -\frac{\sigma_{shk} \ln 10}{10n_p} \quad (4)$$

$$M_k = \frac{(P_0 - P_k) \ln 10}{10n_p} + \ln d_0 \quad (5)$$

Then, by computing the gradients of these likelihood functions, we obtain easily the different ML estimators for respectively RSSI and TOA [6], [7].

$$\begin{cases} \nabla f_{\text{RSSI}} = \sum_{k=1}^p \frac{1}{S_k^2} \frac{((M_k - S_k^2) - \ln d_k)}{d_k^2} (\hat{\mathbf{X}} - \mathbf{X}_k) = \mathbf{0} \\ \nabla f_{\text{TOA}} = \sum_{k=p+1}^q \frac{1}{\sigma_k^2} \frac{(c\tau_k - d_k)}{d_k} (\hat{\mathbf{X}} - \mathbf{X}_k) = \mathbf{0} \end{cases} \quad (6)$$

Based on the results from [8] and with taking into consideration the measurement variances, we propose to simplify the problem (6) in the form of a minimax approximation for respectively RSSI and TOA. These approximations are

supported by the so-called equivalence between both 2-norm and ∞ -norm. The solutions for these two different problems are given respectively by:

$$\hat{\mathbf{X}} = \underset{\mathbf{X}}{\text{argmin}} \max_{k=1, \dots, p} \frac{1}{\sigma_{shk}} |P_k - P_0 + 10n_p \log_{10} \frac{\|\mathbf{X} - \mathbf{X}_k\|}{d_0}| \quad (7)$$

$$\hat{\mathbf{X}} = \underset{\mathbf{X}}{\text{argmin}} \max_{k=p+1, \dots, q} \frac{1}{\sigma_k} |(c\tau_k)^2 - \|\mathbf{X} - \mathbf{X}_k\|^2| \quad (8)$$

Let us define a $(m+1) \times 1$ vector $\bar{\mathbf{X}} = [\mathbf{X}^T \ 1]^T$ and for each k in $(1, 2, \dots, K)$ a matrix \mathbf{Q}_k by:

$$\mathbf{Q}_k = \begin{bmatrix} \mathbf{I} & -\mathbf{X}_k \\ -\mathbf{X}_k^T & \mathbf{X}_k^T \mathbf{X}_k \end{bmatrix} \quad (9)$$

For RSSI cases, we define β_k by:

$$\beta_k = d_0^{-2} 10^{\frac{P_k - P_0}{5n_p}} \quad (10)$$

Using $\bar{\mathbf{X}}$ and \mathbf{Q}_k , we express the minimax problems defined by (7) and (8) in the form of constrained SDP programs. Hence, we obtain:

$$\begin{aligned} & \underset{\mathbf{t}}{\text{minimize}} \quad \|\mathbf{t}\|_2 \\ & \text{s.t.} \quad \bar{\mathbf{X}}(m+1) = 1 \\ & \quad -\sigma_{shk} \mathbf{t}[k] < \log_{10}(\beta_k \bar{\mathbf{X}}^T \mathbf{Q}_k \bar{\mathbf{X}}) < \sigma_{shk} \mathbf{t}[k] \end{aligned} \quad (11)$$

$$\begin{aligned} & \underset{\mathbf{t}}{\text{minimize}} \quad \|\mathbf{t}\|_2 \\ & \text{s.t.} \quad \bar{\mathbf{X}}(m+1) = 1 \\ & \quad -\sigma_k \mathbf{t}[k] < (c\tau_k)^2 - \bar{\mathbf{X}}^T \mathbf{Q}_k \bar{\mathbf{X}} < \sigma_k \mathbf{t}[k] \end{aligned} \quad (12)$$

\mathbf{t} is a vector of length equal to the number of used LDPs. Now, let us denote $\chi = \bar{\mathbf{X}} \bar{\mathbf{X}}^T$. After applying semidefinite relaxation [8], [9], [10], [11], the problems can be reformulated into:

$$\begin{aligned} & \underset{\chi}{\text{minimize}} \quad \|\mathbf{t}\|_2 \\ & \text{s.t.} \quad \chi \succeq \mathbf{0} \\ & \quad \chi(m+1, m+1) = 1 \\ & \quad -\sigma_{shk} \mathbf{t}[k] < \text{Trace}(\beta_k \mathbf{Q}_k \chi) - 1 < \sigma_{shk} \mathbf{t}[k] \end{aligned} \quad (13)$$

$$\begin{aligned} & \underset{\chi}{\text{minimize}} \quad \|\mathbf{t}\|_2 \\ & \text{s.t.} \quad \chi \succeq \mathbf{0} \\ & \quad \chi(m+1, m+1) = 1 \\ & \quad -\sigma_k \mathbf{t}[k] < (c\tau_k)^2 - \text{Trace}(\mathbf{Q}_k \chi) < \sigma_k \mathbf{t}[k] \end{aligned} \quad (14)$$

$\chi \succeq \mathbf{0}$ denotes symmetric positive semidefinite. Equations (13) and (14) are convex optimization problems. Their global optimal solutions can be found using modern SDP solvers like *CVXOPT* and *CVXMOD* [9].

The SDP program for the fusion of RSSI and TOA can be written as follows:

$$\begin{aligned} & \underset{\chi}{\text{minimize}} \quad \|\mathbf{t}\|_2 \\ & \text{s.t.} \quad \chi \succeq \mathbf{0} \\ & \quad \chi(m+1, m+1) = 1 \\ & \quad -\sigma_{shk} \mathbf{t}[k] < \text{Trace}(\beta_k \mathbf{Q}_k \chi) < \sigma_{shk} \mathbf{t}[k] \\ & \quad -\sigma_k \mathbf{t}[k] < (c\tau_k)^2 - \text{Trace}(\mathbf{Q}_k \chi) < \sigma_k \mathbf{t}[k] \end{aligned} \quad (15)$$

IV. CRLBS OF RSSI, TOA, AND RSSI+TOA SCHEMES

The Fisher information matrix (FIM) for respectively RSSI- and TOA- based localization schemes is defined by:

$$\mathbf{J}_{\text{RSSI}} = E \left[\nabla \mathbf{f}_{\text{RSSI}} \cdot \nabla \mathbf{f}_{\text{RSSI}}^T \right] \quad (16)$$

$$\mathbf{J}_{\text{TOA}} = E \left[\nabla \mathbf{f}_{\text{TOA}} \cdot \nabla \mathbf{f}_{\text{TOA}}^T \right] \quad (17)$$

where $\nabla \mathbf{f}_{\text{RSSI}}$ and $\nabla \mathbf{f}_{\text{TOA}}$ are given in 6.

Since calculations are similar in the three cases, we will only present the case of TOA. The CRLB of RSSI based localization scheme can be obtained similarly as in the TOA case. Using the expression of $\nabla \mathbf{f}_{\text{TOA}}$ defined in (6) and assuming independence between TOA measurements, the FIM given by (17) can be written as [7]:

$$\mathbf{J}_{\text{TOA}} = E \left[\sum_{k=p+1}^q \frac{1}{\sigma_k^4} \frac{(c\tau_k - d_k)^2}{d_k^2} (\mathbf{X} - \mathbf{X}_k) (\mathbf{X} - \mathbf{X}_k)^T \right] \quad (18)$$

Developing this expression leads to:

$$\mathbf{J}_{\text{TOA}} = \sum_{k=p+1}^q \frac{1}{\sigma_k^2} \begin{bmatrix} \frac{(x-x_k)^2}{d_k^2} & \frac{(x-x_k)(y-y_k)}{d_k^2} \\ \frac{(x-x_k)(y-y_k)}{d_k^2} & \frac{(y-y_k)^2}{d_k^2} \end{bmatrix} \quad (19)$$

The CRLB, defined as the trace of the inverse of the FIM [12], is then given by:

$$\sigma_{\text{TOA}}^2 = \frac{\sum_{k=p+1}^q \frac{1}{\sigma_k^2}}{\frac{1}{2} \sum_{k=p+1}^q \sum_{l=p+1}^q \frac{((x-x_k)(y-y_l) - (x-x_l)(y-y_k))^2}{\sigma_k^2 \sigma_l^2 d_k^2 d_l^2}} \quad (20)$$

Let φ_k be the angle between the d_k and the line $(y=0)$. We get $\cos(\varphi_k) = \frac{x-x_k}{d_k}$ and $\sin(\varphi_k) = \frac{y-y_k}{d_k}$. Introducing that in (20), using the trigonometric equality $\cos(\varphi_k) \sin(\varphi_l) - \cos(\varphi_l) \sin(\varphi_k) = \sin(\varphi_l - \varphi_k)$, and defining $\varphi_{lk} = \varphi_l - \varphi_k$ the angle between d_k and d_l lead to the simplified expression of the CRLB.

$$\sigma_{\text{TOA}}^2 = \frac{\sum_{k=p+1}^q \frac{1}{\sigma_k^2}}{\sum_{k=p+1}^q \sum_{l=p+1}^q \frac{\sin^2(\varphi_{lk})}{2\sigma_k^2 \sigma_l^2 d_k^2 d_l^2}} \quad (21)$$

Similarly to (19 and 21), the FIM and the CRLB of RSSI case are respectively given by:

$$\mathbf{J}_{\text{RSSI}} = \sum_{k=1}^p \frac{(1+S_k^2)}{S_k^2} \begin{bmatrix} \frac{(x-x_k)^2}{d_k^4} & \frac{(x-x_k)(y-y_k)}{d_k^4} \\ \frac{(x-x_k)(y-y_k)}{d_k^4} & \frac{(y-y_k)^2}{d_k^4} \end{bmatrix} \quad (22)$$

$$\sigma_{\text{RSSI}}^2 = \frac{\sum_{k=1}^p \frac{(1+S_k^2)}{S_k^2 d_k^2}}{\sum_{k=1}^p \sum_{l=1}^p \frac{(1+S_k^2)(1+S_l^2) \sin^2(\varphi_{lk})}{2S_k^2 S_l^2 d_k^2 d_l^2}} \quad (23)$$

In the case of heterogeneous scenario and assuming independence between all measurements, the FIM can be defined as the sum of the FIMs of different LDP implied in the scenario. That is, for a scenario that implies RSSI and TOA:

$$\mathbf{J}_{\text{RSSI+TOA}} = \mathbf{J}_{\text{RSSI}} + \mathbf{J}_{\text{TOA}} \quad (24)$$

Consequently, the CRLB is given by :

$$\sigma_{\text{RSSI+TOA}}^2 = \text{tr} \left\{ (\mathbf{J}_{\text{RSSI+TOA}})^{-1} \right\} \quad (25)$$

The expressions of these different CRLBs show that the positioning accuracy depends on the nature, the number, and

the precision of used LDPs. The accuracy depends also on the position of the targeted device with respect to anchors: i.e. the relative geometry of the localization problem. In fact, this geometry is represented in the CRLB expressions by φ_{lk} . This is very interesting when the system has to choose an additional LDP in order to enhance the positioning accuracy. It must choose the LDP that minimizes the CRLB.

V. SIMULATION RESULTS AND DISCUSSIONS

In order to evaluate the performances of the proposed SDP scheme and to show the importance of hybrid fusion of RSSI and TOA, Monte-Carlo simulations have been carried out within the scenario described in Fig.1. The statistical models used in simulations are extracted from an UWB measurement campaign [13]. A summary of these models is given in Table I:

TABLE I: Statistical models used for simulations.

RSSI	$P_0 = -36.03 \text{ dB}_m, n_p = 2.38, d_0 = 1 \text{ m}, \sigma_{shk} = 3.98 \text{ dB}_m$
TOA	$\sigma_k = 1.142 \text{ m}$

A. Performances of the SDP technique

In order to evaluate the proposed SDP technique, we compare the positioning accuracy achieved by this technique to those achieved by the weighted least-squares (WLS) [14] and the maximum likelihood (ML) techniques and also to the CRLB. For 1000 drawn iterations of the targeted MS position, the cumulative distribution functions (CDFs) of absolute positioning error are plotted in Fig. 2-(a)-to-(c) for respectively the different estimators (WLS, SDP, and ML) in both non-hybrid (i.e. sole RSSI and sole TOA) and hybrid (i.e. RSSI+TOA) cases. These CDFs are also compared to the CDF of the root-square of the CRLB. All these CDFs are computed using 4 RSSIs and 4 TOAs as described in Fig.1.

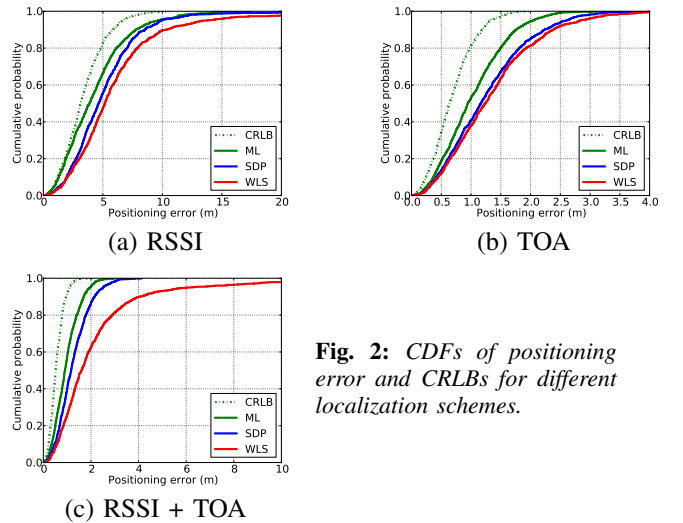


Fig. 2: CDFs of positioning error and CRLBs for different localization schemes.

The comparison between these CDFs reveals that the SDP technique outperforms the WLS technique but does not reach the performances of the ML technique. This is mainly due

to both the linearization and the semidefinite approximation of the localization problem. When doing these steps, some informations are lost which affects the estimation result. In contrast, these informations are still considered by the ML non linearized objective function. Nevertheless, the ML technique may suffer from some outliers (For example in Fig. 2-(c) where the CDF tends toward infinity because of the presence of singularities which result in large positioning errors). These outliers are due to the non convexity of the ML objective functions, a problem that is overcome by the SDP technique.

B. Importance of the hybrid fusion of RSSI and TOA

Fig.3 plots, using all the anchors defined in the generic scenario, the CDFs of the two non-hybrid schemes (4 RSSI and 4 TOA respectively) and the hybrid scheme (4 RSSI + 4 TOA). This figure shows that adding TOAs to RSSIs drastically enhances the positioning accuracy (i.g. at 80%, we have an average gain of 4.5m). By contrast, adding RSSIs to TOA does not enhance the positioning accuracy. This is because of the high precision and the sufficient amount of used TOAs. Hence, the use of RSSIs is justified by one or more of the following facts:

- 1) The number of TOAs is not sufficient to perform localization (i.e. less than 3 TOAs in 2D scenario);
- 2) The precision of TOAs is not accurate;
- 3) RSSIs are usually available without additional cost.

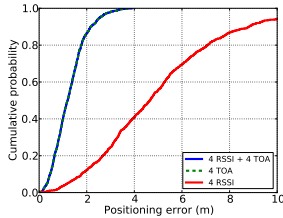


Fig. 3: Contribution of RSSIs and TOAs.

Since RSSIs are usually available on each radio link without any additional cost, we assume here that the four RSSIs defined in the generic scenario are usually available and we gradually increment the number of TOAs. We plot then, in Fig.4-(a), the CDFs of the absolute positioning error for all obtained schemes (4 RSSIs + n TOAs, $n=1,2,3$, or 4). Fig.4-(b) plots the average absolute positioning error with respect to the number of added TOAs. These two figures show that the fusion of TOA, which are usually very precise, with RSSIs enhance the positioning accuracy. The reached accuracy is factor of the number of added TOA. Hence, the system may add only the sufficient number of TOAs which allow to reach the accuracy requested by the service or the application. The objective must be to reduce TOA ranging attempts because they may cause network congestion and reduce network throughput.

In order to show the effect of the LDPs precisions on the positioning accuracy and the fusion of RSSI and TOA, we plot the average positioning error as a function of the shadowing standard deviation in Fig.5-(a) and as a function of the TOA ranging error in Fig.5-(b). The first figure shows that the

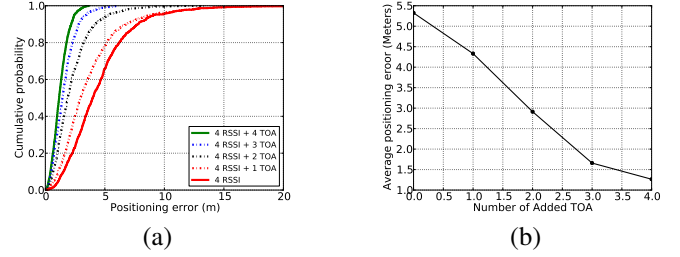


Fig. 4: Effect of gradual addition of TOAs to 4 RSSIs: (a)- CDFs, (b)- Average error.

adding of TOAs to RSSIs is as beneficial as the shadowing is more severe which is the case especially in indoor and complex environments. The second figure itself reveals that adding RSSI to TOA can be of interest when the precision of TOA is lower.

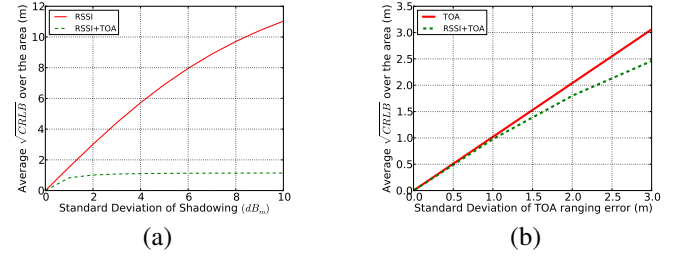


Fig. 5: Effect of the LDP precision on the positioning accuracy: (a)- RSSI Shadowing, (b)- TOA ranging.

The CRLBs for RSSI, TOA, and RSSI+TOA are shown respectively in Fig. 6. These figures show the geometric distribution of the CRLB over the assumed area. The comparison between these three figures obviously shows that time based technique (TOA) has better overall localization accuracy than power based technique (RSSI). Moreover, these three figures show that the CRLB depends on the position of targeted MS with respect to the configuration of anchors. This dependency is quite different from one LDP to another. In the RSSI case, the positioning accuracy is degraded as the MS approaches an anchor or an edge that links two anchors. In the case of TOA, the difficulties of localization are located around the anchors and the positioning accuracy is as better as the MS moves toward the center of the area (i.e. equidistant to all ANs). Fig. 6-(c) shows that the overall positioning accuracy is enhanced when fusing these two LDPs. Table II gives the average CRLB values for each localization scheme. Like in Fig.3, this table confirms the fact that when TOAs have good precision the addition of RSSIs is marginal (a gain of only 10 cm when adding RSSIs to TOAs, a gain of 3.313 meters when adding TOAs to RSSI).

TABLE II: Average $\sqrt{\text{CRLB}}$ values over the L -by- L area for non hybrid and hybrid schemes.

LDP	Average $\sqrt{\text{CRLB}}$ (m)
RSSI	4.378
TOA	1.165
RSSI+TOA	1.065

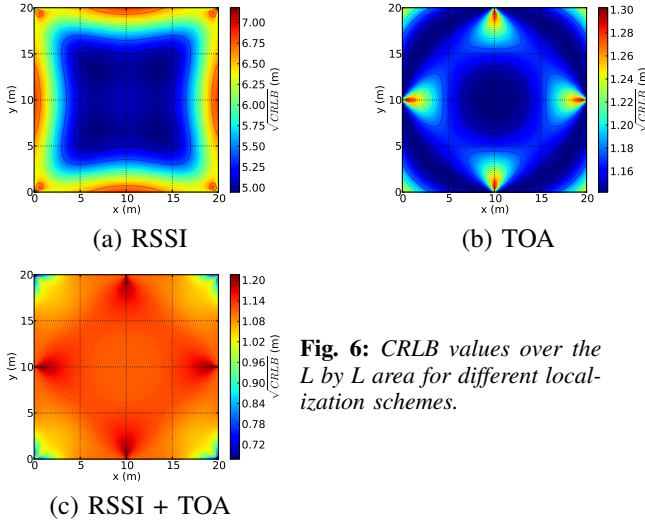


Fig. 6: CRLB values over the L by L area for different localization schemes.

To conclude this section, the Monte-Carlo simulations and the CRLBs reveals that:

- 1) The SDP technique is able to fuse RSSI and TOA, outperforms the WLS technique, but does not reach the ML and the CRLB
- 2) Fusion of RSSI and TOA enhances the localization accuracy compared to sole RSSI or TOA based techniques
- 3) Since TOAs are more reliable than RSSI, the addition of TOA to RSSI enhances drastically the positioning accuracy while the addition of RSSIs to TOAs is either marginal or useless
- 4) The enhancement performed by the fusion of RSSI a TOA is factor of their precisions
- 5) The CRLBs show that the positioning accuracy depends also on the position of the targeted MS with respect to anchors, this is in relation with geometric dilution of precision

VI. CONCLUSION

This paper presented an application of semidefinite programming technique to the hybrid localization based on the fusion of RSSI and TOA. The semidefinite programs have been developed for sole RSSI, sole TOA, and the fusion of RSSI and TOA. Using Monte Carlo simulation within a generic scenario and using realistic radio parameters, the proposed techniques are compared to the WLS and the ML techniques. The proposed technique outperforms the WLS for both non-hybrid and hybrid cases and approaches the ML and the CRLB which is also developed in order to assess the theoretical performances of different schemes. Simulations and CRLBs show that RSSIs are usually not reliable and that their contribution (in hybrid schemes) is factor of the precision and the number of TOAs. If the sufficient number of TOAs having a good precision (i.g. from UWB) is available, there is no interest to use RSSIs. Otherwise, the use of RSSIs is strongly recommended because of their free availability. Nevertheless, more enhanced modeling of RSSIs (i.g. multi-slope models to better model shadowing) and more enhanced RSSI based

ranging techniques may make RSSIs more interesting for localization.

ACKNOWLEDGMENT

This work has been performed in the framework of the ICT project ICT-248894 WHERE2, which is partly funded by the European Union.

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