Effects of dark matter self-interaction on the evolution of the Sagittarius stream

Connor Hainje
Advised by Professor Mariangela Lisanti

May 2021

Abstract

asdf

Contents

Chapter 1

Lambda-CDM

1.1 Evidence for dark matter

In 1932, Jan Oort studied the velocities of stars near the Sun, and he found that the velocities of these stars were systematically larger than expected (Oort 1932). From the velocities of stars at a given radius in a galaxy, one can measure the gravitational mass of all objects inside that radius. As such, Oort's velocity findings were really a statement about the mass of the Milky Way. In Oort's own words, these velocities implied that the amount of gravitating matter in the Milky Way must be larger than what one can estimate by simply counting stars, with the latter falling short by roughly 30-50% (Trimble 1987).

The next year, Fritz Zwicky studied velocity dispersions in the Coma galaxy cluster (Zwicky 1933). From his measurements, he found that the velocity dispersions required "10 to 100 times more mass" for the cluster to remain bound than could be accounted for by luminous matter. He concluded that there must be some large source of non-luminous matter present to account for the difference, calling the non-luminous matter "dunkle Materie", or "dark matter". This is often cited as the origin of the term.

Over the next few decades, evidence continued to mount for the existence of dark matter. Rotation curves began being considered, first by Babcock in 1939 (Babcock 1939) and Oort in 1940 (Oort 1940). In the words of Oort, "the distribution of mass in this object [the M31 galaxy] appears to bear almost no resemblance to that of light." Studies of globular clusters in the Milky Way indicated that at least two-to-three times as much mass must lie outside the orbit of the Sun as inside (Kunth 1952, Lohman 1956). Studies of binary galaxies revealed large mass-to-light ratios (Page 1960, van den Bergh 1961, Holmberg 1954). More galaxy clusters were studied, and the trends noted by Zwicky were found to hold generally (Trimble 2013). This mounting evidence largely took the form of acknowledgement of a large mass-to-light ratio; it was not widely thought that a yet unknown form of matter was responsible.

By the 1970s, however, that would change. This was in large part due to developments on a few different fronts. First, the 1960s and 70s saw new methods for measuring the masses of galaxy clusters. Both X-ray measurements and gravitational lensing first began being used in the 1960s to obtain measurements of the masses of galaxy clusters. These studies independently confirmed the large, non-luminous masses that were indicated by the previous velocity analyses.

Second, the cosmic microwave background was discovered in 1965 (cite), and found to be remarkably isotropic (to better than a part in 10^4). However, strictly baryonic models of the universe suggest anisotropies in the CMB on the order of $\sim 3 \times 10^{-4}$. If the Universe contained some non-baryonic matter which interacted with photons only gravitationally, however, the predicted anisotropies are revised downward, closer to 10^{-5}, consistent with observation. Dark matter seems to fit this role perfectly. Anisotropies on this scale have been observed by more recent studies, beginning with the COBE satellite in 1992.

Third, data from rotation curves became quite convincing. Rubin and Ford published improved optical data from the galaxy M31, the same galaxy studied by Oort in 1940 (RF 1970). They found that, to even farther radii than had been previously considered, the rotation curve did *not* decrease outside the optically bright region of the galaxy, an even stronger indicator of the existence of mass not accounted for by luminous matter. Similarly, Roberts and Rots (1971) and Roberts and Whitehurst (1975) used measurements of 21cm hydrogen emissions to draw very similar conclusions.

Lastly, 1974 saw two watershed papers by independent research groups, each concluding that galaxies have dark halos. Einasto, Kaasik, and Saar wrote “the mass of galactic coronae halos exceeds the mass of populations of known stars by one order of magnitude, as do the effective dimensions” (EKS 1974). Similarly, Ostriker, Peebles, and Yahil wrote “the very large mass-to-light ratio and the very great extent of the spiral galaxies can perhaps most plausibly be understood as due to a giant halo of faint stars” (OPY 1974). They also note that the year prior, Ostriker and Peebles had come to a similar conclusion from considerations of stability, finding the traditional disk galaxy model to be unstable without a spherical disk (OP 1973).

1.2 Lambda-CDM

Since these discoveries, it has been widely believed among astronomers and cosmologists that non-baryonic dark matter exists, that it comprises large, spherical halos around galaxies, and that it played an integral role in the formation of the structures we can see today. Evidence has become increasingly strong with improved measurements of the anisotropy in the CMB (todo cite someone), improved rotation curves for more galaxies (todo cite someone), and better X-ray and gravitational lensing measurements.

Another major question of the twentieth century which came to be largely settled toward its end was that of the cosmological constant, Λ. This constant was introduced by Einstein into his theory of general relativity (todo explain why). Physically, this constant can be interpreted as the vacuum energy density of empty space. It is often called "dark energy" for this reason (Schneider 2015). In the late 1990s, measurements of Type Ia supernovae strongly constrained Λ to be positive, giving empty space a significant, constant vacuum energy (Riess 1998, Perlmutter 1999).

These findings, combined with large amounts of evidence for the Big Bang hypothesis, are all most simply described by the ΛCDM model, sometimes called the standard model of cosmology or the concordance model (Dodelson Schmidt 2021). The model describes the Universe as consisting of three components—the cosmological constant Λ, cold dark matter (CDM), and standard matter—operating under general relativity and coming from an origination event roughly 14 Gyr ago (the Big Bang). This rather simple model is remarkably successful at explaining the Universe.

Through its inclusion of the Big Bang event, the ΛCDM model gives an explanation for the origin of the CMB. In this model, the CMB is a residual radiation left-over from a period shortly after the Big Bang, where the Universe was hot (>10,000 K). As the Universe cooled and structures began to form, the photons from this period began to propagate freely in all directions.

The ΛCDM model also provides a well-tested account of the formation of structures in the Universe (Dodelson Schmidt 2021). In the early Universe, small gravitational perturbations caused matter to collapse into small structures. These small structures—effectively just perturbations in the mass density of the Universe—created gravitational potential wells, attracting other small structures and assembling together to produce larger ones. This works especially well for describing the distribution of dark matter in the Universe, as small pockets of dark matter converge to form subhalos, halos, and larger. The case for baryonic matter and galaxy formation is more complicated, involving baryonic processes, gas dynamics, and more, but is also well-explained in the régime of hierarchical structure formation. Simulations of hierarchical structure formation have found that it is able to account well for the observed distribution of structures on the scales of galaxies, galaxy clusters, and larger. The implications of this model of structure formation are considered in more detail in the next section.

In fact, the success of this theory of structure formation serves as reason to believe that dark matter is cold (hence the "C" in ΛCDM). Dark matter being "cold" means that it was non-relativistic at the time of decoupling (the period in the early Universe when matter began to fall out of thermal equilibrium) (Mohanty 2020). By contrast, "hot" dark matter (HDM) would have been relativistic at the time of decoupling. Given these higher velocities, the velocity dispersion of HDM would have been non-negligible (differing from the negligible velocity dispersion of CDM). A finite velocity dispersion, however, would have prevented the dark matter particles from being bound to shallow gravitational potential wells, further preventing the formation of small-scale structures (Schneider 2015). Thus, HDM fails to form structure.

On its own, ΛCDM is remarkably able to explain most of our observations of the Universe. However, there are a few cosmological problems which have arisen. One is known as the horizon problem (Bergstrom Goobar 2008, Schneider 2015), which points out the the CMB is, to a high degree, isotropic, indicating that the photons making up this radiation are roughly in thermal equilibrium across the sky. However, this means that regions of the sky which are too far separated to be causally connected have remained in equilibrium. This is not explained by ΛCDM.

Another problem is the flatness problem, which points out that there is potentially a fine-tuning problem with the vanilla ΛCDM model (Bergstrom Goobar 2008, Schneider 2015). The Universe as we observe it is approximately flat today, meaning that its curvature is approximately unity. If the curvature were not unity, its deviation from flatness is expected to grow with time. As such, if the Universe is not flat, it must have been *very* close to unity at early times, meaning that this parameter may need to be "fine-tuned" to achieve a reasonable explanation. The problem is that our model should be able to describe parameters like these *without* fine-tuning.

Combining the ΛCDM model with the theory of cosmic inflation, however, solves both of these problems (Bergstrom Goobar 2008). Inflation posits that the Big Bang was followed by a period of rapid (exponential!) inflationary expansion. Such a phenomenon would solve the horizon problem, as the whole observable Universe would have originated from a much smaller, thermally-connected region before inflation. It also solves the flatness problem, as inflation would have significantly stretched and flattened any curvature in the Universe, providing a natural explanation for the observed flatness today. In fact, this creates a sort of inverse fine-tuning problem: in this theory, any curvature which is *far* from unity requires fine-tuning.

1.3 Structure formation

As stated above, the ΛCDM model predicts structures to form in the Universe as a result of gravitational perturbations at early times. As expansion slows, the pockets of matter created by these perturbations host small gravitational potential wells, causing nearby matter to collapse inward and the growth of structure to occur. With the majority of matter in the Universe being dark matter, the first structures which occur are pockets of dark matter halos. The halos observable today are the result of the hierarchical combination of smaller halos.

The abundance of these halos can be described by extended Press-Schechter

(EPS) theory (Bullock and Boylan-Kolchin 2017). This theory is grounded on rather simple assumptions: that one can use a spherical collapse model and that one can extrapolate from linear perturbation theory even into non-linear regimes. Despite these assumptions, it has been shown to give predictions about the mass spectrum of dark matter halos which are in accord with high-resolution numerical simulations.

The halos themselves can be described as virialized objects with mass

$$M_{\rm vir} = \frac{4\pi}{3} R_{\rm vir}^3 \Delta_c \rho_c,$$

where $R_{\rm vir}$ is the virial radius, ρ_c is the critical density of the Universe, and Δ_c is the over-density parameter. In this work, we choose to set $\Delta_c = 200$ and thus mark the virial mass and radius as with a subscript 200 instead of "vir".

The ΛCDM model also provides predictions about the internal structure of these dark matter halos. Dark matter-only N-body simulations were performed as early as 1988 (Frenk et al. 1988), with higher-resolution simulations like (Dubinski and Carlberg 1991) and (NFW 1996) following shortly thereafter. The resulting halos from these simulations were found to agree remarkably well with a *two-power* density profile (Binney and Tremaine)

$$\rho(r) = \frac{\rho_0}{(r/a)^\alpha (1 + r/a)^{\beta-\alpha}},$$

where ρ_0 is a characteristic density for the halo and a is the length scale. Special cases of this model include $(\alpha, \beta) = (1, 4)$, the Hernquist model (Hernquist 1990), and $(\alpha, \beta) = (1, 3)$, the NFW model (NFW 1996, NFW 1997). In general, the NFW model enjoys the most usage as one of the most simple and accurate models of the mass density distribution of smaller halos.

todo Include some figures

The required parameters above, ρ_0 and a, are typically reformulated for the NFW distribution. In particular, we can take a given virial mass M_{200}, determine the corresponding virial radius R_{200}, and define the halo concentration $c = R_{200}/a$. The characteristic density can be found by integrating the density profile up to R_{200} and setting it equal to M_{200}. In this way, the virial mass and concentration are enough to completely specify the NFW distribution. The result is given by

$$\rho_0 = \frac{M_{200}}{4\pi a^3} \frac{1}{\log(1 + c) - c/(1 + c)}.$$

As these halos are formed hierarchically from smaller halos, however, it has been found that the dense centers of the subhalos are able to survive the merging process. A direct result of this finding is that dark matter halos today should be full of substructure with satellite subhalos of varying sizes. In fact, simulations have shown that the number of subhalos within a halo is approximately self-similar for host halo mass (Bullock and Boylan-Kolchin 2017). While it is difficult to determine a method for identifying and counting these subhalos, it does yield a testable prediction of the small-scale structure of dark matter in the ΛCDM paradigm.

todo Discuss galaxy formation with an eye for how this leads to the missing satellites and too-big-to-fail problems

1.4 Small-scale problems

The theory of structure formation that follows from ΛCDM leads us to a few important results. First, we expect the density distributions of dark matter halos to approximately follow the NFW model, specifically with r^{-1} dependence for small r. Second, we expect rich substructure in massive dark matter halos with predictions for the number of subhalos of varying masses contained within. Third, through galaxy formation and the method of abundance matching, we obtain predictions about the number and masses of satellite galaxies that should be hosted in galaxies like the Milky Way. These three key deductions form the basis for the three classic **small-scale problems** in the ΛCDM model.

The first problem we will discuss is known as the **core-cusp problem**, which is primarily related to the first deduction above. Dark matter-only simulations of CDM halos show that the expected distribution of mass follows a density distribution that is very dense and *cuspy*, i.e. one that goes like r^{-1} at small radii (Bullock and Boylan-Kolchin 2017). As early as 1993, however, it was pointed out that such a distribution is inconsistent with observational data. In particular, "[cuspy] density profiles are excluded by gravitational lensing analyses on cluster scales and by the rotation curves of gas-rich, halo-dominated dwarf spirals on small scales" (Flores and Primack 1993). It was also recognized by Navarro, Frenk, and White, who wrote "CDM halos are too concentrated to be consistent with the halo parameters inferred for dwarf irregulars" (NFW 1996).

The second problem is the **missing satellites** problem. CDM simulations predict that galaxies as large as the Milky Way should have as many as $\sim 1_{0}00$ dark subhalos large enough to host dwarf galaxies (Bullock and Boylan-Kolchin 2017). However, as of 2019, less than 60 dwarf galaxies are known within the Milky Way (Simon 2019).

There exists a potential solution to the missing satellites problem, however, in the form of abundance matching. First, one can expect that dark matter halos become less efficient at making galaxies with decreasing mass, and therefore there exists some threshold mass below which these halos remain completely dark. Then, abundance matching allows one to "solve" the missing satellites problem for the subhalos above this threshold (Bullock and Boylan-Kolchin 2017).

This solution to the missing satellites problem thus makes a testable prediction.

The central masses of Milky Way satellites should be consistent with the central masses of the most massive subhalos in ΛCDM simulation (Bullock and Boylan-Kolchin 2017). First probed by Boylan-Kolchin et al. in 2011, however, this does not hold; the most massive subhalos predicted in simulation are significantly more massive than the most massive satellite galaxies. If these subhalos do exist but have just remained dark, then we have a new problem: why did these subhalos fail to form galaxies? This is known as the **too-big-to-fail problem**, as these subhalos ought to be too big to fail to form observable galaxies. If these massive dark subhalos just do not exist, then this is a fundamental failing of the ΛCDM model's predictions in line with the missing satellites problem.

Some believe that these small-scale problems are the result of the omission of baryonic effects in ΛCDM simulations. As described above, the majority of the conflicting predictions come as the result of dark matter-only simulations. Further, in recent years, it has been shown that all of the above problems (and other small-scale issues) can be reduced or eliminated by the inclusion of stellar feedback and other effects. For example, (Pontzen and Governato 2012) show that supernova feedback can alleviate the core-cusp problem, and (Chan et al. 2015) show that stellar feedback can explain the too-big-to-fail problem.

However, prior to these findings, researchers sought modifications to the model that would preserve the successes of ΛCDM on large scales *and* solve the small-scale problems. Some researchers have sought modifications to the model of gravity, but a more popular avenue has been the modification of the dark matter model. While it seems that integration of more sophisticated baryonic processes may be sufficient to solve the small-scale problems, there is no evidence yet that other proposed dark matter models are incorrect descriptions of the particle nature of dark matter. As such, further tests of ΛCDM and these alternative dark matter models are necessary as probes of dark matter particle physics. The alternative dark matter model that we consider in this work is *self-interacting dark matter*, or SIDM.

Chapter 2

Self-interacting dark matter

2.1 Introduction

Self-interacting dark matter (SIDM) was introduced in 2000 by Spergel and Steinhardt (Spergel and Steinhardt 2000). The model was proposed as a solution to the core-cusp and missing satellites problems, as the addition of self-interactions was thought to have three primary effects on the distribution of dark matter. 1. Self-interactions in regions of high density would cause dark matter particles to be unbound, reducing the density of halos primarily in their center region. This would yield a cored density profile rather than a cuspy one, solving the core-cusp problem. 2. It is also expected that interactions would yield a more isotropic velocity dispersion than seen in CDM as well as erase the typical triaxial ellipticity seen in halo shapes. In other words, SIDM halos should be more spherical, a potentially testable prediction. 3. Through the processes of isotropizing the velocity dispersion and reducing density in core regions, it was also expected that substructure would be greatly reduced, lowering the number of dwarf galaxies and thereby solving the missing satellites problem. However, since the dark matter scattering rate would be naturally dependent on the dark matter density, these effects would only be expected in higher density regions. Toward the outermost radii of halos and on larger scales, the effects of self-interaction would be negligible, preserving the large-scale successes of the ΛCDM model.

Shortly thereafter began a wave of numerical simulations to test these predictions. The results of these simulations were mixed. Some were focused on the evolution of galaxies and found confirmation of the predicted effects (more strongly spherical shape, cored density profile) like (Burkert 2000, Dave et al. 2001). Other simulations, like cluster and cosmological simulations, seemed to show that SIDM would be inconsistent with observation (Moore et al. 2000, Yoshida et al. 2000a, Yoshida et al. 2000b).

At the same time, constraints on the allowed self-interaction cross section began being compiled, primarily from clusters. (Meneghetti et al. 2001) used cluster simulations to limit the allowable cross section to mass ratio to < 0.1 cm^2/g. Similarly, gravitational lensing data from the MS 2137-23 cluster was used by (Miralda-Escude 2002) to limit the cross section to mass ratio to $< 10^{-25.5}$ cm^2/GeV, approximately 0.02 cm^2/g. These cross sections were far too small to solve the small-scale problems in galaxies, where simulations like (Dave et al. 2001) suggested the necessary cross section to be in the range 10^{-25} to 10^{-23} cm^2/MeV (0.05 to 5 cm^2/g).

However, more recent simulations with higher resolution and better statistics have relaxed many of these constraints significantly, improving the viability of the model and once again sparking interest in self-interactions. todo detail these new simulations and why they make sidm attractive

todo add a discussion of observations that indicate that the cross-section is velocity-dependent over large scales todo add one of those plots that shows it changing slowly for galactic scales todo add a discussion of what different kinds of astrophysical observations might mean different things about the particle physics properties

2.2 Predicted density profile

todo run through an analytic calculation of core size, etc. todo Jeans approach!

In 2014, Kaplinghat et al. provided a semi-analytic approach to deriving equilibrium solutions for the dark matter density profile using the Jeans equation. This approach allows for the inclusion of the gravitational potential of both the dark matter and baryons, providing the means to make predictions about the effects of baryons on the dark matter distribution.

They begin with the Jeans equation, rewritten via Poisson's equation. They assume constant velocity dispersion σ_0 and that the dark matter density profile is given by $\rho(\mathbf{r}) = \rho_0 \exp(h(\mathbf{r}))$. Plugging these in yields

$$\nabla_x^2 h(\mathbf{x}) + (4\pi G_N r_0^2/\sigma_0^2)[\rho_B(\mathbf{x}) + \rho_0 \exp(h(\mathbf{r}))] = 0,$$

where we have introduced a length scale r_0, a dimensionless length $x = r/r_0$, and the baryonic density profile ρ_B.

As a simple test of this equation, we can consider the case where baryonic matter dominates. Then, the $\exp(h)$ term can be neglected, yielding

todo work through this derivation with a few more steps

The solution in this case can then be written as

$$\rho(\mathbf{x}) = \rho_0 \exp[(\Phi_B(0) - \Phi_B(\mathbf{x}))/\sigma_0^2].$$

The authors then recommend defining the core radius as the radius at which the density is half ρ_0. Such a position would give $h(\mathbf{r}_c) = -\ln 2$, or

$$\Phi(0) - \Phi(\mathbf{r}_c) = -\sigma_0^2 \ln 2.$$

Thus, the core size would depend only on the baryonic potential in the case where it dominates, which follows from these assumptions but stands in marked contrast to observation, where the baryonic contribution does not dominate.

todo finish this discussion

2.3 Particle physics models

Given that very little is known conclusively about the particle physics nature of dark matter, the introduction of the possibility of self-interactions makes way for a wealth of rich new theories. We will cover a few of the most popularly considered models below.

2.3.1 Self-coupled scalar

The first particle model that we consider is the simplest: a scalar particle that interacts with itself through a two-to-two coupling. This can be described by the Lagrangian

$$\mathcal{L}_{\text{int}} = -\frac{\lambda}{4!}\varphi^4.$$

From the Lagrangian, we can read off the Feynman rule for a four-point intersection to have the matrix element $i\mathcal{M} = -i\lambda$, yielding the two-to-two self-interaction differential cross-section

$$\frac{d\sigma}{d\Omega} = \frac{\lambda^2}{64\pi^2(4m^2}.$$

Integrating over the solid angle gives a total cross section

$$\sigma(\varphi\varphi \to \varphi\varphi) = \frac{\lambda^2}{128\pi m^2}.$$

One can easily see that this cross section does not admit any kind of velocity independence. Thus, one could make this model consistent for a small subset of scales (e.g. $\sigma/m \sim 1$ cm^2/g for galaxy scales), but then it would necessarily fail on other scales. This makes the model okay only for analyses of limited scales where the cross-section is not expected to vary, but it is generally infeasible as a solution to the small-scale problems.

2.3.2 Light mediator

Perhaps the simplest model with a theory rich enough to solve all of the observed problems is one wherein dark matter self-interactions are mediated by a light particle. We will consider a model where dark matter is represented by χ and has mass m_χ, and the mediator field is ϕ with mass m_ϕ. This theory works with both scalar and vector mediators, depending on what specific theory one wants to consider. Perhaps the best motivated origin for such a model is one where the dark matter particle is charged under a spontaneously broken $U(1)$ symmetry and the mediator arises as the corresponding gauge boson (Tulin Yu 2018).

Such a model would have an interaction Lagrangian given by

$$\mathcal{L}_{\text{int}} = \cdots$$

where we let the coupling constant be g_χ. In the non-relativistic limit, the interaction is well-approximated by the Yukawa potential (Tulin, Yu, Zurek 2013a,b)

$$V(r) = \pm \frac{\alpha_\chi}{r} e^{-m_\phi r},$$

where $\alpha_c hi \equiv g_\chi^2/4\pi$ is the dark fine structure constant. The $\pm$ will be set depending on whether the interaction is attractive or repulsive. For a scalar ϕ, the potential is attractive and the sign is $(-)$. For vector ϕ, the potential is attractive $(+)$ for $\chi\overline{\chi}$ scattering and repulsive $(-)$ for $\chi\chi$ and $\overline{\chi\chi}$ scattering.

Using the Yukawa potential, we can obtain the Born differential cross section in the limit that $\alpha_\chi m_\chi / m_\phi \ll 1$ to be (Tulin, Yu 2018)

$$\frac{d\sigma}{d\Omega} = \cdots$$

An important implication of this formula is that the mediator mass must be positive, i.e.$m_\phi > 0$. If instead $m_\phi = 0$, we would then find that $d\sigma/d\Omega \propto v_{\text{rel}}^{-4}$, which is far too strong at small velocities to admit a solution which is consistent with observations. A small but nonzero mediator mass m_ϕ, on the other hand, allows us to “soften” this velocity-dependence to admit a more consistent model.

While quite simple, it has been shown in (todo cite) that it is possible for it to simultaneously accommodate all important observations and solve the small scale problems.

2.3.3 Strong interactions

Some of the richest theories for self-interacting dark matter candidates that one can consider are non-Abelian gauge theories where the dark matter candidates arise as composite bound states. In these theories, the self-interaction manifests as a strong interaction.

The motivation for considering such a model comes from our experience with QCD and the visible sector (todo cite). For a dark matter model to be a good candidate, it must be stable over the lifetime of the Universe and be neutral under standard model phenomena. Further, we desire models in which the particles exhibit strong self-interactions. These are all properties exhibited by particles in the visible sector under QCD, so it makes sense to consider a similar theory to describe our dark matter candidate. However, we do not necessarily know the gauge group or particle properties of dark matter, leaving us a great freedom to vary the model significantly. Many of the resulting models thus have interesting and unique new physics, though these details are greatly model-dependent.

The primary free parameters of models of this kind are the confinement scale Λ (different from the cosmological constant), and the dark quark mass(es). In the event that our "dark QCD" contains no analogue to electromagnetic/weak interactions, meson-like bound states of the dark quarks could be stable (todo cite). These mesons can be classified as loosely pion-like, where $m \ll \Lambda$, or quarkonium-like, where $m \gg \Lambda$ (todo cite). There are several proposed models for each of these scenarios; one of the more well-known is the strongly-interacting massive particle, or SIMP, where the dark matter candidate is pion-like and many non-Abelian theories are possible.

Our non-Abelian model may instead look quite similar to visible QCD, wherein the primary stable bound states are baryonic in nature. In (todo cite), it is noted that the advantage of such models is that "dark matter is automatically sufficiently stable, and no further ultraviolet model-building is needed." One such dark baryon model is "Stealth Dark Matter," proposed by the LSD collaboration, which is a scalar dark baryon under a confining $SU(4)$ theory. This theory is named *stealth* dark matter because it is found that the baryons are safe from direct detection, though it does predict a spectrum of lighter meson particles that would be possible to detect at colliders (todo cite).

The third class of candidate particles that has received attention are dark glueballs. Glueballs are bound states of only gluons and are predicted to exist in QCD, but are very difficult to detect. Dark glueballs would then be bound states of dark gluons. Such a model is possible if all the dark fermions in the theory have masses significantly larger than Λ. In this case, glueballs may become stable under an accidental symmetry like baryons, allowing them to be the primary dark matter candidate.e

The observables that could result from the above considerations are as diverse as the models themselves. One aspect of these models that wee have not considered is what the interactions with the standard model could look like. Some models predict the dark matter candidate to be neutral under standard model interactions, but its constituents to be charged. In such a case, the model would have a coupling to the photon, and it would be possible to directly detect the particle. We may also consider the case where our theory predicts fundamental fermions. It is plausible that these fermions would obtain at least part of their mass through a coupling to the Higgs boson, again providing a mechanism by which we could directly detect the particles. Kribs and Neil provide more details of these observables, as well as collider-specific results, in (todo cite).

2.4 In simulation

todo discuss how SIDM is represented in GIZMO following section 2 https://ui.adsabs.harvard.edu/abs/2013MN

In this work, we will be exploring the introduction of self-interaction in predictions about the infall of the Sagittarius dwarf galaxy and, specifically, the formation of its stream. This is done through the use of N-body simulations. As such, we present a description of how these self-interactions are modeled in simulation. We choose to use GIZMO (todo cite) for our simulations, and the implementation of self-interactions therein is the one described by (Rocha et al. 2012).

In our simulation, we consider some number of "macro-particles," each of which represents an ensemble of dark matter particles, a patch of the dark matter phase-space density. We let each macro-particle have mass m_p, and we keep this mass consistent across all dark matter macro-particles. Since we consider the macro-particle as representing a patch of the phase-space density, we consider its position to be centered at some point $\mathbf{x}$ but spread out according to a kernel $W(r, h)$. Here, r is the distance from the center of the macro-particle and h is a smoothing length. In GADGET-2, from which GIZMO is built, the kernel is given by

$$W(r, h) = \cdots$$

todo find whether this is the same for GIZMO. todo find how h is determined in GIZMO. The velocity of the macro-particle, on the other hand, is taken to be a delta function, such that the macro-particles have a single defined velocity.

When the patches represented by two macro-particles overlap, we can compute the interaction rate between them. The rate of scattering of a macro-particle j off a target particle i is given by

$$\Gamma(i|j) = (\sigma/m) m_p |\mathbf{v}_i - \mathbf{v}_j| g_{ji},$$

where σ/m is the familiar cross-section to mass ratio and g_{ij} is a number density factor whose purpose is account for the overlap of the two macro-particles' smoothing kernels. It is given by

$$g_{ji} = \int_0^h d^3\mathbf{x}' \, W(|\mathbf{x}'|, h) \, W(|\delta\mathbf{x}_{ji} + \mathbf{x}'|, h),$$

with $\delta\mathbf{x}_{ji}$ the displacement vector between the macro-particle positions.

Over the course of a time step δt, the probability of an interaction of macro-particle j off target macro-particle i is given by

$$P(i|j) = \Gamma(i|j) \, \delta t.$$

The total probability of interaction between these two particles in this time step, then, would be the average of the two directed probabilities, i.e.

$$P_{ij} = \frac{1}{2}\left(P(i|j) + P(j|i)\right).$$

To actually represent the interaction, then, one draws a random number and adjusts the velocities of the particles if the number lies below the probability. The velocities are adjusted in a manner consistent with an elastic scattering, isotropic in the center of mass frame.

More details are presented in (Rocha et al. 2012), including the derivation of the scattering rate formula from the Boltzmann equation. We use the implementation which is packaged with the publicly-available GIZMO simulation suite.