# Joint Time Delay and DOA Estimation Using 2-D Matrix Pencil Algorithms and IEEE 802.11ac

Abdo Gaber, and Abbas Omar
Chair of Microwave and Communication Engineering
The University of Magdeburg
Magdeburg, Germany

*Abstract*—**The IEEE 802.11ac is an emerging WLAN standard for the 5 GHz band in which multiple input multiple output (MIMO) and wider channel bandwidths are enhanced compared to 802.11n. In this paper, we present the problem of estimating the propagation time delay, and the relative direction of arrival (DOA) associated with signals in a multipath communication channel for wireless indoor positioning. The recent subspace based algorithms, represented by two-dimensional matrix pencil (2-D MP) algorithms, are applied in a new way to estimate these parameters simultaneously from an estimated space channel frequency response (S-CFR). The performance of using multiple antennas and wideband orthogonal multicarrier signals are presented. Non line of sight (NLOS) experimental results show that accuracy in the range of a few hundreds of picoseconds, and a fraction of one degree could be achieved.**

***Keywords-uniform linear array; 2-D matrix pencil algorithms; time delay and DOA estimation; OFDM; IEEE 802.11 standards***

## I. INTRODUCTION

For wireless indoor positioning, signals of opportunity such as those of the IEEE 802.11 standards can be used. However, bandwidths of the order of several hundred megahertz should be used to provide a reasonable protection against extensive multipath environments in indoor areas [1]. In order to support wider channel bandwidths, 802.11ac defines its channelization for 20, 40, 80, and 160 MHz channels [2]. Hence, super resolution algorithms will be used for post processing to reduce bandwidth requirements. The 802.11ac standard provides as a maximum 8×8 multiple-input multiple-output (MIMO) antenna configuration. While the main advantage of MIMO is to enhance data throughput, it can also be used to estimate the direction of arrival (DOA).

The problem of joint estimation of angles and relative time delays of multipath signals has been addressed for narrowband sources in [3]. The ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) was used. The ESPRIT algorithm was also used in [4] to estimate the DOA for OFDM systems. The frequency-domain time of arrival (TOA) estimation was investigated in [1] using MUSIC (Multiple Signal Classification) algorithm. It has been shown in the literature that the conventional Matrix Pencil (MP) algorithm is superior as regards both DOA and TOA estimation compared with other super resolution algorithms such as ESPRIT and Root MUSIC. MP works with the data directly without forming a covariance matrix or performing additional processing in the presence of multipath coherent signals [5]. The one-dimensional (1-D) MP was presented in [6] for estimating frequencies and damping factors of exponentially damped and / or undamped sinusoids in noise using the uniform linear array (ULA). The principle of 1-D MP was enhanced in [7] for estimating two-dimensional (2-D) frequencies using the uniform rectangular array (URA). Later, the 2-D Unitary Matrix Pencil (UMP) was presented in [5] to find the DOAs of the narrowband signals (azimuth and elevation angles), in which the complexity of the computations can be reduced by doing real valued computations using a unitary matrix transformation (UMT) [8]. In all MP algorithms, the most computationally intensive step is to estimate the signal subspace using a Singular Value Decomposition (SVD). By using the UMT, the computational cost is reduced due to the conversion of the complex data matrix into a real matrix, which is very efficient in real time implementations.

In this paper, we will focus on the joint estimation of propagation time delays and relative DOAs of multipath propagation signals using a single source and a single antenna array as an initial step for wireless indoor positioning system based on the time difference of arrival (TDOA) and the DOA methods. Different realizations of the 2-D MP algorithms to estimate these parameters simultaneously are implemented based on the physical MIMO-OFDM system parameters. The key element of our work is to use the preamble of the OFDM frame to measure the channel state for purposes other than demodulation of the data portion [2]. In general, the discrete samples of the channel frequency response (CFR) can be obtained using a multicarrier modulation technique such as OFDM or channel sweeping at different frequencies. The ULA is used, which consists of $M$ identical and omnidirectional antennas. Hence, we have a number of $M$ antennas distributed equally in the space dimension, and a number of $N$ pilots distributed equally in the frequency dimension. In fact, using the ULAs and the multicarrier signals for joint estimation of these parameters can resolve a larger number of paths in case two or more paths have equal time delays or DOAs. The performance when using 20, 40, 80, and 160 MHz bandwidths (BWs) and when using 4, 6, and 8 antennas of 802.11ac are investigated in the presence of non line of sight (NLOS) multipath coherent signals.

The rest of the paper is organized as follows: system model and space channel frequency response (S-CFR) concept are discussed in section II. The 2-D MP and the 2-D UMP algorithms are summarized in sections III and IV, respectively. The derivations of these algorithms can be obtained from the literature. We will focus here on the main steps and our modifications to estimate DOAs and time delays. In section V, the complexity and NLOS measurements are presented to show the performance of various 2-D MP algorithms. Finally, conclusions are drawn in section VI.

## II. SYSTEM MODEL

In the subsequent analysis, we will assume a multipath channel impulse response (CIR) $h(t)$, which is given by [3]

$$h(t)=\sum\nolimits_{l=1}^{L}\alpha_l a(\theta_l)\delta(t-\tau_l) \qquad (1)$$

where $L$ is the number of distinct propagation paths, $\alpha_l$ and $\tau_l$ represent the complex gain and the propagation delay of the $l$th path, respectively. $a(\theta_l)$ is the array response vector to the $l$th path from direction $\theta_l$ . The array response of the $m$th antenna to the $l$th path can be represented as, $a_m(\theta_l)=e^{-j2\pi f\tau_m(\theta_l)}$, where $\tau_m(\theta_l)=md\sin\theta_l/c$ represents the different propagation time that the plane wave impinging from direction $\theta_l$ needs to span the different distance between the antenna $m$ and the reference antenna in the antenna array as shown in Fig. 1; $c$ is the speed of light. In a mobile communication, arrival angles and time delays are relatively stationary, where the amplitude and the relative phase of each path are highly nonstationary and subject to Rayleigh fading [3]. For positioning purposes, our concern is to estimate the time delay and the relative angle of the shortest path, which represents a key element in this work to reduce the complexity and to mitigate the pairing problem in case two or more paths have equal DOAs or time delays.

At the receiver side, once the FFT window has been adjusted; the $k$th subcarrier output at the reference antenna can be represented as

$$R_{0,k}=X_k\cdot\sum\nolimits_{l=1}^{L}\alpha_{0,l,k}e^{-j(\omega_c+\omega_k)\tau_l}+w_{0,k} \quad ;-N_u/2\le k\le N_u/2 \quad (2)$$

where $X_k$ is the transmitted symbol (could be a pilot, data, or a null symbol) of the $k$th subcarrier, $\omega_k=2\pi k/T_u$, $1/T_u=\Delta f$ is the OFDM subcarrier spacing, $\omega_c=2\pi f_c$ is the carrier phase

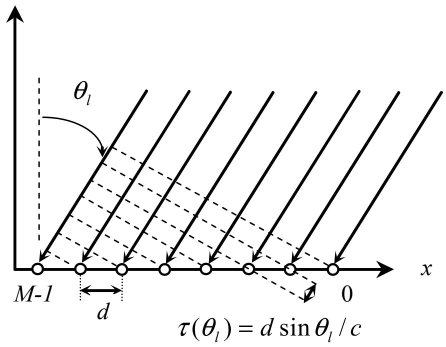


Figure 1. A uniform linear antenna array.

velocity, $w_{0,k}$ is additive white Gaussian noise (AWGN) at the reference antenna and the $k$th subcarrier, and $N_u+1$ is the number of useful subcarriers at the central spectrum. It has been assumed that the Doppler shifts are much smaller than the subcarrier spacing. From Fig.1, the signals captured by the antenna elements differ from each other by a phase offset due to the different propagation path. Hence, the received signal by an antenna $m$ in the antenna array is

$$R_{m,k}=X_k\cdot\sum\nolimits_{l=1}^{L}\alpha_{m,l,k}e^{-j(\omega_c+\omega_k)\tau_l}e^{-j(\omega_c+\omega_k)\tau_m(\theta_l)}+w_{m,k} \qquad (3)$$

### A. OFDM System Sensors

In wireless communication systems, the training sequences are used for synchronization and estimating channel parameters. $N_p$ pilot subcarriers per OFDM symbol are assumed $\{S_k: k=0,...,N_P-1\}$. The OFDM symbol structure of the training sequence has $N_p$ symbols, and $N_{dc}$ zero values at dc in the middle of the training sequence [2], [9]. These zero values at dc lead to an array discontinuity of the estimated CFR. Hence, an interpolation is used to mitigate the array discontinuity [9]. For simplicity, let us mention to the minimum frequency among subcarriers by $f_0$ and $f_k=f_0+k\Delta f$, where $k=0,...,N-1$, and $N=N_p+N_{dc}$. The least square estimate of the CFR can be obtained from (3) for the $m$th antenna as

$$H_{m,k}=\sum\nolimits_{l=1}^{L}e^{-j2\pi k\Delta f\tau_l}e^{-j2\pi f_k md\sin\theta_l/c}.A_{m,l,k}+w_{m,k}/S_k \qquad (4)$$

where $A_{m,l,k}=\alpha_{m,l,k}.e^{-j2\pi f_0\tau_l}$ . To show the phase differences across the frequency dimension and the space dimension, which represent the key element to use the 2-D MP algorithm, let us write the noiseless S-CFR in a matrix from as

$$\mathbf{H}=\begin{bmatrix}\sum_{l=1}^{L}A_{0,l,0} & \sum_{l=1}^{L}A_{0,l,1}e^{-j2\pi\Delta f\tau_l} & \cdots & \sum_{l=1}^{L}A_{0,l,N-1}e^{-j2\pi\Delta f(N-1)\tau_l}\\ \sum_{l=1}^{L}A_{1,l,0}e^{-j2\pi f_0 d\sin\theta_l/c} & \sum_{l=1}^{L}A_{1,l,1}e^{-j2\pi\Delta f\tau_l}e^{-j2\pi f_1 d\sin\theta_l/c} & \cdots & \sum_{l=1}^{L}A_{1,l,N-1}e^{-j2\pi\Delta f(N-1)\tau_l}e^{-j2\pi f_{N-1}d\sin\theta_l/c}\\ \vdots & \vdots & \ddots & \vdots\\ \sum_{l=1}^{L}A_{M-1,l,0}e^{-j2\pi f_0(M-1)d\sin\theta_l/c} & \sum_{l=1}^{L}A_{M-1,l,1}e^{-j2\pi\Delta f\tau_l}e^{-j2\pi f_1(M-1)d\sin\theta_l/c} & \cdots & \sum_{l=1}^{L}A_{M-1,l,N-1}e^{-j2\pi\Delta f(N-1)\tau_l}e^{-j2\pi f_{N-1}(M-1)d\sin\theta_l/c}\end{bmatrix}_{\mathrm{M\times N}} \quad (5)$$

The noiseless $H_{m,k}$ sample in terms of $f_c$ is

$$H_{m,k} = \sum_{l=1}^{L} e^{-j2\pi k\Delta f(\tau_l + md\sin\theta_l / c)} e^{-j2\pi f_c md\sin\theta_l / c} . A'_{m,l,k} \tag{6}$$

where $-(N-1)/2 \le k \le (N-1)/2$, and $A'_{m,l,k} = \alpha_{m,l,k} . e^{-j2\pi f_c \tau_l}$. From (6), it is clear that the effect of time delay coming from the antenna array is negligible compared with $\tau_l$ and it can be reduced by set the coordinates of the base station to the antenna array center. Therefore, the multipath channel poles, which should be estimated, can be assumed as, $x_l = e^{-j\mu_l}$, and $z_l = e^{-j\upsilon_l}$, where $\mu_l = 2\pi f_c d\sin\theta_l / c$ and $\upsilon_l = 2\pi\Delta f \tau_l$.

## III. 2-D MATRIX PENCIL ALGORITHM

In this work, the 2-D MP algorithms are used to estimate the time delays and the relative DOAs using the ULA, where the first dimension is the OFDM pilots, distributed equally in the frequency dimension, and the second dimension is the antenna elements, distributed equally in the space dimension. The S-CFR was defined in (5), where the rank of $\mathbf{H}$ is no longer than $L$; i.e., $rank(\mathbf{H}) \le L$. Therefore, $z_l$ and $x_l$ cannot be both obtained from the principle left or right singular vectors of $\mathbf{H}$, and the principle singular vectors of $\mathbf{H}$ do not contain sufficient information to carry out the pairing between $x_l$ and $z_l$ [7]. To enhance the rank condition of a matrix, in another way to restore the dimensionality of the signal subspace, a partition and stacking process should be used besides the frequency-space smoothing. By using the principle of 1-D MP algorithm [6], the Hankel matrix $\mathbf{Y}_m$ can be created for each antenna by windowing each row in (5) individually as follows

$$\mathbf{Y}_m = \begin{bmatrix} H_{m,0} & H_{m,1} & \cdots & H_{m,N-P} \\ H_{m,1} & H_{m,2} & \cdots & H_{m,N-P+1} \\ \vdots & \vdots & \ddots & \vdots \\ H_{m,P-1} & H_{m,P} & \cdots & H_{m,N-1} \end{bmatrix}_{P\times(N-P+1)} \tag{7}$$

where $P$ is the pencil parameter used to obtain the Hankel matrix. For 2-D MP algorithm, an enhanced matrix can be written in a Hankel block matrix as follows

$$\mathbf{Y}_e = \begin{bmatrix} \mathbf{Y}_0 & \mathbf{Y}_1 & \cdots & \mathbf{Y}_{M-K} \\ \mathbf{Y}_1 & \mathbf{Y}_2 & \cdots & \mathbf{Y}_{M-K+1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Y}_{K-1} & \mathbf{Y}_K & \cdots & \mathbf{Y}_{M-1} \end{bmatrix}_{KP\times(M-K+1)(N-P+1)} \tag{8}$$

**w**here $K$ is the pencil parameter used to obtain the Hankel block matrix. $P$ and $K$ are like two tuning parameters, which can be adjusted to increase the estimation accuracy. To select the pencil parameters $P$ and $K$, some necessary conditions have been investigated in [7] should be satisfied as $(K-1)P \ge L$, $K(P-1) \ge L$, and $(M-K+1)(N-P+1) \ge L$.

The modified 2-D MP algorithm for joint time delays and relative DOAs estimation can be summarized as follows

*Step1:* From the estimated CFR of antenna m, find the Hankel matrix $\mathbf{Y}_m$ as in (7). Then, the enhanced matrix $\mathbf{Y}_e$ is defined in the Hankel block matrix as in (8). To increase the number of snapshots inherently, an extended matrix $\mathbf{Y}_{ex}$ can also be defined in forward and backward as follows

$$\mathbf{Y}_{ex} = [\mathbf{Y}_e \;\; : \underset{KP}{\mathbf{\Pi}} \mathbf{Y}_e^* \underset{(M-K+1)(N-P+1)}{\mathbf{\Pi}}] \tag{9}$$

where $\mathbf{\Pi}$ is the exchange matrix that reverses the ordering of the rows. $\mathbf{Y}_{ex}$ is a centro-Hermitian matrix.

*Step2:* The SVD is used for noisy data to reduce part of the noise effect. The matrix $\mathbf{Y}_e$ of (8) can then be decomposed as follows [10]

$$\mathbf{Y}_e = \mathbf{U\Sigma V}^H \tag{10}$$

where the superscript $H$ denotes the conjugate transpose, $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices, and $\Sigma$ is a diagonal matrix with the singular values (SVs) of $\mathbf{Y}_e$ as, $\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_B$, where $B = \min(KP, (M-K+1)(N-P+1))$. The matrix $\Sigma$ can be split into two submatrices $\Sigma_s$ and $\Sigma_n$. $\Sigma_s$ is $L\times L$ diagonal matrix with the $L$ largest SVs which characterize the signal subspace as $\lambda_k = \lambda_k^s + \sigma_k^2$, where *k=1,...,L*. $\Sigma_n$ is *(B-L)×(B-L)* diagonal matrix with the *(B-L)* smallest SVs which characterize the additive noise as $\lambda_k = \sigma_k^2$, where $k = L+1,...,B$. If the forward-backward matrix $\mathbf{Y}_{ex}$ of (9) is used in (10), $B = \min(KP, 2(M-K+1)(N-P+1))$; it will be called the 2-D MP-Ex algorithm.

*Step3:* Based on the Information Theoretic Criteria (ITC), the modified Minimum Descriptive Length (MDL) criterion is used to estimate the signal subspace dimension $\hat{L}$ by eliminating the noise components [9], [11].

*Step4:* The SVD of (8), or (9), can then be decomposed as

$$\mathbf{Y}_e = \mathbf{Y}_s + \mathbf{Y}_n = \mathbf{U}_s \Sigma_s \mathbf{V}_s^H + \mathbf{U}_n \Sigma_n \mathbf{V}_n^H \tag{11}$$

where $\mathbf{U}_s$ and $\mathbf{V}_s$ are the submatrices of $\mathbf{U}$ and $\mathbf{V}$ corresponding to the SVs of $\Sigma_s$ and span the signal subspace, and $\mathbf{U}_n$ and $\mathbf{V}_n$ are corresponding to the SVs of $\Sigma_n$ and span the noise subspace.

*Step5:* Extracting $x_l$: The matrix pencil can be written along the space dimension as, $\mathbf{U}_{s2} - x\mathbf{U}_{s1}$, where $\mathbf{U}_{s1}$ and $\mathbf{U}_{s2}$ are obtained from $\mathbf{U}_s$ by deleting the last and the first $P$ rows, respectively. The desired $x_l$'s can be obtained as the eigenvalues of $\mathbf{U}_{s1}^{\dagger}\mathbf{U}_{s2}$, which represents a generalized eigenvalue problem of dimension $\hat{L} \times \hat{L}$, where $\mathbf{U}_{s1}^{\dagger}$ is the Moore-Penrose pseudo-inverse of $\mathbf{U}_{s1}$. The $l$th DOA can then be calculated from $x_l = e^{-j2\pi d \sin\hat{\theta}_l / \lambda}$, where $l = 1,...,\hat{L}$, and $\lambda = c / f_c$, as $\hat{\theta}_l = \sin^{-1}(\arg(x_l^*).\lambda / 2\pi d)$.

*Step6:* Extracting $z_l$: In order to estimate $\{z_l; l = 1,...,\hat{L}\}$, the shuffling matrix is used to introduce the permutation as, $\mathbf{U}_{sp} = \mathbf{P}\mathbf{U}_s$, where the shuffling matrix $\mathbf{P}$ is defined as,

$$\begin{aligned}\mathbf{P} = [&\mathbf{s}(1), \mathbf{s}(1+P), \cdots, \mathbf{s}(1+(K-1)P), \mathbf{s}(2), \mathbf{s}(2+P), \cdots \\ &, \mathbf{s}(2+(K-1)P), \cdots, \mathbf{s}(P), \mathbf{s}(P+P), \cdots, \mathbf{s}(P+(K-1)P)]^T,\end{aligned} \quad (12)$$

where $\mathbf{s}(i)$ is a column vector of size $KP$ with one at the $i$th position and zero everywhere else. The matrix pencil can be written along the frequency dimension as, $\mathbf{U}_{sp2} - z\mathbf{U}_{sp1}$, where $\mathbf{U}_{sp1}$ and $\mathbf{U}_{sp2}$ are obtained from $\mathbf{U}_{sp}$ by deleting the last and the first $K$ rows, respectively. The desired $z_l$'s can be obtained as the eigenvalues of $\mathbf{U}_{sp1}^{\dagger}\mathbf{U}_{sp2}$, which represents a generalized eigenvalue problem of dimension $\hat{L} \times \hat{L}$. The $l$th time delay can then be calculated from $z_l = e^{-j2\pi\Delta f \hat{\tau}_l}$ as $\hat{\tau}_l = \arg(z_l^*) / 2\pi\Delta f$, where $l = 1,...,\hat{L}$.

The order of poles in each set, namely $\mathbf{Z} = \{z_l; l = 1,...,\hat{L}\}$ and $\mathbf{X} = \{x_l; l = 1,...,\hat{L}\}$, is still unknown. Several efforts have been made to pair the unknown parameters in 2-D scenarios, which could be multiple frequencies or azimuth and elevation angles in the literature. In our work, we will use the principle of both the wireless indoor positioning and the suggested method in [12]. A pairing technique was presented in [12], which provides more accurate pairing results compared with the others in the literature and requires less computational complexity. However, if there are a number of repeated poles, the eigenvalue decomposition (EVD) problem should be solved a number of times as described in [12]. In case of wireless positioning, our concern is to estimate the time delay of the shortest path and the corresponding DOA. Therefore, the eigenvalue problem of $\mathbf{U}_{sp1}^{\dagger}\mathbf{U}_{sp2}$ is computed first to find the eigenvalues $\mathbf{Z}_d$ (diagonal matrix) and the eigenvectors $\mathbf{W}$ as, $\mathbf{U}_{sp1}^{\dagger}\mathbf{U}_{sp2} = \mathbf{W}\mathbf{Z}_d\mathbf{W}^{-1} \Rightarrow \mathbf{Z}_d = \mathbf{W}^{-1}\mathbf{U}_{sp1}^{\dagger}\mathbf{U}_{sp2}\mathbf{W}$. Then, the estimated propagation time delays of the estimated multipath channel are rearranged in an ascending order. After that, the eigenvectors of $\mathbf{W}$ are rearranged such as the previous order to get $\mathbf{W}'$. For example, the corresponding column of the shortest path is put as the first column in $\mathbf{W}'$; our concern is the DOA of the shortest path. Finally, compute $\mathbf{X}_d$ by premultiplying $\mathbf{U}_{s1}^{\dagger}\mathbf{U}_{s2}$ with $\mathbf{W}'^{-1}$, and then postmultiplying with $\mathbf{W}'$. It is clear that a single EVD is used instead of two, and the problem of repeated poles is mitigated for wireless positioning applications.

## IV. The 2-D Unitary Matrix Pencil Algorithm

The modified 2-D UMP algorithm for time delays and relative DOAs estimation can be summarized as follows.

*Step1:* Such as the first step of 2-D MP algorithm, define the centro-Hermitian matrix $\mathbf{Y}_{ex}$ as in (9).

*Step2:* Calculate the real data matrix $\mathbf{Y}_{\mathrm{Re}}$, using the centro-symmetry of the antenna arrays or any complex matrix could be written in a centro-hermitian matrix form as

$$\mathbf{Y}_{\mathrm{Re}} = \mathbf{Q}_{K_1}^H \mathbf{Y}_{ex} \mathbf{Q}_{K_2} \quad (13)$$

where $\mathbf{Q}_{K_1}$ and $\mathbf{Q}_{K_2}$ are unitary matrices whose columns are conjugate symmetric and have a sparse structure as follows

$$\mathbf{Q}_{K(even)} = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{I} & j\mathbf{I} \\ \mathbf{\Pi} & -j\mathbf{\Pi} \end{bmatrix} \quad \mathbf{Q}_{K(odd)} = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{I} & \mathbf{0}^T & j\mathbf{I} \\ \mathbf{0} & \sqrt{2} & \mathbf{0} \\ \mathbf{\Pi} & \mathbf{0}^T & -j\mathbf{\Pi} \end{bmatrix} \quad (14)$$

For $\mathbf{Q}_{K(even)}$, the identity matrix $\mathbf{I}$ and the exchange matrix $\mathbf{\Pi}$ have the dimension of $K/2$, for $\mathbf{Q}_{K(odd)}$, $\mathbf{I}$ and $\mathbf{\Pi}$ have the dimension of *(K-1)/2,* and $\mathbf{0}$ is a *(K-1)/2* zero row vector.

*Step3:* Perform a SVD on $\mathbf{Y}_{\mathrm{Re}}$ as in (10), and estimate the number of effective paths $\hat{L}$ using the modified MDL criterion.

*Step4:* Determine $\mathbf{U}_s$ as the left singular vectors of $\mathbf{U}$, which span the signal subspace and correspond to the largest $\hat{L}$ singular values of $\mathbf{Y}_{\mathrm{Re}}$.

*Step5:* Calculate the selection matrices $\mathbf{K}_{\mathrm{Re}1}$ and $\mathbf{K}_{\mathrm{Im}1}$ to extract $z_l$'s poles just once and store them as constant matrices according to

$$\mathbf{K}_{\mathrm{Re}1} = \mathrm{Re}(\mathbf{Q}_{K_1}^H \mathbf{J}_1 \mathbf{Q}_{K_2}) \quad \mathbf{K}_{\mathrm{Im}1} = \mathrm{Im}(\mathbf{Q}_{K_1}^H \mathbf{J}_1 \mathbf{Q}_{K_2}) \quad (15)$$

where $\mathbf{J}_1$ is a selection matrix constructed from an identity matrix and a zero matrix as $\mathbf{J}_1 = [\mathbf{I}_{KP-K} \;\; : \;\; \mathbf{0}_{(KP-K)\times K}]$.

In addition, calculate the selection matrices $\mathbf{K}_{\mathrm{Re}2}$ and $\mathbf{K}_{\mathrm{Im}2}$ to extract $x_l$'s poles just once and store them as constant matrices according to

$$\mathbf{K}_{\mathrm{Re}2} = \mathrm{Re}(\mathbf{Q}_{K_1}^H \mathbf{J}_2 \mathbf{Q}_{K_2}) \quad \mathbf{K}_{\mathrm{Im}2} = \mathrm{Im}(\mathbf{Q}_{K_1}^H \mathbf{J}_2 \mathbf{Q}_{K_2}) \qquad (16)$$

where $\mathbf{J}_2$ is a selection matrix constructed from an identity matrix and a zero matrix as $\mathbf{J}_2 = [\mathbf{I}_{KP-P} \ : \ \mathbf{0}_{(KP-P)\times P}]$.

*Step6:* Extracting $z_l$ : Compute the EVD of $\Psi_v = [\mathbf{K}_{\mathrm{Re}1}\mathbf{U}_{sp}]^{\dagger}\mathbf{K}_{\mathrm{Im}1}\mathbf{U}_{sp}$ as, $\mathbf{Z}_d = \mathbf{W}^{-1}\Psi_v\mathbf{W}$, to get $\mathbf{Z}_d = diag\{\Lambda_l = \tan(v_l/2); l = 1,\cdots,\widehat{L}; v_l = 2\pi\Delta f\widehat{\tau}_l\}$. Like in 2-D MP, $\mathbf{U}_{sp}$ is shuffled by the shuffling matrix as, $\mathbf{U}_{sp} = \mathbf{P}'\mathbf{U}_s$, where $\mathbf{P}'$ is $\mathbf{Q}_{K_1}^H\mathbf{P}\mathbf{Q}_{K_2}$ due to using the UMT. Then, calculate the time delays according to $\widehat{\tau}_l = 1/\pi\Delta f \times \tan^{-1}(\Lambda_l)$, where $l = 1,\cdots,\widehat{L}$.

*Step7:* Extracting $x_l$ : By using the same method in the previous section, compute $\mathbf{X}_d$ by premultiplying $\Psi_\mu = [\mathbf{K}_{\mathrm{Re}2}\mathbf{U}_s]^{\dagger}\mathbf{K}_{\mathrm{Im}2}\mathbf{U}_s$ with inverse $\mathbf{W}'$, and then postmultiplying with $\mathbf{W}'$ to get $\{\Omega_l = \tan(\mu_l/2); l = 1,\cdots,\widehat{L}; \mu_l = 2\pi d \sin\widehat{\theta}_l/\lambda\}$. After that, calculate the DOAs according to $\sin\widehat{\theta}_l = \lambda/\pi d \times \tan^{-1}(\Omega_l)$, where $l = 1,\cdots,\widehat{L}$.

Fig. 2 shows a block diagram of the proposed joint propagation time delay and DOA estimation algorithms.

## V. COMPLEXITY AND MEASUREMENTS ANALYSIS

### A. Computational Complexity

In all MP algorithms, the most computationally intensive step is to estimate the signal subspace using the SVD. In the conventional 2-D MP algorithm, the enhanced matrix $\mathbf{Y}_e$ is a complex matrix of size $KP \times (M-K+1)(N-P+1)$. To compute the singular values and left singular vectors of $\mathbf{Y}_e$, it requires $17K^3P^3/3 + K^2P^2(M-K+1)(N-P+1)$ complex multiplications, where $(M-K+1)(N-P+1) > KP$ [10]. In case of using $\mathbf{Y}_{ex}$, the number of columns increases by a factor of two. In 2-D UMP algorithm, the centro-hermitian matrix $\mathbf{Y}_{ex}$ is transformed by the UMT into a real matrix with negligible computational effort. The size of $\mathbf{Y}_{\mathrm{Re}}$ is $KP \times 2(M-K+1)(N-P+1)$, and it requires $17K^3P^3/3 + 2K^2P^2 \times (M-K+1)(N-P+1)$ real multiplications. Hence, the SVD computational complexity of 2-D UMP algorithm decreases by a factor of 4. The pencil parameter values of *P* and *K* were selected to be *N/3* and *M/3*, respectively, to reduce the complexity and to stay in the appropriate range. Table I shows the CFR length of each BW [2], [9].

### B. Measurements Analysis

The experimental results are presented to show the capability of 2-D MP, 2-D MP-Ex, and 2-D UMP algorithms to estimate the propagation time delays and the relative DOAs using multi-antenna multi-carrier systems. The network analyzer, Agilent ENA E5071C, was used. It was used to measure the indoor CFR in the frequency range of 802.11ac. In this paper, a frequency range of 5170 MHz to 5330 MHz is used, which corresponds to a 160 MHz BW. The measurement system is shown in Fig. 3. Omnidirectional antennas were used, which have 3 dBi gain and 0.668 ns time delay, measured using the wideband Time Domain Transmission (TDT), Agilent 86100A. At the receiver, the number of antenna elements in the ULA could be 4, 6, or 8. The separation distance between antenna elements was designed to be 2.857 cm, calculated from the half wavelength of the carrier frequency 5.25 GHz. As shown in Fig. 3, the transmitter antenna at the location point in the corridor was connected to port 1 of ENA through cable 1 of length 12 m and time delay 45.64 ns. At the receiver side in the labor, one antenna from the antenna array elements was used to take the measurements at one time, which was connected to port 2 through cable 2 of length 1 m and time delay 4.79 ns. The remaining antenna elements in the antenna array were terminated by 50 ohm resistance loads. The connection between antenna elements and port 2 was changed manually. The transmitted power of ENA was 10 dBm.

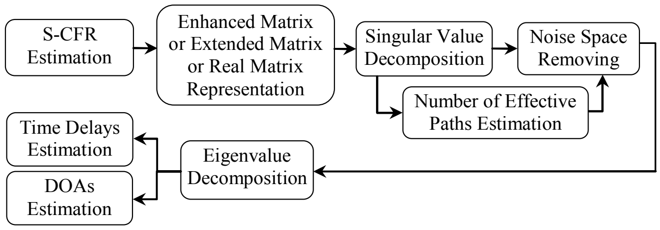


Figure 2. Operational flow of the proposed joint propagation time delay and DOA estimation using 2-D MP algorithms.

TABLE I. CFR VECTOR LENGTH REGARDING 802.11AC BANDWIDTHS

| **Bandwidth (MHz)** | **20** | **40** | **80** | **160** |
|---|---|---|---|---|
| **FFT / IFFT order** | 64 | 128 | 256 | 512 |
| $N = N_p + N_{dc}$ | 57 | 117 | 245 | 501 |

The complex CFR of each antenna element can be obtained by sweeping the channel at uniformly spaced frequencies. The frequency responses were collected at NLOS positions; there is a wall between Tx and Rx of thickness 13 cm. The antenna height in both Tx and Rx was 152 cm. The real and imaginary parts of the forward transmission coefficient S21 were measured and stored for further processing. For each position, 60 measurements were recorded for averaging purposes during two days. Each measurement covers a 160 MHz BW with a sampling interval set to 312.5 KHz. The number of samples of the CFR is 512 samples, which is equal to the maximum IFFT/FFT order of 802.11ac. A threshold was used to mitigate the large over estimation of a number of effective paths using the modified MDL.

By using the 80 MHz BW and 8 antenna elements, Fig. 4 shows the root mean square error (RMSE) of both time delay (in nanoseconds) and DOA (in degrees) of various 2-D MP algorithms. It is clear that the 2-D UMP is the best option regarding complexity and accuracy. By using the 2-D UMP, Fig. 5 shows the RMSE of both time delay and DOA estimation in case of using 4, 6, or 8 antenna elements, and in

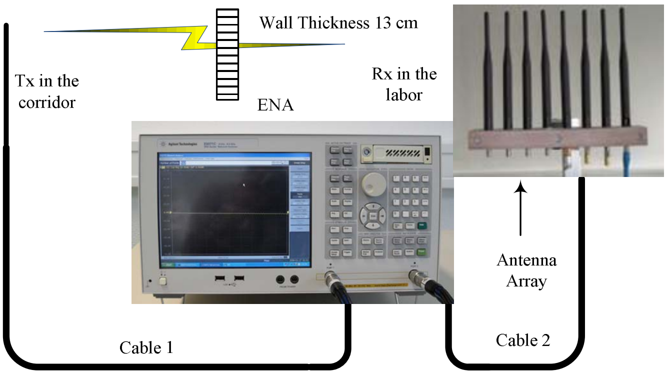


Figure 3. Frequency response measurment system using ENA.

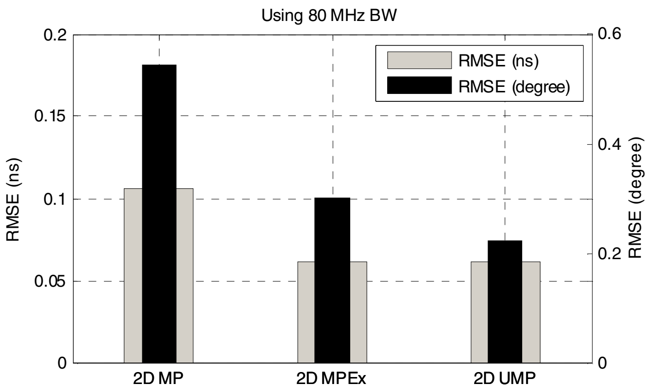


Figure 4. Comparison of time delay and DOA estimation accuracy of various 2-D MP algorithms using 80 MHz BW and 8 antenna elements.

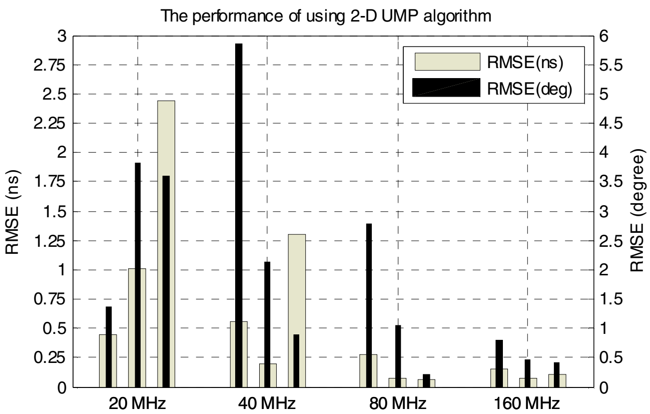


Figure 5. Comparison of time delay and DOA estimation accuracy of 2-D UMP using various 802.11ac BWs and number of antenna elements which could be 4, 6, or 8 plotted from left to right for each BW.

case of using 20, 40, 80, or 160 MHz BW. From Fig. 5, if the BW and / or the number of antennas increase(s), the accuracy of time delay and DOA estimation increases. It should be noted that if the number of subcarriers increases (BW), the necessary number of antennas for accurate DOA decreases, which can reduce the receiver complexity.

## VI. Conclusion

In this paper, we implemented the 2-D MP algorithms in different realizations to estimate the propagation time delays and the relative DOAs simultaneously using the multi-antenna multi-carrier systems for wireless positioning. Using multi-antenna multi-carrier principles can successfully enhance the dimensionality of the signal subspace for joint time delay and DOA estimation; it represents a robust technique versus multipath channel fading. Using the priori information of wireless positioning (our concern is to estimate the time delay and the DOA of the shortest path) mitigates the problem of repeated poles and hence reduces the complexity of calculating extra EVD problems. The necessary number of antennas for accurate DOA estimation can be reduced by increasing the number of subcarriers (BW), which reduces the receiver complexity. The performance of the 2-D MP, the 2-D MP-Ex and the 2-D UMP algorithms have been investigated and compared using 802.11ac system parameters. The 2-D UMP is the best choice for wireless positioning regarding complexity and accuracy. Based on the NLOS experimental results, accuracy in the range of a few hundreds of picoseconds, and a fraction of one degree has been achieved.

## References


[1] Xinrong Li, Pahlavan K., "Super-resolution TOA estimation with diversity for indoor geolocation," *Wireless Communications, IEEE Transactions on*, vol. 3, no. 1, pp. 224- 234, Jan. 2004.

[2] IEEE P802.11ac. Specification framework for TGac. IEEE 802.11-09/0992r21, Jan. 2011.

[3] Vanderveen M.C., Van der Veen A.-J., Paulraj A., "Estimation of multipath parameters in wireless communications," *Signal Processing, IEEE Transactions on*, vol. 46, no. 3, pp. 682-690, Mar. 1998.

[4] Shiann-Shiun Jeng, Chen Wan Tsung, "Multipath direction finding with frequency allocation subspace smoothing for an OFDM wireless communication system," *Personal Indoor and Mobile Radio Communications, PIMRC 2005. IEEE 16th International Symposium on*, vol. 4, pp. 2471-2475 , Sept. 2005.

[5] N. Yilmazer, T.K. Sarkar, "2-D unitary matrix pencil method for efficient direction of arrival estimation," Digital Signal Processing, vol. 16, pp. 767–781, 2006.

[6] Y. Hua, T.K. Sarkar, "Matrix pencil method for estimating parameters of exponentially damped/undamped sinusoids in noise," *Acoustics, Speech and Signal Processing, IEEE Transactions on*, vol. 38, no. 5, pp. 814-824, May 1990.

[7] Y. Hua, "Estimating two-dimensional frequencies by matrix enhancement and matrix pencil," *Signal Processing, IEEE Transactions on*, vol. 40, no. 9, pp. 2267-2280, Sep. 1992.

[8] Keh-Chiarng Huarng, Chien-Chung Yeh, "A unitary transformation method for angle-of-arrival estimation," *Signal Processing, IEEE Transactions on*, vol. 39, no. 4, pp. 975-977, Apr. 1991.

[9] A. Gaber, A. Omar, "A Study of TDOA Estimation Using Matrix Pencil Algorithms and IEEE 802.11ac," *Ubiquitous Positioning Indoor Navigation and Location Based Service, UPINLBS 2012,* vol., no., pp. 1-8 , Oct. 2012.

[10] Gene H. Golub, Charles F. Van Loan, Matrix Computations. Johns Hopkins University Press, Baltimore, 3rd Edition 1996.

[11] M. Wax, T. Kailath, "Detection of signals by information theoretic criteria," *Acoustics, Speech and Signal Processing, IEEE Transactions on*, vol. 33, no. 2, pp. 387- 392, Apr. 1985.

[12] Fang-Jiong Chen, Carrson C. Fung, Chi-Wah Kok, Sam Kwong, "Estimation of Two-Dimensional Frequencies Using Modified Matrix Pencil Method," *Signal Processing, IEEE Transactions on*, vol. 55, no. 2, pp. 718-724, Jan. 2007.