Galactic Rotation Curve of the Milky Way

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In an effort to better explain the rotation of galaxies, which has been known to behave in a way not consistent with a standard Keplerian model, we used data taken from the Camp Evans satellite dish in New Jersey, USA to construct the galactic rotation curve of the Milky Way from 21-cm emissions spectra. This rotation curve exhibits a very interesting phenomenon, in that instead of velocity falling away with increasing distance, it becomes roughly flat at galactic radii above 3 kiloparsecs. With this rotation curve, we examine three different theoretical models to determine which model best fits our data. In the end, it is clear from the rotation curve that a Keplerian model does not work; however, modeling the baryonic mass distribution of the universe as a disk, surrounded by a cored isothermal sphere of some additional unseen non-baryonic mass (as we might expect from the long-ago work of Fritz Zwicky), we find that we can model this galactic rotation curve quite well. Finally, we use this model to obtain an estimate for the total mass of the Milky Way.

INTRODUCTION

The issue of discrepancies in mass measurements of galaxies has long been an issue plaguing astronomers, with some of the earliest indications that something is wrong with our picture of galactic rotation coming from researchers Fritz Zwicky and Jan Oort. In the 1930s, Zwicky studied the Coma galaxy cluster. He found that by measuring the radius and root-mean-squared velocity of the cluster, one could obtain an estimate for the mass of the galaxy purely from its kinematics. Problematically, however, this estimate is an order of magnitude greater than the sum of all the visible mass in the Coma cluster, so where is all the extra mass predicted by kinematics?

Another large discrepancy was discovered by Jan Oort. He discovered that one can reduce the motion of galaxies and galaxy clusters for the most part down to two constants, known as Oort's Constants. Here, the issue is that, theoretically, a Keplerian model for galactic rotation would lead one to expect, for the Milky Way, the constants to be roughly $A \approx 21$ km/s/kpc and $B \approx -7$ km/s/kpc. Experimental measurements, however, put these two constants at roughly $A \approx 15$ km/s/kpc and $B \approx -12$ km/s/kpc, which is far more consistent with a model predicting a flat rotation curve (i.e. a model in which velocity is constant relative to changes in radius).

In this paper, then, we address these strange phenomena and provide more data that may help to answer these questions. Firstly, we obtain, from 21-cm emissions spectra, a rotation curve for the Milky Way galaxy. It appears to be approximately a flat rotation curve (as is roughly consistent with Oort's constants). Then, we consider several theoretical models with respect to our data, and find that if we posit that there is some kind of unseen matter found in roughly the proportions predicted by Zwicky's work, then we obtain a surprisingly accurate model for said flat rotation curve.

METHODOLOGY

First, using spectral data from the 60' Camp Evans satellite dish, we analyze data taken at 43 different galactic longitudes. The data measured were power values for various emissions frequencies near 1420 MHz (corresponding to 21-cm wavelength emissions). The frequencies correspond directly to Doppler-velocities, via the Doppler relation

$$f_{obs} = \frac{1 + v_D}{c} f_{21} \implies v_D = c \frac{f_{obs}}{f_{21} - 1},$$

where f_{obs} is the measured frequency, f_{21} is the frequency of 21-cm emissions (precisely 1420.41 MHz), c is the speed of light, and v_D is the deduced Doppler velocity.

Power data corresponds proportionally to the temperatures measured ($P \propto T$), so all power data was converted to temperature. The various sensor channels of the satellite used different power gains, so each channel's power measurements were proportionally adjusted by their respective gain. Spurious data from problematic sensors were removed by hand from all data sets. Finally, each data set was adjusted for background noise by modeling the background linearly, and subtracting the resulting line from the data.

ANALYSIS AND RESULTS

The immediate result of the measurements is a heatmap, where each galactic longitude-Doppler velocity pair is associated with a specific temperature measurement. Find this heatmap in Figure 1.

For each longitude, the velocity of the Earth projected onto the direction in which the telescope points was also recovered, as well as the minimum relative velocity of approach toward the Earth of other detected objects. The minimum relative velocity of approach toward

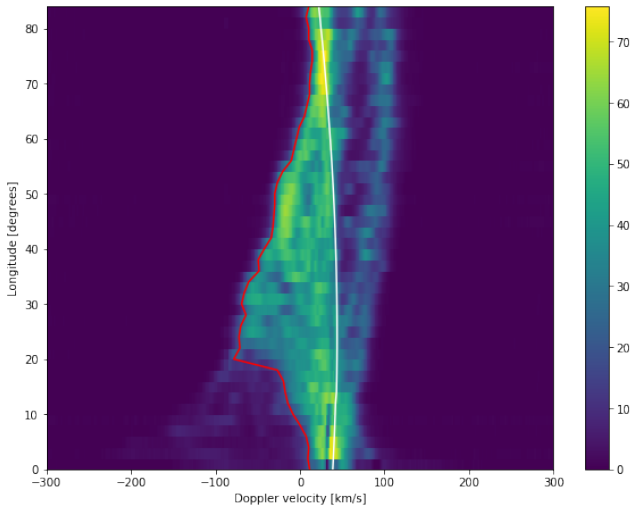

FIG. 1. Heatmap showing temperature [K] for each Doppler velocity/longitude pair. White line indicates Earth's velocity for each longitude. Red line indicates the minimum relative velocity of approach toward Earth for objects in the galaxy.

the Earth is important because we can deduce from it rather immediately the absolute velocity of recession of objects in the Milky Way, which are the velocities we are most interested for in constructing our rotation curve. The Earth's velocity projections and minimum relative velocities of approach are plotted on the heatmap in white and red respectively.

By subtracting the minimum relative velocities of approach from the velocity of the Earth, we obtain the maximum velocities of recession, corrected for our own velocity as observers on Earth; in particular, for minimum relative velocity of approach v_{min}, the projected velocity of Earth v_E, and maximum velocity of recession v_{max}, we find

$$v_{max} = -(v_{min} - v_E) = v_E - v_{min}$$

The absolute circular velocity of a target at radius r is $v(r) = r\Omega(r)$, where $\Omega(r)$ is the object's angular velocity. The relative circular velocity measured by an Earth-bound observer is $v_{max} = r\Omega(r) - r\Omega_\odot$ ($\Omega_\odot$ is the sun's galactic angular velocity). Combining these, we can write $v(r) = v_{max} - r\Omega_\odot$. Further, galactic radius may be measured by an Earthly observer as $r = R_\odot \sin\ell$, where $R_\odot$ is the sun's galactic radius and ℓ is the galactic longitude of measurement from Earth. Thus, we obtain

$$v = v_{max} + R_\odot \Omega_\odot \sin\ell,$$

which can be used to immediately deduce absolute velocities of recession from our measured data.

By then plotting the obtained values of rotational velocity versus their galactic radii, we obtain a rotation curve of the Milky Way (see Figure 2). This rotation curve exhibits interesting phenomena, in that it is roughly linear until approximately 3 kiloparsecs, at which point it holds nearly constant for increasing distance.

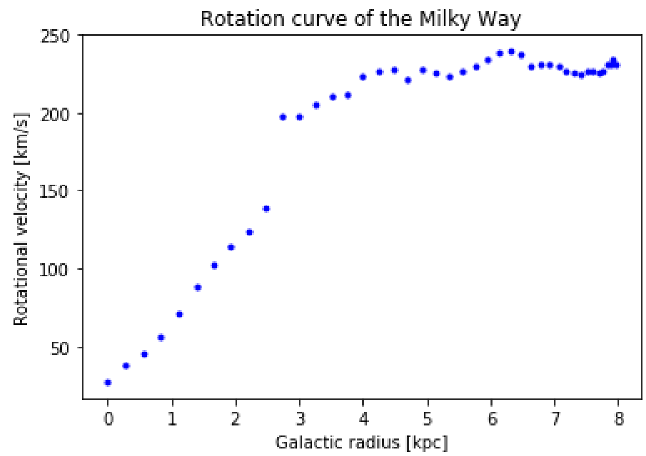

FIG. 2. The rotation curve of the Milky Way: gives rotational velocity [km/s] about the galactic center as a function of galactic radius [kpc].

RESULTS

With our rotation curve of the Milky Way, we now examine various theoretical models for Galactic Rotation and see which explains the rotation curve the best.

Keplerian Motion

The first we will consider is simple Keplerian motion. Assuming that all the mass of the Milky Way is concentrated at its center, we would find that the galaxy obeys Kepler's laws as they apply to our solar system; namely, we would find the velocities of objects at a galactic radius of r to be

$$v = \sqrt{GM/r}$$

where G is Newton's gravitational constant and M is the concentrated mass at the galactic center. Considering data for galactic radius of 3 kpc and above, we find that this model does not fit our rotation curve very well. See a plot of the Keplerian rotation curve compared to our data in Figure 3.

Exponential Disk Mass Distribution

The next model we will consider is the "Exponential Disk" model. Instead of assuming that a galaxy's mass is concentrated at its center, we instead assume that a galaxy's mass is concentrated cylindrically symmetrically about the center, with a certain exponential falloff as radius increases. This would give velocities of objects at a

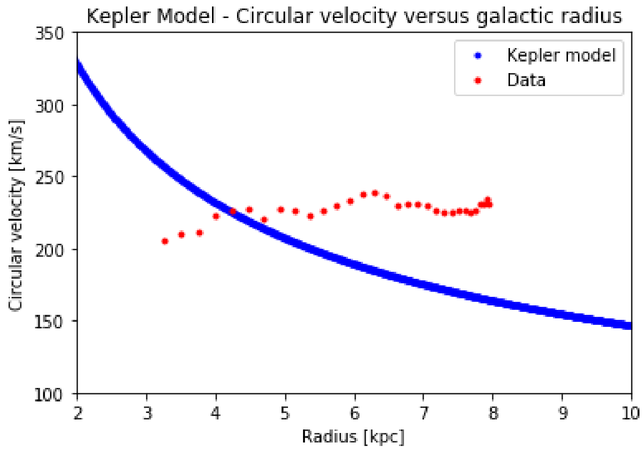

FIG. 3. The rotation curve as predicted by the Keplerian model, compared to the rotation curve found by our measurements.

radius of r to be given by

$$v(r)^2 = \frac{GMr^2}{2R_D^3}(I_0(u)K_0(u)I_1(u)K_1(u))$$

with G Newton's gravitational constant, M the mass of all objects interior to the current radius r, R_D being the characteristic radius of this exponential disk, $u \equiv r/2R_D$, and I_n and K_n being the nth-order modified Bessel functions of the first and second kind. Using this model, we obtain a rotation curve that more accurately fits the data. Find a plot of the Keplerian and exponential disk models compared to the data in Figure 4.

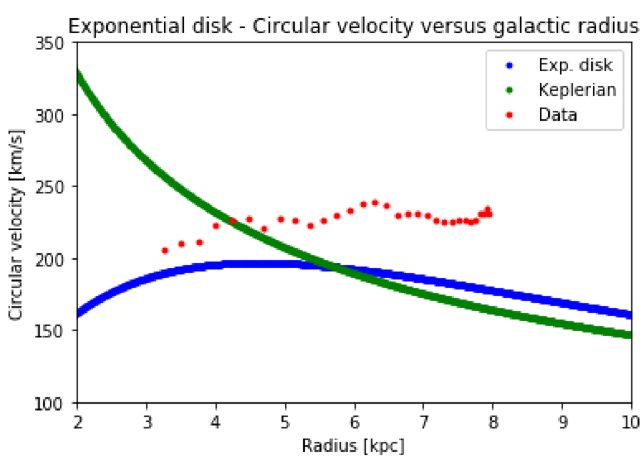

FIG. 4. Here, the Keplerian model is compared to the "exponential disk" model, which are plotted alongside the data from experimentation.

Cored Isothermal Sphere of Dark Matter

While the disk model is certainly closer, it still is not a great fit. Thus, we consider a third model: take the exponential disk for baryonic matter in the galaxy, but assume there exists also a cored isothermal sphere of some kind of unseen matter, like a halo about the galaxy. (This unseen matter will hereafter be named "dark matter," because it is not visible.) Suppose the dark matter halo has mass density

$$\rho(r) = \rho_0 \frac{R_\odot^2 + a^2}{r^2 + a^2}$$

where $R_\odot$ is the Sun's galactic radius and a is the characteristic radius of the halo, to be determined experimentally as a best-fit parameter. Thus, the amount of mass given inside a radius r would be

$$M_{iso}(r) = 4\pi\rho_0(R_\odot^2 + a^2)(r - a\tan^{-1}\tfrac{r}{a})$$

and we define ρ_0 as

$$\rho_0 = \frac{M_{iso,\odot}}{4\pi(R_\odot^2 + a^2)(R_\odot - a\tan^{-1}\frac{R_\odot}{a})}$$

Here, we introduce a second free parameter, $M_{iso,\odot}$, which is $M_{iso}(R_\odot)$ and will be determined again experimentally as a best-fit parameter. Now, to find the total circular velocity, we must add the effects of both the baryonic disk and dark matter halo. Because centripetal forces add as $F_{tot} = F_{disk} + F_{halo}$, we find the total circular velocity to be

$$v_T = \sqrt{v_{disk}^2 + v_{halo}^2}$$

We calculate v_{disk} in the same way as for the previous model, and we calculate $v_{halo} = \sqrt{GM_{iso}(r)/r}$. Then, by trying various values of the free parameters a and $M_{iso,\odot}$, it appears that letting $M_{iso,\odot} = 4\times10^{10}$ solar masses and $a = 4$ yields a very accurate model for our experimental rotation curve.

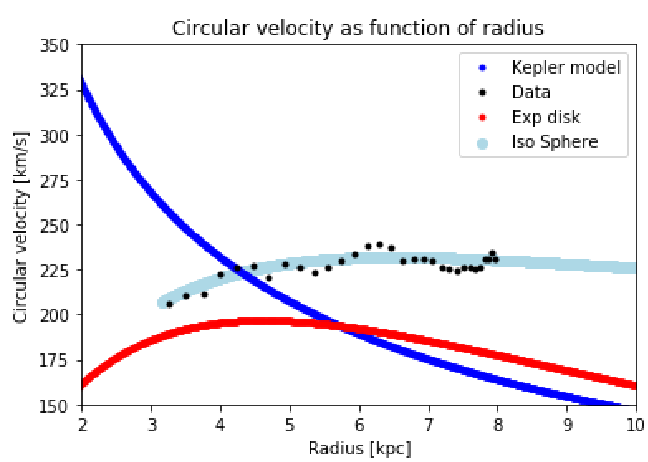

FIG. 5. A comparison of all considered models. The "dark matter" model uses best-fit parameters $a = 4$ and $M_{iso}(R_\odot) = 4 \times 10^{10}$ solar masses.

As can be seen in Figure 5, the model which assumes the existence of dark matter most accurately represents the data in the $r > 3$ kpc range, providing strong evidence for the potential existence of this matter and reinforcing the findings of Zwicky and Oort.

Assuming this model to be accurate, we can thus estimate the mass of the Milky Way including the mass contributed by dark matter! Using the previous formula for $M_{iso}(r)$, we simply use $r = R_{galaxy}$, the radius of the Milky Way (120 kpc) and the above best-fit values for the free parameters to find that the mass of the Milky Way is approximately 1.28×10^{12} solar masses.

CONCLUSIONS

To summarize, we have here been able to plot the rotation curve for the Milky Way galaxy using data recently taken from the Camp Evans satellite dish. This rotation curve exhibits an interesting phenomenon: at around 3 kpc, the measured velocities for objects no longer depend on distance, but instead remain approximately constant. We then compare our rotation curve with experimental models of galactic rotation, finding that a Keplerian explanation of the galaxy fails, and that the most useful explanation for the galactic rotation we see is that the baryonic matter in the Milky Way has a exponential disk-like distribution, complemented by a halo of some unseen matter called “dark matter,” modeled by a cored isothermal sphere. This model fits the data remarkably well, indicating that perhaps a rich topic for further research is this unseen dark matter. Further, we can estimate, using the best fit parameters for this dark matter halo model, that the total mass of the Milky Way is around 1.28×10^{12} solar masses. Compare that to the mass of the Milky Way’s baryonic matter at around 5×10^{10} solar masses, and we can see that this posited unseen matter must be present in relatively substantial amounts. Further work must be done, however, into the nature of this non-baryonic “dark” matter. Further research should also be performed to ensure that this rotation curve is not simply a unique feature of the Milky Way, but is instead present generally in galactic rotation.

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