A Sliding Mode Observer for Permanent-Magnet Synchronous Motor

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This observer described here after is an example extracted from [Utkin 09].

1 Motor model

General motor model:

$$\begin{aligned} \boldsymbol{U} &= \boldsymbol{R}\boldsymbol{I} + \frac{d\boldsymbol{\Psi}}{dt} \\ \boldsymbol{\Psi} &= \boldsymbol{L}\boldsymbol{I} + \boldsymbol{\Psi_M} \end{aligned} \tag{1}$$

with $\boldsymbol{\Psi_M}$ the magnetic flux from the permanent magnet, $\boldsymbol{\Psi}$ the flux vector, $\boldsymbol{R}$ and $\boldsymbol{L}$ respectively the resistance and inductances of the motor coils, and $\boldsymbol{U}$ and $\boldsymbol{I}$ the voltage and current for each coils.

1.1 Within the a,b,c frame

The flux parts generated by the permanent magnet flowing within each coils is given as:

$$\begin{aligned} \Psi_{ma} &= \lambda_0 \cos(\theta_e) \\ \Psi_{mb} &= \lambda_0 \cos\left(\theta_e - \frac{2\pi}{3}\right) \\ \Psi_{mc} &= \lambda_0 \cos\left(\theta_e + \frac{2\pi}{3}\right) \end{aligned} \tag{2}$$

The current is modeled in each coils with:

$$\begin{aligned} \frac{di_a}{dt} &= -\frac{R}{L} i_a - \frac{1}{L} e_a + \frac{1}{L} u_a \\ \frac{di_b}{dt} &= -\frac{R}{L} i_b - \frac{1}{L} e_b + \frac{1}{L} u_b \\ \frac{di_c}{dt} &= -\frac{R}{L} i_c - \frac{1}{L} e_c + \frac{1}{L} u_c \end{aligned} \tag{3}$$

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with the back EMF voltage derived from (2):

$$
\begin{aligned}
e_a &= \frac{d\Psi_{ma}}{dt} = -\lambda_0 w_e \sin(\theta_e) \\
e_b &= \frac{d\Psi_{mb}}{dt} = -\lambda_0 w_e \sin\left(\theta_e - \frac{2\pi}{3}\right) \\
e_c &= \frac{d\Psi_{mc}}{dt} = -\lambda_0 w_e \sin\left(\theta_e + \frac{2\pi}{3}\right)
\end{aligned}
\tag{4}
$$

1.2 Within the α, β frame

(Through a linear transformation from $\Re^3$ to $\Re^2$)

Current model from equation (3) becomes:

$$
\begin{aligned}
\frac{di_\alpha}{dt} &= -\frac{R}{L} i_\alpha - \frac{1}{L} e_\alpha + \frac{1}{L} u_\alpha \\
\frac{di_\beta}{dt} &= -\frac{R}{L} i_\beta - \frac{1}{L} e_\beta + \frac{1}{L} u_\beta
\end{aligned}
\tag{5}
$$

and the back EMF from (4) becomes:

$$
\begin{aligned}
e_\alpha &= \frac{d\Psi_{m\alpha}}{dt} = -\lambda_0 w_e \sin(\theta_e) \\
e_\beta &= \frac{d\Psi_{m\beta}}{dt} = -\lambda_0 w_e \cos(\theta_e)
\end{aligned}
\tag{6}
$$

1.3 Sliding mode Observer for the Back-EMF e_α and e_β

Sliding mode observer equation:

$$
\begin{aligned}
\frac{d\hat{i}_\alpha}{dt} &= -\frac{R}{L} \hat{i}_\alpha + \frac{1}{L} u_\alpha - \frac{l_1}{L}\ sign(\hat{i}_\alpha - i_\alpha) \\
\frac{d\hat{i}_\beta}{dt} &= -\frac{R}{L} \hat{i}_\beta + \frac{1}{L} u_\beta - \frac{l_1}{L}\ sign(\hat{i}_\beta - i_\beta)
\end{aligned}
\tag{7}
$$

A Simulink subsystem implement this equation on figure 1.

After a limited time, sliding mode is enforced if $l_1 > max\left(|e_\alpha|, |e_\beta|\right)$.

Defining errors as $\bar{i}_\alpha = \hat{i}_\alpha - i_\alpha$ and $\bar{i}_\beta = \hat{i}_\beta - i_\beta$, we obtain with (5) and (7):

$$
\begin{aligned}
\frac{d\bar{i}_\alpha}{dt} &= -\frac{R}{L} \bar{i}_\alpha - \frac{1}{L} e_\alpha - \frac{l_1}{L}\ sign(\bar{i}_\alpha) \\
\frac{d\bar{i}_\beta}{dt} &= -\frac{R}{L} \bar{i}_\beta - \frac{1}{L} e_\beta - \frac{l_1}{L}\ sign(\bar{i}_\beta)
\end{aligned}
\tag{8}
$$

When sliding mode occurs, the error is null thus $\bar{i}_\alpha = 0$ and $\bar{i}_\beta = 0$ and maintained null thus $\frac{d\bar{i}_\alpha}{dt} = 0$ and $\frac{d\bar{i}_\beta}{dt} = 0$.

The switching quantities equivalent value are the desired Back EMF quantities e_α and e_β:

$$\begin{aligned} \Big(l_1\ sign(\bar{i}_\alpha)\Big)_{eq} &= e_\alpha \\ \Big(l_1\ sign(\bar{i}_\beta)\Big)_{eq} &= e_\beta \end{aligned} \tag{9}$$

However, they could not be used as is. Consider their low pass filtered result z_a and z_b:

$$\begin{aligned} z_\alpha(t) + \Delta_\alpha(t) &= e_\alpha \\ z_\beta(t) + \Delta_\beta(t) &= e_\beta \end{aligned} \tag{10}$$

The Δ_α and Δ_β represents the error due to the high frequency residual, and to the delay from the low pass filter.

Remark regarding the AN1078: In the Application Note 1078 [Zambada 07], the enforced sliding mode is different and provides a continuous delayed BEMF $[e_\alpha\ e_\beta]$ estimation. A low pass filter is used to smoother even more the estimated BEMF and an offset is subtracted to the estimated rotor angle θ_e to compensate for the filter delay. In an updated implementation [Zambada 10], an adaptive low pass filter is used that maintain the filter delay constant for the motor running speed: the cut off frequency is function of the rotor spin frequency. The offset subtracted to the estimated rotor angle θ_e is then constant.

1.4 Observer to smooth e_α, e_β, and estimate w_e

Here, an observer for e_α, e_β and w_e is designed instead of filtering the resulting switching quantities. This observer have an important filtering property for the estimated e_α and e_β.

We assume that variations of the rotor speed (w_e) are slow (i.e. $\dot{w}_e = 0$) in comparisons to others quantities involved in the observer. Under this assumption, the time derivative for equation (6) is:

$$\begin{aligned} \dot{e}_\alpha &= \lambda_0 w_e^2 \cos(\theta_e) \\ \dot{e}_\beta &= -\lambda_0 w_e^2 \sin(\theta_e) \end{aligned}$$

which can be expressed as:

$$\begin{aligned} \dot{e}_\alpha &= -w_e e_\beta \\ \dot{e}_\beta &= w_e e_\alpha \end{aligned} \tag{11}$$

The observer smoothing e_α, e_β is based on the dynamics system (11). The switching quantities from (7) used in (11) are integrated to resolve e_α and e_β. This integration has a important low pass filter action.

The observer equations are:

$$\begin{aligned}
\dot{\hat{e}}_\alpha &= -\hat{w}_e \hat{e}_\beta - l_2(\hat{e}_\alpha - z_\alpha) \\
\dot{\hat{e}}_\beta &= \hat{w}_e \hat{e}_\alpha - l_2(\hat{e}_\beta - z_\beta) \\
\dot{\hat{w}}_e &= (\hat{e}_\alpha - z_\alpha)\hat{e}_\beta - (\hat{e}_\beta - z_\beta)\hat{e}_\alpha
\end{aligned} \tag{12}$$

A Simulink subsystem implment this equation on figure 2.

Note that the filter used z_a and z_b might be a throughput filter. We consider $z_\alpha(t) \to e_\alpha(t)$ and $z_\beta(t) \to e_\beta(t)$.

Defining errors quantities: $\bar{e}_\alpha = \hat{e}_\alpha - e_\alpha$, $\bar{e}_\beta = \hat{e}_\beta - e_\beta$, and $\bar{w}_e = \hat{w}_e - w_e$, equation (12) can be rewritten as:

$$\begin{aligned}
\dot{\bar{e}}_\alpha &= -\hat{w}_e \hat{e}_\beta + w_e e_\beta - l_2(\hat{e}_\alpha - e_\alpha) \\
\dot{\bar{e}}_\beta &= \hat{w}_e \hat{e}_\alpha - w_e e_\alpha - l_2(\hat{e}_\beta - e_\beta) \\
\dot{\bar{w}}_e &= (\hat{e}_\alpha - e_\alpha)\hat{e}_\beta - (\hat{e}_\beta - e_\beta)\hat{e}_\alpha
\end{aligned} \tag{13}$$

A Lyapunov function candidate V is used to prove the observer convergence:

$$V = \frac{1}{2}\left(\bar{e}_\alpha^2 + \bar{e}_\beta^2 + \bar{w}_e^2\right) \tag{14}$$

its derivative is expressed using (13):

$$\begin{aligned}
\dot{V} &= \dot{\bar{e}}_\alpha \bar{e}_\alpha + \dot{\bar{e}}_\beta \bar{e}_\beta + \dot{\bar{w}}_e \bar{w}_e \\
&= -\hat{w}_e \hat{e}_\beta e_\alpha + w_e e_\beta e_\alpha - l_2(\hat{e}_\alpha - e_\alpha) e_\alpha \\
&+ \hat{w}_e \hat{e}_\alpha \bar{e}_\beta - w_e e_\alpha \bar{e}_\beta - l_2(\hat{e}_\beta - e_\beta)\bar{e}_\beta + \dot{\bar{w}}_e \bar{w}_e \\
&= -l_2(\bar{e}_\alpha^2 + \bar{e}_\beta^2)
\end{aligned} \tag{15}$$

Implying that $\bar{e}_\alpha$ and $\bar{e}_\beta$ tend to 0 thus $\hat{e}_\alpha$ and $\hat{e}_\beta$ converge respectively to e_α and e_β.

Rewriting (13) as:

$$\begin{aligned}
\dot{\bar{e}}_\alpha &= -\bar{w}_e \hat{e}_\beta - w_e \bar{e}_\beta - l_2 \bar{e}_\alpha \\
\dot{\bar{e}}_\beta &= \bar{w}_e \hat{e}_\alpha + w_e \bar{e}_\alpha - l_2 \bar{e}_\beta \\
\dot{\bar{w}}_e &= \bar{e}_\alpha \hat{e}_\beta - \bar{e}_\beta \hat{e}_\alpha
\end{aligned} \tag{16}$$

and considering $\bar{e}_\alpha = 0$ and $\bar{e}_\beta = 0$, we finally obtain

$$\begin{aligned}
-\bar{w}_e \hat{e}_\beta &= 0 \\
\bar{w}_e \hat{e}_\alpha &= 0
\end{aligned}$$

implying that $\bar{w}_e$ tend to 0 thus $\hat{w}_e$ converge to w_e.

1.5 Simulink Implementation

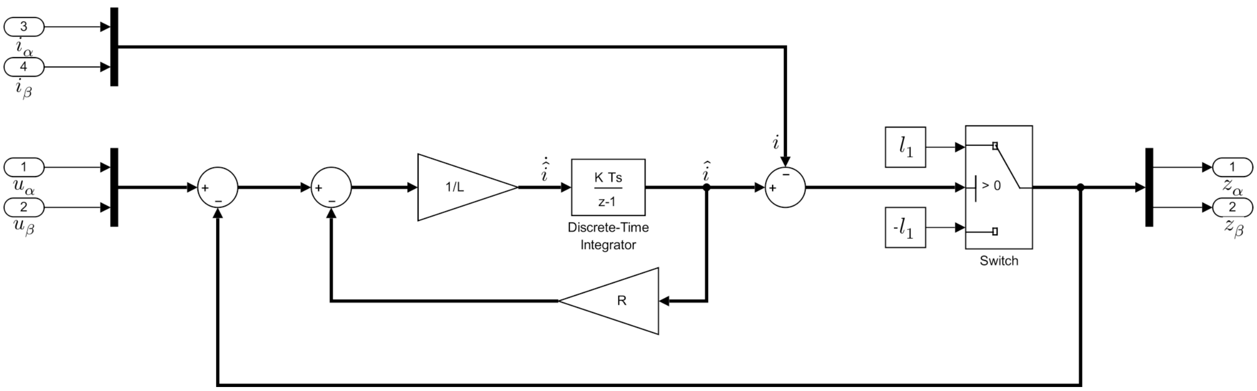

Figure 1: Simulink implementation of a Sliding Mode Observer. This system is equivalent to the equation (7). The two inputs quantities are the current measurement $[i_\alpha \ i_\beta]$ and the voltage applied $[u_\alpha \ u_\beta]$. The output values $[z_\alpha \ z_\beta]$ are "equivalent" to the back EMF. However theses two quantities are noisy (switching quantitie) and should not be used as is. The switch block mimick the sign function. It could be replaced by a gain block with a high enough value and a saturated output.

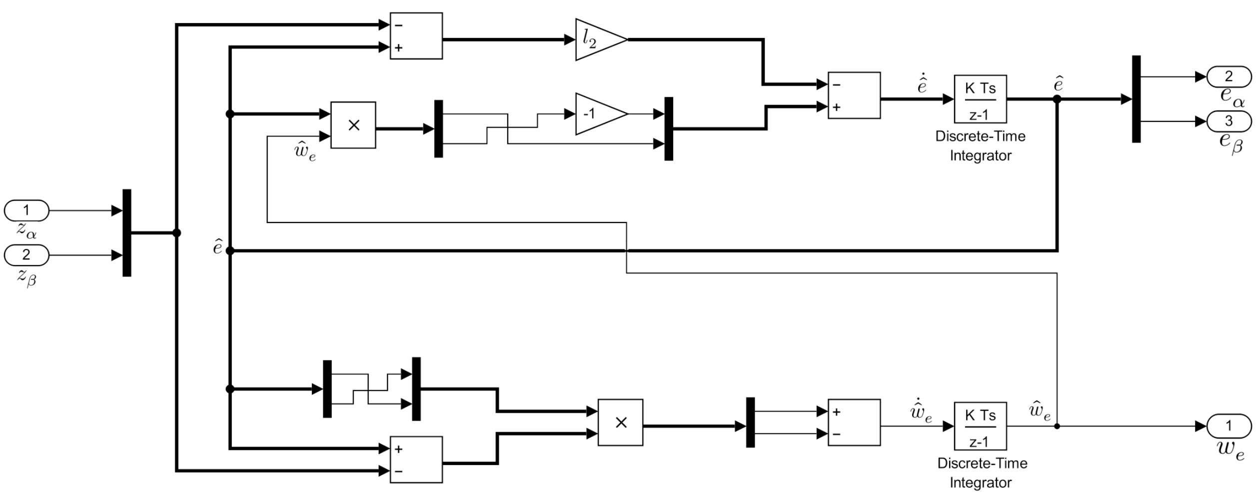

Figure 2: Observer smoothing the estimated back EMF estimated from fig.1. This system is equivalent to the equation (12).

References

[Utkin 09] Vadim Utkin, Juergen Guldner & Jingxin Shi. Sliding mode control in electromechanical systems, second edition. Automation and Control Engineering. Taylor & Francis, 2009.

[Zambada 07] Jorge Zambada. *Sensorless Field Oriented Control of PMSM Motors.* http://ww1.microchip.com/downloads/en/AppNotes/01078A.pdf, March 2007. Microchip Technology Inc, AN1078A.

[Zambada 10] Jorge Zambada & Debraj Deb. *Sensorless Field Oriented Control of PMSM Motors.* http://ww1.microchip.com/downloads/en/AppNotes/01078B.pdf, March 2010. Microchip Technology Inc, AN1078B.