Here’s a clear explanation of **why models with only random intercepts** vs. **models with both random slopes and random intercepts** can have **very different degrees of freedom (df)** in a mixed model framework.

**1. Understanding Degrees of Freedom in Mixed Models**

In linear mixed-effects models (LMMs), **degrees of freedom** are used to:

* Estimate **uncertainty** in fixed-effect parameters (confidence intervals, -values),
* Determine how much information is effectively available to estimate fixed and random effects.

Unlike simple linear models, the df are not straightforward counts of observations minus parameters because:

* Random effects introduce **hierarchical structure**,
* Groups are not independent,
* Variance components (e.g., intercept variance, slope variance) are also estimated.

Mixed models use approximate df calculations (e.g., **Satterthwaite** or **Kenward-Roger** methods) that account for the structure of the data.

**2. Random Intercepts-Only Model**

Consider a random-intercepts-only model:

lmer(y ~ x + (1 | group), data = data)

This model:

* Allows each group to have its **own baseline level (intercept)**,
* Assumes **the effect of predictor** (the slope) is the **same across all groups**.

**Implications for df:**

* There is **only one variance component** to estimate (intercept variance),
* The model treats group-to-group variability as a single random factor,
* Fewer parameters → **higher residual df** for estimating fixed effects.

Essentially, this structure **borrows strength across groups**, so each group contributes information to the overall slope estimation.

**3. Adding Random Slopes**

Now consider a random intercepts **and** random slopes model:

lmer(y ~ x + (1 + x | group), data = data)

This model:

* Still allows group-specific intercepts,
* **Also allows each group to have its own slope** for predictor .

It must now estimate **three variance components**:

1. Random intercept variance,
2. Random slope variance,
3. Covariance between intercept and slope.

**4. Why Degrees of Freedom Drop Dramatically**

When random slopes are added:

1. **More parameters are estimated per group**
   * Instead of estimating one variance parameter (intercepts only), you now estimate three.
   * This uses up more of the data's information.
2. **Correlation between slope and intercept increases complexity**
   * The intercept-slope covariance matrix has to be estimated,
   * This further reduces the df available for fixed effects.
3. **Less pooling, more independence**
   * Random slopes make each group's relationship between and **partially independent**,
   * There is less "borrowing of strength" across groups because each slope can deviate freely,
   * This reduces the effective sample size for estimating the fixed effect of .

**5. Conceptual Example**

Imagine a study with:

* 10 groups,
* 5 observations per group.

| **Model type** | **Parameters per group** | **Total random effects** |
| --- | --- | --- |
| Random intercepts only | 1 (intercept variance) | 1 |
| Random intercepts + slopes | 3 (intercept variance, slope variance, covariance) | 3 |

The **random slopes model has 3× as many group-level parameters**, which "consumes" more df, leaving fewer for testing fixed effects.

**6. Practical Consequences**

**a) Inflated standard errors**

* With lower df, the standard errors for fixed-effect estimates (like the slope of ) increase.
* This can make fixed effects appear **less significant**, even if the point estimate hasn't changed much.

**b) Model comparison penalties**

* When comparing models via likelihood ratio tests or AIC:
  + The random slopes model has a higher penalty for complexity,
  + Making it harder to justify unless supported by data.

**c) Overfitting risk**

* With small sample sizes per group, random slopes may overfit,
* This is why convergence warnings are common with random slopes.

**7. Key Takeaways**

| **Feature** | **Random intercepts only** | **Random intercepts + slopes** |
| --- | --- | --- |
| Group baseline variation | Yes | Yes |
| Group slope variation | No (shared slope) | Yes (unique slope per group) |
| # variance components | 1 | 3 |
| Degrees of freedom (df) | Higher | Lower |
| SE for fixed effects | Lower | Higher |

**Summary:**  
Adding random slopes dramatically reduces df because it increases the number of variance and covariance parameters estimated, reduces pooling of information across groups, and makes the estimation problem more complex.

**8. References**

* Barr, D. J., Levy, R., Scheepers, C., & Tily, H. J. (2013). *Random effects structure for confirmatory hypothesis testing: Keep it maximal*. Journal of Memory and Language, 68(3), 255–278.
* Bolker, B. M., et al. (2009). *Generalized linear mixed models: a practical guide for ecology and evolution*. Trends in Ecology & Evolution, 24(3), 127–135.
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**9. Bottom Line**

Adding random slopes increases model flexibility but also **uses up more information**, reducing degrees of freedom.

The trade-off:

* **Random intercepts only** → simpler, higher df, more pooling.
* **Random intercepts + slopes** → more realistic, but lower df and higher uncertainty in fixed-effect estimates.