Sensor fusion for relative altimetry using a hybrid Gaussian mixture filter

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***Abstract*—We consider the problem of merging measurements when the sensors are likely to be affected by multiple malfunctions. An hybrid model is introduced which describes the dynamic behavior of the various sensors not only under each operating mode, but also during mode transitions.**
The data fusion problem is then written within the Bayesian probabilistic framework, as an estimation problem. Its optimal solution can be approximated numerically in multiple ways. The Gaussian mixture approach is well adapted because of the multiple hypothesis context. This led us to the development of an hybrid Gaussian mixture filter.
The application we dealt with is the improvement of the relative altitude sensing function onboard aircraft. It aims to deliver the height of the aircraft above the ground. Currently this function is only supported by the radio-altimeters system along with specific thresholding and voting logics. Our approach leads to a joint estimation of the relative height and of the sensors operating modes. It thus allows to combine various sensors while monitoring their modes easily. We particularly focus on the case of two radio altimeters, which is the current situation.

I. Introduction

Within the framework of the continuous improvement of aircrafts operational reliability, and in the context of increase in the air traffic, AIRBUS intends to increase the reliability and the regularity of its operations, in particular during critical flight phases such as the approach, landing and takeoff. Indeed, during these flight phases, the Radio Altimeter (RA) system is the only onboard sensor which directly measures the height of the aircraft above the ground. And this height is used in automatic guidance loops (like the flare function) and in flight control laws (like the anti-tailstrike function) of the aircraft.

Therefore, in the frame of new navigation architecture activities, we study solutions allowing to increase the reliability of the critical parameter "Height of the aircraft above the ground", currently measured by the Radio Altimeter. Our main objective is to remove the undesired events related to misbehaviour of the Radio Altimeter [1] which could have a significant operational impact during critical flight phases (like go-around leading to a significant increase of crew workload during landing phase). We also try to take into account new operational needs, for which an architecture only based on Radio Altimeter could not achieve the expected targets. And finally we want to enhance the estimation of the relative altitude h_{ra} with respect to accuracy, integrity and availability.

Classical solutions are based on material redundancy (architectures with 2 or 3 RA), and on thresholding and voting logics for the monitoring. However they are highly dependent on the involved components and no longer valid after the lost of one sensor.

The approach which has been followed is to identify new sensors (not affected by the same misbehaviors) that could participate to the relative altitude function and study how to fuse this new information with the RA one. These new sensors could be already available on board, but not yet used by the relative altitude function (like INS, barometer, GPS). They could be also new technologies applied to classical sensing (like Digital or Single Antenna RA). Or they could be new sensors (like LIDAR for example).

Fusing the various (but related) information obviously improve the accuracy of the h_{ra} estimation, at least as long as no sensor dysfunction occurs. However it is far much more difficult to improve availabilty, reliability and integrity. This requires to detect and manage sensor dysfunctions. In order to solve this problem we proposed to develop an hybrid sensor model dedicated to data fusion.

In section II the hybrid modeling approach dedicated to sensor fusion is described.

The first part of the model is the description of the environment where the sensors are embedded, that is to say the aircraft trajectory and the ground variations. It is described by a continuous state vector. Various models may be used, but a tradeoff between accuracy and complexity shall be found.

Then, for each sensor, the hybrid model involves a (vector) continuous state x and a (scalar) discret state s. This discret state represents the sensor operating mode. For example the RA operating mode may be 'nominal', 'biased', 'frozen', 'fixed', 'non computed data', etc... Note that this description is based on dysfunction symptoms. The dynamic of this discret state is simply modeled by a transition probability table. The continuous state x is used to model the dynamic of the measurement within each operating mode, but also when mode transition occurs. This representation of the dynamical behaviour along mode changes seems to be new. It will allow to fasten the detection of dysfunction occurences, because it does not assume any frozen operating mode.

Based on this hybrid model structure we developed a data fusion algorithm that delivers an estimate of the relative altitude h_{ra} but also provides estimates of the operating modes of all involved sensors. Let us recall that within the Bayesian framework which is adopted here, all unknowns are modeled as probability density functions (pdf). The formal expression of the optimal solution to this estimation problem is recalled.

However only numerical approximations may be practically carried out. We choosed to use a Gaussian Mixture approximation (see section III) mainly because of our multi-hypothesis context. Furthermore the multi-modal probability density functions are also usefull to model some specific situations (for example an unknown bias significantly different from zero). Our data fusion algorithm is thus an Hybrid Gaussian Mixture Filter (HGMF).

The key points of our estimation algorithm are as follow:

• It compute recursively along with time a multi-Gaussian approximation of the pdf of all the states variables (continuous and discret).

• Based on these computed pdf approximations, several state estimates can be obtained at any time. We mainly consider the least mean squares estimation for the continuous states, and the maximum a posteriori estimates of the discret ones.

• The use of a multi-modal representation of the pdf allows to monitor operating modes trends, to indicate potential failures, to validate / invalidate these hypothesis with mastered error probabilities.

• The signal status message (SSM) associated to each sensor may be used to process the current sensor measurement or not (when $SSM =' NCD'$, i. e. No Computed Data). This asynchronous behavior can also handle measurements with different sampling rates.

• The algorithm we develop is modular in the sense that the models of the various sensors are independent of each other. The environment model used, however, must be developed in relation to the set of sensors available. It must allow to express the dynamic redundancy that exists between measures, and also ensure observability.

The application of this methodology to our relative altimetry problem is presented in section IV. The simulator used to clear the fusion algorithm is briefly presented and data fusion results are commented.

II. HYBRID SYSTEM MODELING AND FILTERING

An hybrid system may be defined as a dynamic stochastic system having an hybrid state, that is a continuous component $x_t \in \mathbb{R}^n$ among with a discrete component $s_t \in \{1, \ldots, m\}$. In most cases, one makes the assumption that systems are Markovian with discrete time $t \in \mathbb{N}$. In theses cases, one has to consider only the so-called transition probability[1]:

$$p\left(x_t, s_t = i | x_{t-1}, s_{t-1} = j\right)$$

[1]This writing is some what abusive as the pdf of a discrete random variable cannot be defined. A better writing, but some what heavy, should be $\mathbb{P}\left(dx_t, s_t = i | x_{t-1}, s_{t-1} = j\right)$.

Addressing filtering issues, one considers a memoryless observation process y_t defined by its probability density function (pdf): $p\left(y_t | x_t, s_t = i\right)$. Using the Bayes rule, one can rewrite the transition pdf as follows

$$\begin{aligned}
& p\left(x_t, s_t = i | x_{t-1}, s_{t-1} = j\right) \\
= \;& p\left(s_t = i | x_{t-1}, s_{t-1} = j\right)\; p\left(x_t | x_{t-1}, s_t = i, s_{t-1} = j\right) \\
= \;& p\left(s_t = i | s_{t-1} = j\right)\; p\left(x_t | x_{t-1}, s_t = i, s_{t-1} = j\right)
\end{aligned}$$

assuming, although it is not necessary, that the discrete variable transitions are independent from the continuous state x_{t-1} and that the continuous transition probability admits a pdf.

Note that, unlike classical definitions, one allows the transition pdf $p\left(x_t | x_{t-1}, s_t = i, s_{t-1} = j\right)$ to depend on both s_{t-1} and s_t. Indeed, hybrid systems are often defined using state equations as follows [2]:

$$\begin{aligned}
x_t &= f\left(x_{t-1}, s_t, w_t\right) \\
y_t &= h\left(x_t, s_t, v_t\right)
\end{aligned}$$

where w_t and v_t are white noises, a necessary condition to generate a Markovian process. This representation is clearly less general that ours and it will appear in the sequel that it is not adapted to our application issue since we are interested in the detection of mode transitions.

In the classical (continuous) Bayesian estimation scheme, the aim is to compute the a posteriori pdf $p\left(x_t | y_{0:t}\right)$ where $y_{0:t} \triangleq \{y_0, \ldots, y_t\}$. In our context, the aim is to compute the hybrid pdf $p\left(x_t, s_t = i | y_{0:t}\right)$ from the knowledge of $p\left(x_{t-1}, s_{t-1} = j | y_{0:t-1}\right)$. The update of this a posteriori pdf is made using the classical prediction/correction steps as follows:

• **Prediction**: the so-called Chapman-Kolmogorov equation leads to:

$$\begin{aligned}
p\left(x_t, s_t = i | y_{0:t-1}\right) &= \sum_{j=1}^{m} \int p\left(x_t, x_{t-1}, s_t = i, s_{t-1} = j\right) dx_{t-1} \\
&= \sum_{j=1}^{m} p\left(s_t = i | s_{t-1} = j\right) \int p\left(x_t | x_{t-1}, s_t = i, s_{t-1} = j\right) \\
&\quad \times\, p\left(x_{t-1}, s_{t-1} = j | y_{0:t-1}\right) dx_{t-1}
\end{aligned} \tag{1}$$

• **Correction**: the so-called Bayes rule leads to:

$$p\left(x_t, s_t = i | y_{0:t}\right) = \frac{p\left(y_t | x_t, s_t = i\right) p\left(x_t, s_t = i | y_{0:t-1}\right)}{p\left(y_t | y_{0:t-1}\right)} \tag{2}$$

III. GAUSSIAN MIXTURE FILTERING

A Gaussian mixture (GM) is a pdf defined as a weighted sum of Gaussian pdf [3]:

$$p\left(x\right) = \sum_{k=1}^{N} \rho^k\, \Gamma\left(x - \hat{x}^k, P^k\right)$$

where Γ stands for the Gaussian pdf

$$\Gamma\left(x, P\right) = \frac{1}{\sqrt{\left(2\pi\right)^n \left|P\right|}} \exp\left(-\frac{1}{2} x^T P^{-1} x\right)$$

The Gaussian mixture filtering (GMF) technique is well adapted to linear systems with non Gaussian white noises

$$\begin{aligned} x_t &= F x_{t-1} + w_t \\ y_t &= H x_t + v_t \end{aligned}$$

Indeed, one shows that, if the prior is a GM and if all white noises are GM, then the a posteriori pdf is also a GM.

$$p\left(x_t|y_{0:t}\right) = \sum_{k=1}^{N_t} \rho_t^k \, \Gamma\left(x_t - \hat{x}_{t|t}^k, P_{t|t}^k\right)$$

The Gaussian pdf components are obtained by computing N_t Kalman filters in parallel.

This technique can be extended easily to non linear systems providing the use of extended Kalman filters. In the case of linear systems, the GM filter can be seen as a particulate filter, each of the Gaussian pdf being viewed as a set of particles. In [4] the efficiency of the GM filtering with respect to the particle approach has already been pointed out for terrain-aided navigation.

Note that if at time $t-1$ the a posteriori pdf is represented by N_{t-1} Gaussian pdf and if the numbers of Gaussian pdf representing the noises w_t and v_t are p and q respectively, then the number of Gaussian pdf needed at time t is equal to $N_t = N_{t-1} \times p \times q$. That is the reason why a selection process must be used to manage complexity.

In our context, the same scheme can be used assuming that

$$\begin{aligned} p\left(x_t|x_{t-1}, s_t = i, s_{t-1} = j\right) &= \Gamma\left(x_t - F^{i,j} x_{t-1}, Q^{i,j}\right) \\ p\left(y_t|x_t, s_t = i\right) &= \Gamma\left(y_t - H^i x_t, R^i\right) \end{aligned}$$

Here, for clarity, one considers only one Gaussian pdf for each discrete transition. It is straightforward to expend this to a GM.

Assume that at time $t-1$, for each discrete state, the a posteriori pdf is defined by a Gaussian mixture as follows:

$$\begin{aligned} & p\left(x_{t-1}, s_{t-1} = j|y_{0:t-1}\right) \\ &= \sum_{k=1}^{N_{t-1}^i} \rho_{t-1}^{j,k} \, \Gamma\left(x_{t-1}^j - \hat{x}_{t-1|t-1}^{j,k}, P_{t-1|t-1}^{j,k}\right) \end{aligned}$$

The filtering process leads then to the following prediction / correction / selection steps:

• **Prediction.** Equation 1 writes as

$$p\left(x_t, s_t = i|y_{0:t-1}\right) = \sum_{j=1}^{m} \sum_{k=1}^{N_{t-1}^i} p\left(s_t = i|s_{t-1} = j\right) \rho_{t-1}^{j,k}$$

$$\int \Gamma\left(x_t - F^{j,i} x_{t-1}, Q^{j,i}\right) \Gamma\left(x_{t-1} - \hat{x}_{t-1|t-1}^{j,k}, P_{t-1|t-1}^{j,k}\right) dx_{t-1}$$

The Gaussian convolution may be rewritten (Kalman filter formulae) $\Gamma\left(x_t - \hat{x}_{t|t-1}^{i,j,k}, P_{t|t-1}^{i,j,k}\right)$ with

$$\begin{aligned} \hat{x}_{t|t-1}^{i,j,k} &= F^{j,i} \hat{x}_{t-1|t-1}^{j,k} \\ P_{t|t-1}^{i,j,k} &= F^{j,i} P_{t-1|t-1}^{j,k} \left(F^{j,i}\right)^T + Q^{j,i} \end{aligned}$$

As a consequence, the predicted pdf takes the following form

$$p\left(x_t, s_t = i|y_{0:t-1}\right) = \sum_{j=1}^{m} \sum_{k=1}^{N_{t-1}^i} \tilde{\rho}_{t-1}^{i,j,k} \, \Gamma\left(x_t - \hat{x}_{t|t-1}^{i,j,k}, P_{t|t-1}^{i,j,k}\right)$$

with $\tilde{\rho}_{t-1}^{i,j,k} = p\left(s_t = i|s_{t-1} = j\right) \rho_{t-1}^{j,k}$, which is also a GM. Let us rearrange this sum and note

$$p\left(x_t, s_t = i|y_{0:t-1}\right) = \sum_{k=1}^{m \times N_{t-1}^i} \tilde{\rho}_{t-1}^{i,k} \, \Gamma\left(x_t - \hat{x}_{t|t-1}^{i,k}, P_{t|t-1}^{i,k}\right)$$

• **Correction.** Equation 2 leads to

$$\begin{aligned} p\left(x_t, s_t = i|y_{0:t}\right) \quad \propto \quad & \sum_{k=1}^{m \times N_{t-1}^i} \Gamma\left(y_t - H^i x_t, R^i\right) \tilde{\rho}_{t-1}^{i,k} \\ & \times \Gamma\left(x_t - \hat{x}_{t|t-1}^{i,k}, P_{t|t-1}^{i,k}\right) \end{aligned}$$

Again, using the Kalman filter formulae, each Gaussian pdf product may be rewritten $\Gamma\left(y_t - H^i \hat{x}_{t|t-1}^{i,k}, \Sigma_t^{i,k}\right) \times \Gamma\left(x_t - \hat{x}_{t|t}^{i,k}, P_{t|t}^{i,k}\right)$where

$$\begin{aligned} \hat{x}_{t|t}^{i,k} &= \hat{x}_{t|t-1}^{i,k} + K_t^{i,k} \left(y_t - H^i \hat{x}_{t|t-1}^{i,k}\right) \\ \Sigma_t^{i,k} &= H^i P_{t|t-1}^{i,k} \left(H^i\right)^T + R^i \\ K_t^{i,k} &= P_{t|t-1}^{i,k} \left(H^i\right)^T \left(\Sigma_t^{i,k}\right)^{-1} \\ P_{t|t}^{i,k} &= P_{t|t-1}^{i,k} - K_t^{i,k} H^i P_{t|t-1}^{i,k} \end{aligned}$$

The a posteriori pdf is then proportional to

$$p\left(x_t, s_t = i|y_{0:t}\right) \propto \sum_{k=1}^{m \times N_{t-1}^i} \bar{\rho}_t^{i,k} \, \Gamma\left(x_t^i - \hat{x}_{t|t}^{i,k}, P_{t|t}^{i,k}\right)$$

with $\bar{\rho}_t^{i,k} = \Gamma\left(y_t - H^i \hat{x}_{t|t-1}^{i,k}, \Sigma_t^{i,k}\right) \tilde{\rho}_{t-1}^{i,k}$
Finally, the normalized a posteriori pdf is

$$p\left(x_t, s_t = i|y_{0:t}\right) = \sum_{k=1}^{m \times N_{t-1}^i} \rho_t^{i,k} \, \Gamma\left(x_t - \hat{x}_{t|t}^{i,k}, P_{t|t}^{i,k}\right)$$

with $\rho_t^{i,k} = \frac{\bar{\rho}_t^{i,k}}{\sum_k \bar{\rho}_t^{i,k}}$

• **Selection.** Obviously, due to limited computational capacity, the maximal number of Gaussian components must be fixed. We chose to simply prune the Gaussian pdf with lowest weights.

IV. Application to relative altimetry filtering

A. *Relative altimetry problematic*

We study solutions allowing to increase the reliability of the critical parameter "Height of the aircraft above the ground", currently measured by the Radio Altimeter. This parameter is used during critical flight phases like takeoff, approach and landing. Our main objective is to remove the undesired events related to misbehaviour of the Radio Altimeter.

We identified new sensors that could participate to the relative altitude function and studied how to fuse this new information with the RA.

Fusing the various (but related) information obviously improve the accuracy of the h_{ra} estimation. To improve availabilty, reliability and integrity requires to detect and manage sensor dysfunctions. In order to solve this problem we applied the data fusion algorithm presented just before. An hybrid sensor model dedicated to data fusion was improved and we developed a data fusion algorithm that delivers an estimate of the relative altitude h_{ra} but also provides estimates of the operating modes of all involved sensors.

The description of the environment where the sensors are embedded is necessary. Then, for each sensor, the hybrid model is built. All the sensor operating modes are described with dysfunction symptoms and a probability table which describe the possibilities of transition between all the operating modes. This modeling is disclosed in part in the following paragraph. Then the result of our approach on chosen scenarios are commented.

B. Modeling

First, one has to model the aircraft behavior together with the altitude of the ground below the aircraft. The simplest model of the aircraft altitude should be

$$\begin{aligned} z_t^a &= z_{t-1}^a + v_{t-1}^z \Delta t \\ v_t^z &= v_{t-1}^z + \varepsilon_t^v \end{aligned}$$

where z_t^a stands for the aircraft altitude, v_t^z for its vertical speed, Δt the sampling period and ε_t^v a Gaussian white noise. Let us define the aircraft state $x_t^a = [z_t^a, v_t^z]^T$. One then has

$$p\left(x_t^a | x_{t-1}^a\right) = \Gamma\left(x_t^a - F^a x_{t-1}^a, Q^a\right)$$

with $F^a = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}$, $Q^a = \begin{bmatrix} 0 & 0 \\ 0 & (\sigma^v)^2 \end{bmatrix}$

Concerning the altitude of the ground below the aircraft, one may consider that it behaves like a Brownian motion when the ground is smooth and that, sometimes, there are a jump due, for example, to a flight over a cliff or buildings. This behavior can be modeled by a GM as follows

$$p\left(z_t^s | z_{t-1}^s\right) = (1 - \lambda \Delta t)\, \Gamma\left(z_t^s - z_{t-1}^s, q^s\right) + \lambda \Delta t \Gamma\left(z_t^s - z_{t-1}^s, Q^s\right)$$

where λ stands for the frequency of jumps and where $q^s \ll Q^s$.

For each RA measurement μ_t^k, one has to introduce a discrete state s_t^k representing its own operating mode. Let us say that $s_t^k = 1$ when the RA is nominal and $s_t = 2$ when a bias is present. The bias is assume to have the mean $\bar{b}$, the standard deviation $\bar{b}$ and to fluctuate slowly with a standard growth σ^b. Let us introduce the all state X_t including the aircraft model, the ground model and the RA models (the biases). One then have:

1) Nominal mode: $s_{t-1}^k = 1$, $s_t^k = 1$. The observation is faithful:

$$p\left(\mu_t^k | X_t, s_t^k = 1\right) = \Gamma\left(\mu_t^k - \left(z_t^a - z_t^s\right), R\right)$$

2) Bias advent $s_{t-1}^k = 1$, $s_t^k = 2$. An unknown bias b_t^k occurred:

$$p\left(b_t^k | X_{t-1}, s_t^k = 2, s_{t-1}^k = 1\right) = \Gamma\left(b_t^k - \bar{b}, (\bar{b})^2\right)$$

$$p\left(\mu_t^k | X_t, s_t^k = 2\right) = \Gamma\left(\mu_t^k - \left(z_t^a - z_t^s\right) - b_t^k, R\right)$$

3) The bias is still present and slowly fluctuating $s_{t-1}^k = 2$, $s_t^k = 2$

$$p\left(b_t^k | X_{t-1}, s_t^k = 2, s_{t-1}^k = 2\right) = \Gamma\left(b_t^k - b_{t-1}^k, \left(\sigma^b\right)^2\right)$$

$$p\left(\mu_t^k | X_t, s_t^k = 2\right) = \Gamma\left(\mu_t^k - \left(z_t^a - z_t^s\right) - b_t^k, R\right)$$

4) The bias have disappeared $s_{t-1}^k = 2$, $s_t^k = 1$. The observation returns to nominal

$$p\left(\mu_t^k | X_t, s_t^k = 1\right) = \Gamma\left(\mu_t^k - \left(z_t^a - z_t^s\right), R\right)$$

It appears clearly here our need to differenciate the transitions 2 and 3. Indeed, the transition pdf of biais do not depends only on s_t but also on s_{t-1}.

C. Simulation results

We consider a relative altitude system that is made of two RA. To illustrate the filtering performance, three scenarios are discussed:

• The first scenario deals with a takeoff phase. The output of one of the RA is kept locked on a -6ft value during the first part of the flight. This dysfunction has been observed in case of heavy rain, when the water covers the aircraft fuselage like a skin at the RA antennas. The measured distance is almost zero, but the output is -6ft because of the offset which is always added to compensate for the landing gear heigth.

• Within the second scenario we consider a landing phase. One RA output signal is frozen during 15 seconds when approching the runway threshold. This failure may appear as a result of system degradation. The measured height keeps constant along a few second, then jumps to the correct value when the dysfunction disappears.

• The third scenario is an approach over a non flat ground. During the descent from 6000ft down to 1200ft two turns (45) are made to align the aircraft on the runway. At the second turn, a bias appears on one RA measurement. This symptom is characteristic of the presence of a bright spot on the ground in the vicinity of the airport.

The hybrid model which is used by the fusion algorithm is made of the concatenation of components chosen among the ones described hereabove (IV-B). The used nomenclature is as depicted in table I. Each RA model involves 5 operating modes.

The measurement values that are processed by the filter are not onboard records, but simulated ones. We use a simulator which has been developed for that purpose. In addition to a simplified representation of the environment (air trajectory and attitude, ground elevation, ...) the behavior of the various

n	Mode	Remark
1	Nominal	
2	Biased	Bright spot
3	Fixed at 25ft	Locked on landing gear
4	Fixed at -6ft	Antennas coupling
5	Frozen on last value	Damaged installation

TABLE I
NOMENCLATURE

sensors in the environment and in their modes of operation is also simulated. This model is obviously independent of the one used by the fusion algorithm. It is dedicated to the generation of pseudo-measurements. The occurrence of faults can be specified in each scenario so that even highly improbable events can be studied.

Scenario 1: The results are depicted on figures 1 and 2. The first one shows the true height above the ground h_{ra}, and the RA measurements. The output of RA number 1 is locked on -6ft because of an antennas coupling, probably due to heavy rains. This misbehavior lasts 30s. Then its output becomes consistent with that of the second RA. During all the simulation, the RA number 2 indications are correct. The considered ground is flat due to the proximity to the airport.

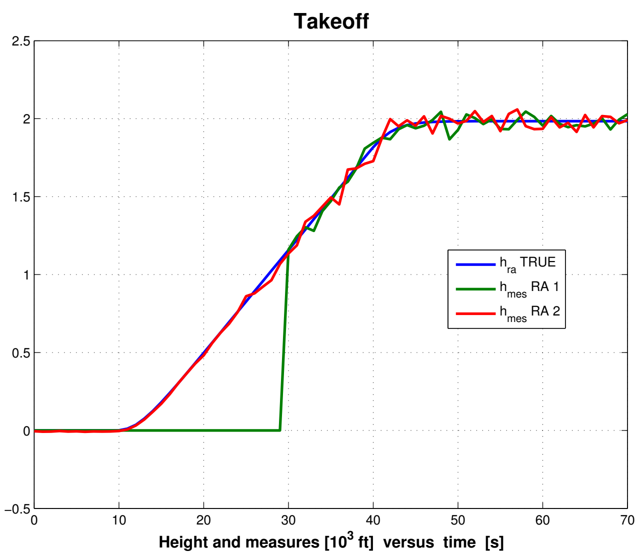

Fig. 1. Scenario 1: Takeoff

Figure 2 gives the estimated height outputted by our filter compared to the true height. It also shows the estimates of the operating modes for both sensors.

Note that during the first 10 seconds the aircraft is taxiing so that several combinations of RA operating modes can explain the observations. This is the reason why the RA mode estimates are not correct before the take-off. Nevertheless, the estimated height is very correct all along the flight.

Scenario 2: The results for the landing scenario simulation are depicted on figures 3 and 4. The output of RA number 1 remains constant, fixed to the last correct measure, during about 15s. This misbehavior is well detected by our filter. Once again the estimated height is correct all along the flight.

However it could be noted that at the beginning of

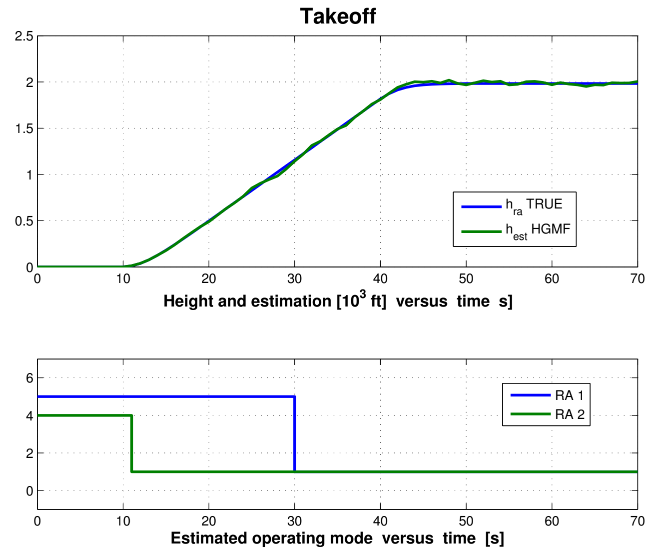

Fig. 2. Scenario 1: Takeoff

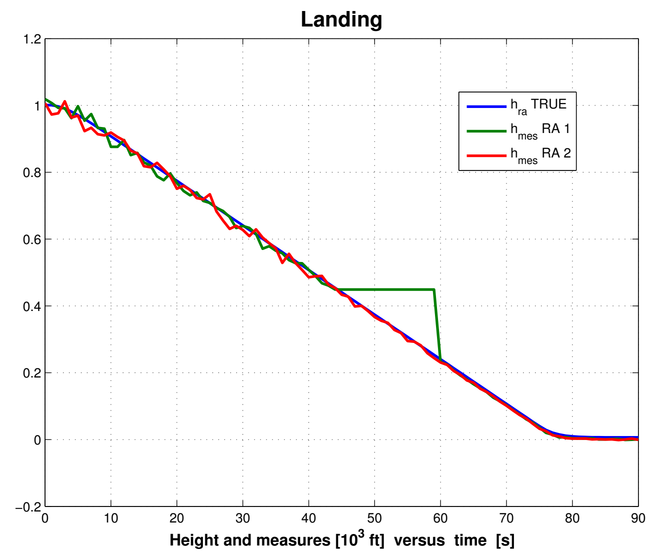

Fig. 3. Scenario 2: Landing

the simulation, the filter is experiencing some difficulties in properly estimating the RA operating modes. Indeed, the filter initialization has not been worked so far. The initialization of the a priori estimated state is always fixed to zero and assuming proper operation. This could be improved, for example using the first measures. Nevertheless this has no impact on the estimated height.

Scenario 3: During the approach, the RA number 1 output signal deliver a biased value during the final turns make to align the aircraft on the runway. The simulation uses here a ground profile that is no longer flat. It was built as a chosen smooth profile superimposed on a Brownian fractional motion [5] that represents the ground rougthness.

The results depicted on figures 5 and 6 shows that obvious a non flat ground may be confused with sensor failures. The filter reaches to recover coherency between the measurements

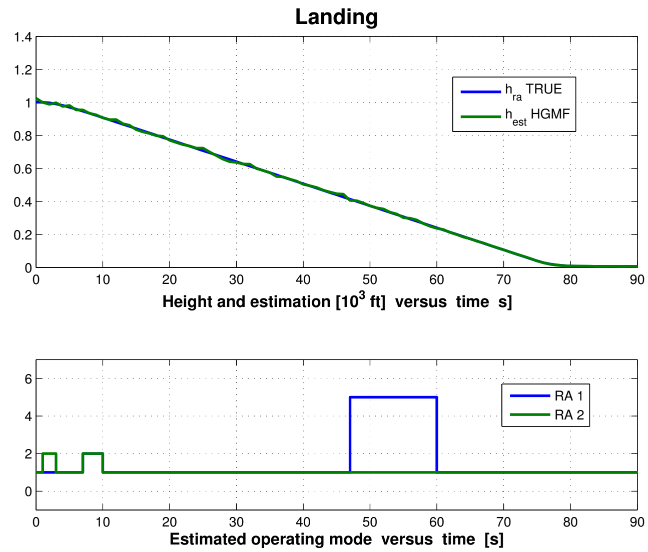

Fig. 4. Scenario 2: Landing

and the fusion model, and the estimated height is still correct. Nevertheless it will be necessary to improve this behavior in order to deal with all types of profil including cliffs.

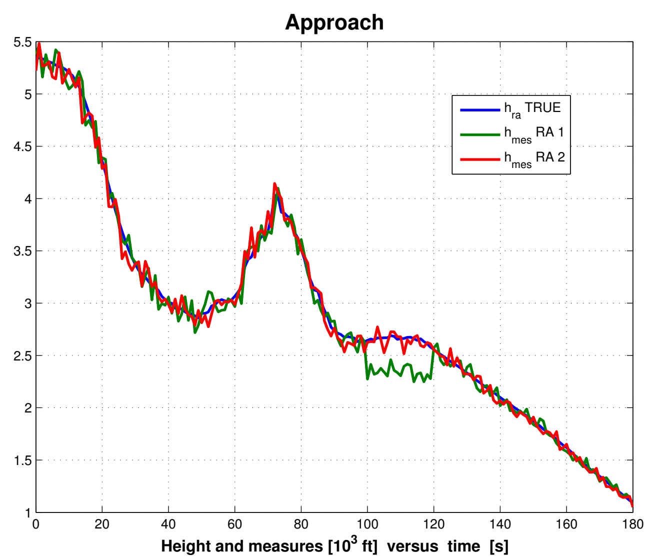

Fig. 5. Scenario 3: Approach

One can wonder how the results depend on changes in the setting of the transition matrix between sensor states. We found that not to small probability values must be chosen. Nevertheless results are not sensitive over a quite large range of tuning.

V. Conclusion

In order to improvement of the relative altitude sensing function onboard aircraft, we developed an Hybrid Gaussian Mixture Filtering framework dedicated to sensors fusion.
Hybrid means that the fusion model involves both continuous and discret states. Indeed each sensor model has one discret

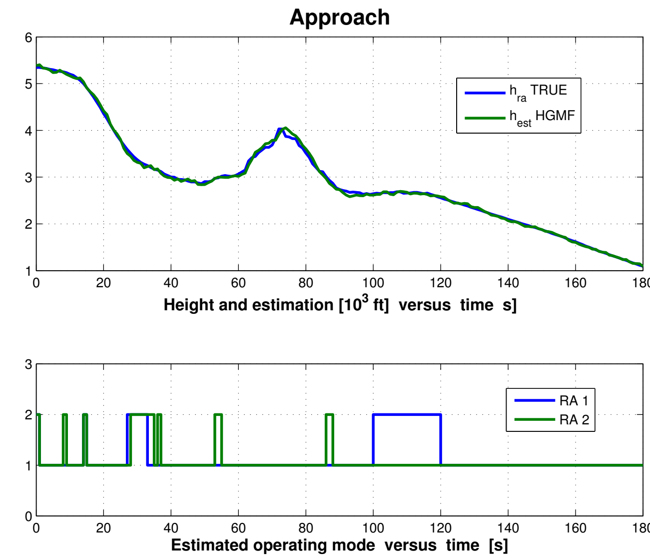

Fig. 6. Scenario 3: Approach

state that represents its operational mode. A key point was to model the dynamical behavior of the continuous state not only within each operational mode but also at the times where transitions take place.
Gaussian Mixture is relative to the approximation set where the probability density functions of the states are looked for. Such a set allows to easily manage the tradeoff between algorithm accuracy and complexity.
The *Filter* thus delivers an approximation of the state pdf. From this approximation, an optimal state estimation can be computed at any time, including for the discrete states. The sensors monitoring is thus an integrated part of the estimation process.

Some simulation results illustrate the behavior of the algorithm. The case of a sensing system made of two RA has been studied. We have shown that the HGM Filter can correct malfunctions and thus improve the integrity, availability and reliability of the relative height sensing function.

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