

COMP5211: Machine Learning

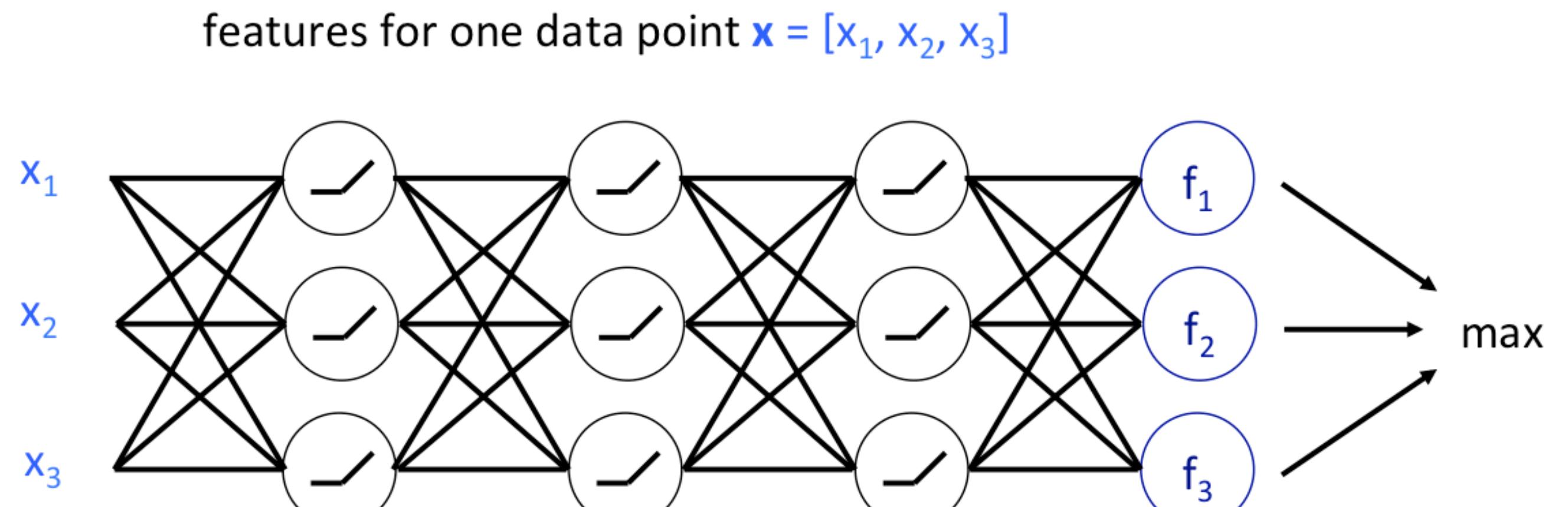
Lecture 13

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Neural Network

Multiclass Classification

- K classes: K neurons in the final layer
- Output of each f_i is the score of class i
 - Taking $\arg \max_i f_i(x)$ as the prediction



Neural Network

Multiclass loss

- Softmax function: transform output to probability:

- $[f_1, \dots, f_K] \rightarrow [p_i, \dots, p_K]$

- Where $p_i = \frac{e^{f_i}}{\sum_{j=1}^K e^{f_j}}$

- Cross-entropy loss:

- $L = - \sum_{i=1}^K y_i \log(p_i)$

- Where y_i is the i -th label

Convolutional Neural Network

Image classification without CNN

- Input an image
- Extract “interesting points” (e.g., corner detector)
- For each interesting point, extract 128-dimensional SIFT descriptor
- Clustering of SIFT descriptor to get “visual vocabulary”
- Then transform image to a feature vector (bag of visual words)
- Run classification (SVM)

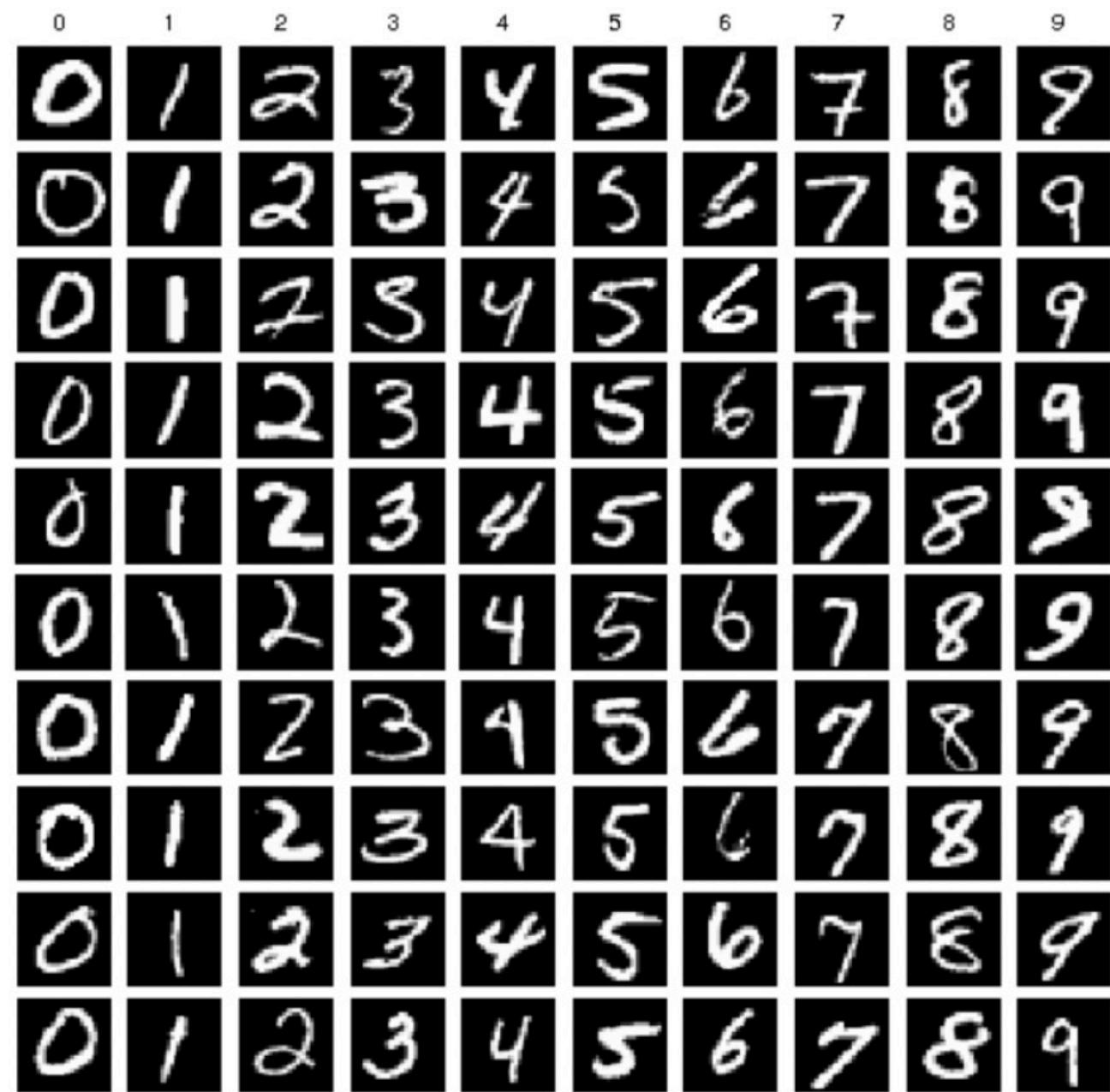


* Mushroom image by Tifred25 (http://commons.wikimedia.org/wiki/File:Bolet_Orange_01.jpg)

Dataset

MNIST

- Hand-written digits (0 to 9)
 - Total 60,000 samples, 10-class classification



Dataset

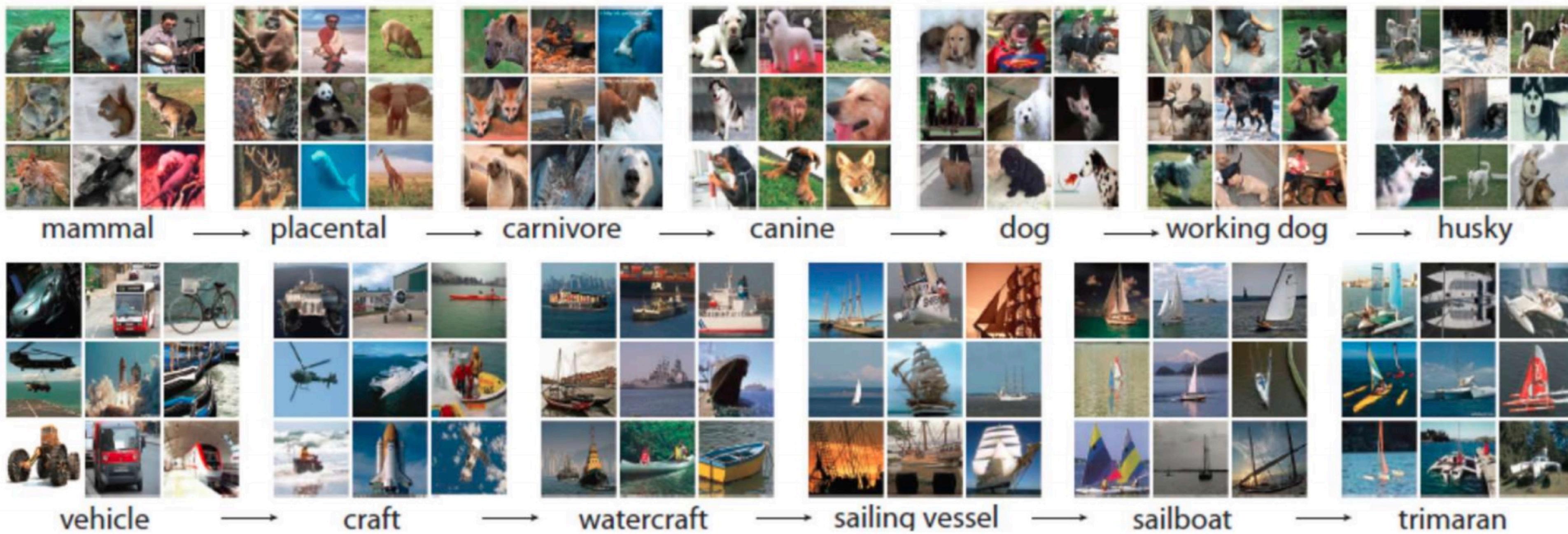
MNIST Classification Accuracy

- See the website by Yann LeCun: <http://yann.lecun.com/exdb/mnist/>

Classifier	Test Error
Linear classifier	12.0 %
SVM, Gaussian kernel	1.4%
SVM, degree 4 polynomial	1.1%
Best SVM result	0.56%
2-layer NN	~ 3.0%
3-layer NN	~ 2.5%
CNN, LeNet-5 (1998)	0.85%
Larger CNN (2011, 2012)	~ 0.3%

Dataset

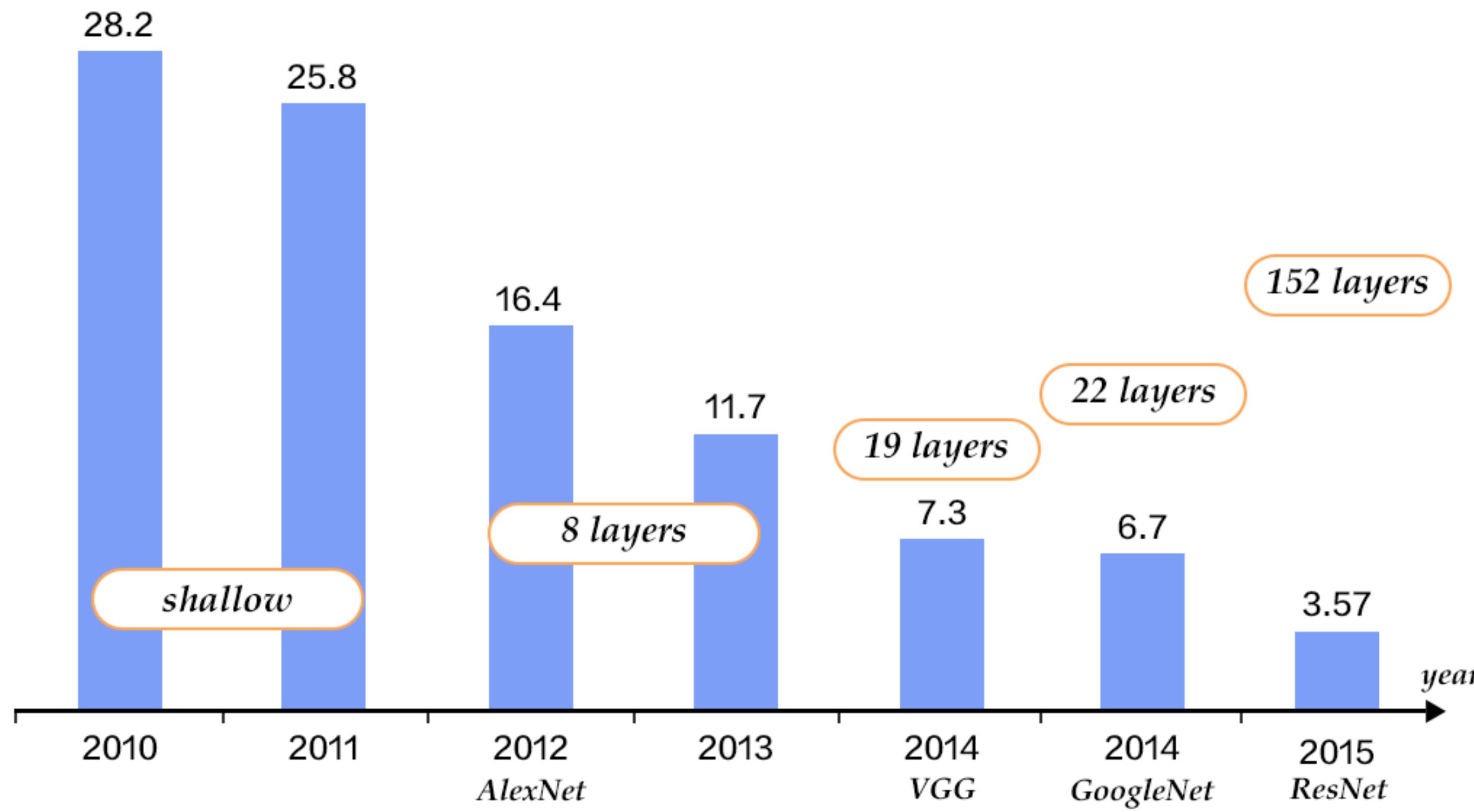
ImageNet Data



- ILSVRC competition: 1000 classes and about 1.2 million images
- Full imagenet: >20,000 categories, each with about a thousand images.

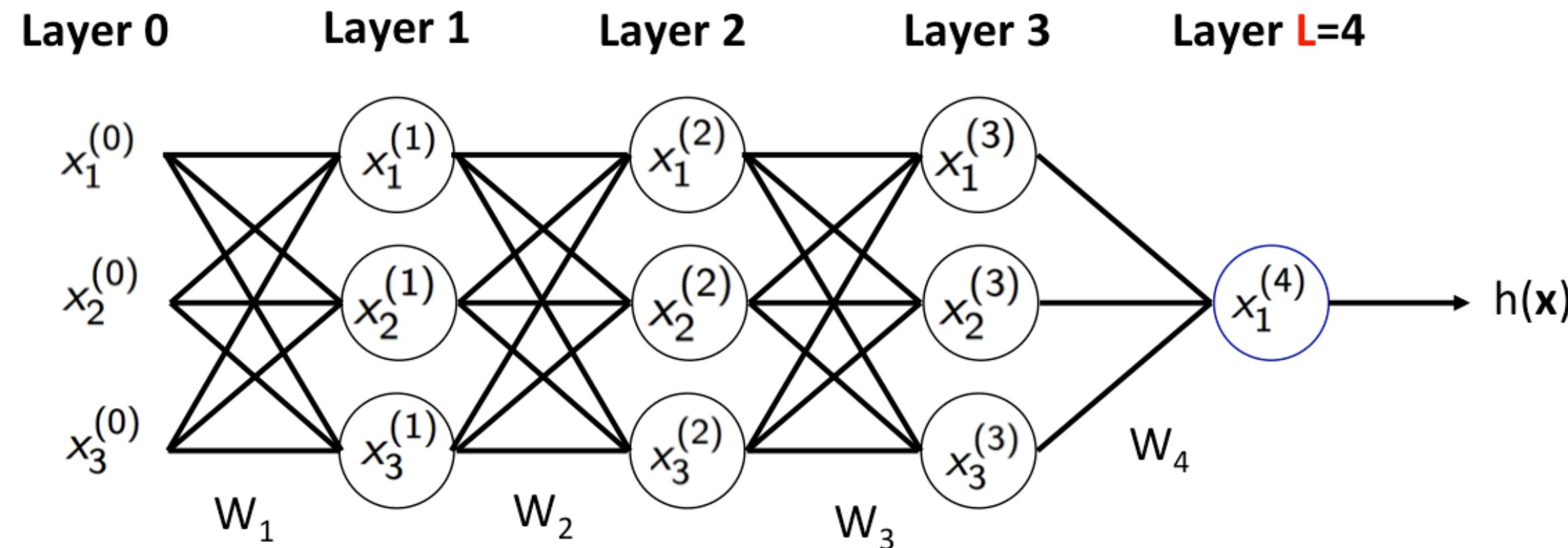
Dataset

ImageNet Results



Convolutional Neural Network

Neural Networks



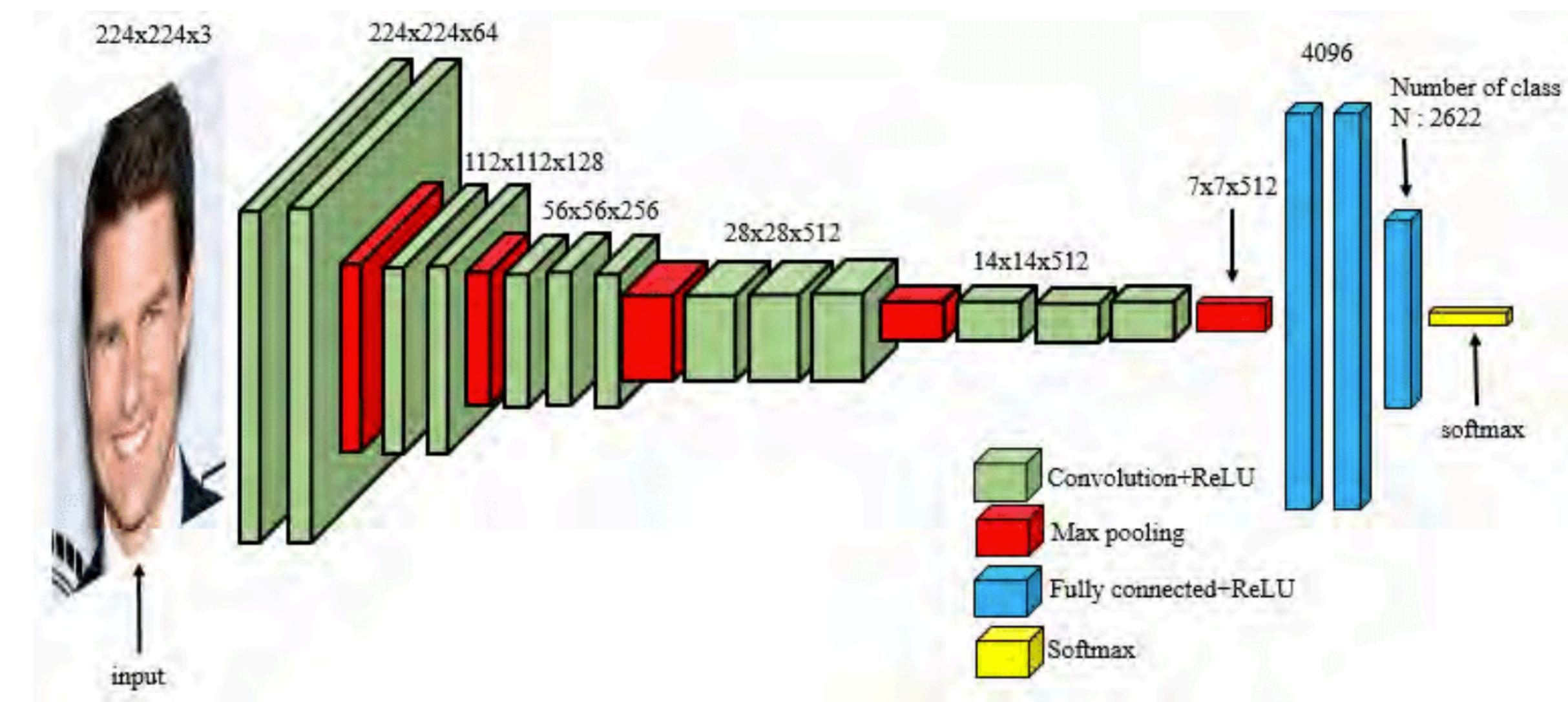
$$\begin{aligned} h(\mathbf{x}) &= x_1^{(4)} = \theta(W_4 \mathbf{x}^{(3)}) = \theta(W_4 \theta(W_3 \mathbf{x}^{(2)})) \\ &= \dots = \theta(W_4 \theta(W_3 \theta(W_2 \theta(W_1 \mathbf{x})))) \end{aligned}$$

- Fully connected networks \Rightarrow doesn't work well for computer vision applications

Convolutional Neural Network

The structure of CNN

- Structure of VGG



- Two important layers:
 - Convolution
 - Pooling

Convolutional Neural Network

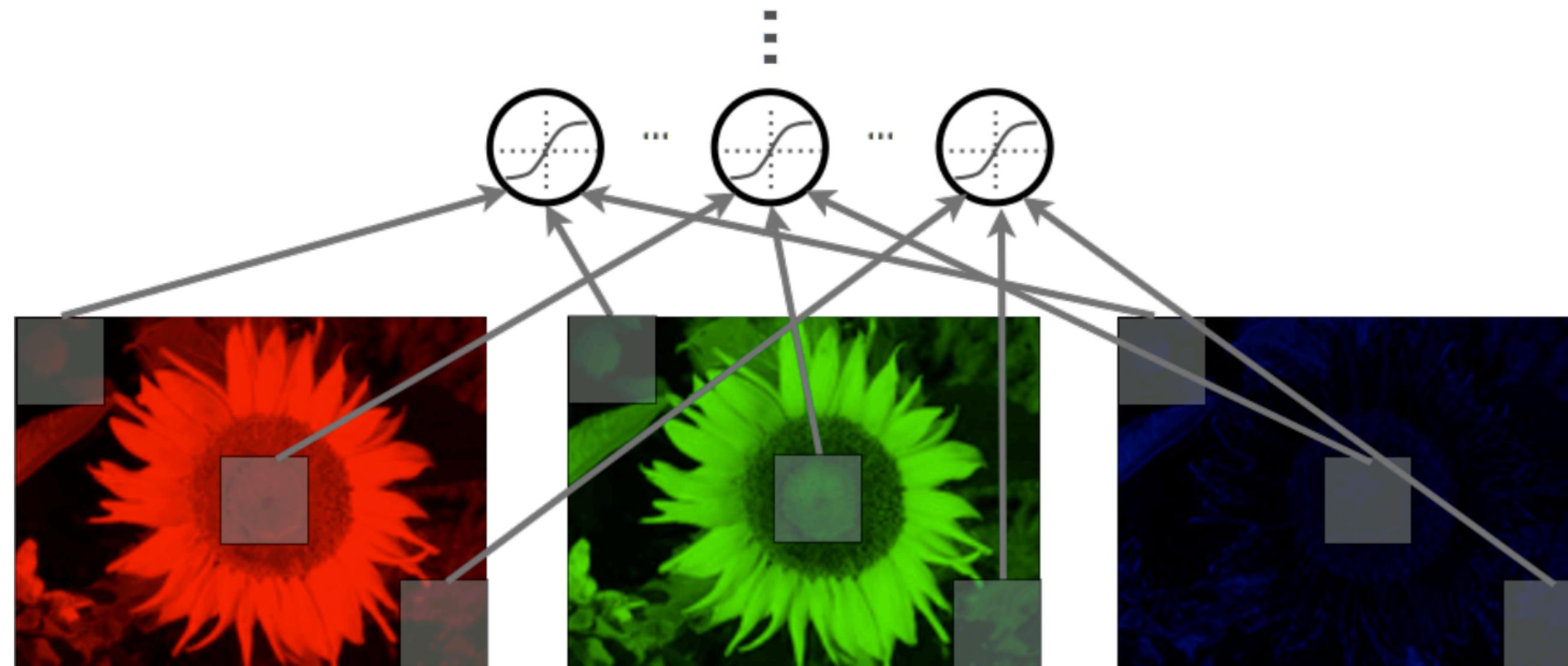
Convolution Layer

- Fully connected layers have too many parameters
 - \Rightarrow poor performance
- Example: VGG first layer
 - Input: $224 \times 224 \times 3$
 - Output: $224 \times 224 \times 64$
 - Number of parameters if we use fully connected net:
 - $(224 \times 224 \times 3) \times (224 \times 224 \times 64) = 483 \text{ billion}$
 - Convolution layer leads to:
 - Local connectivity
 - Parameter sharing

Convolutional Neural Network

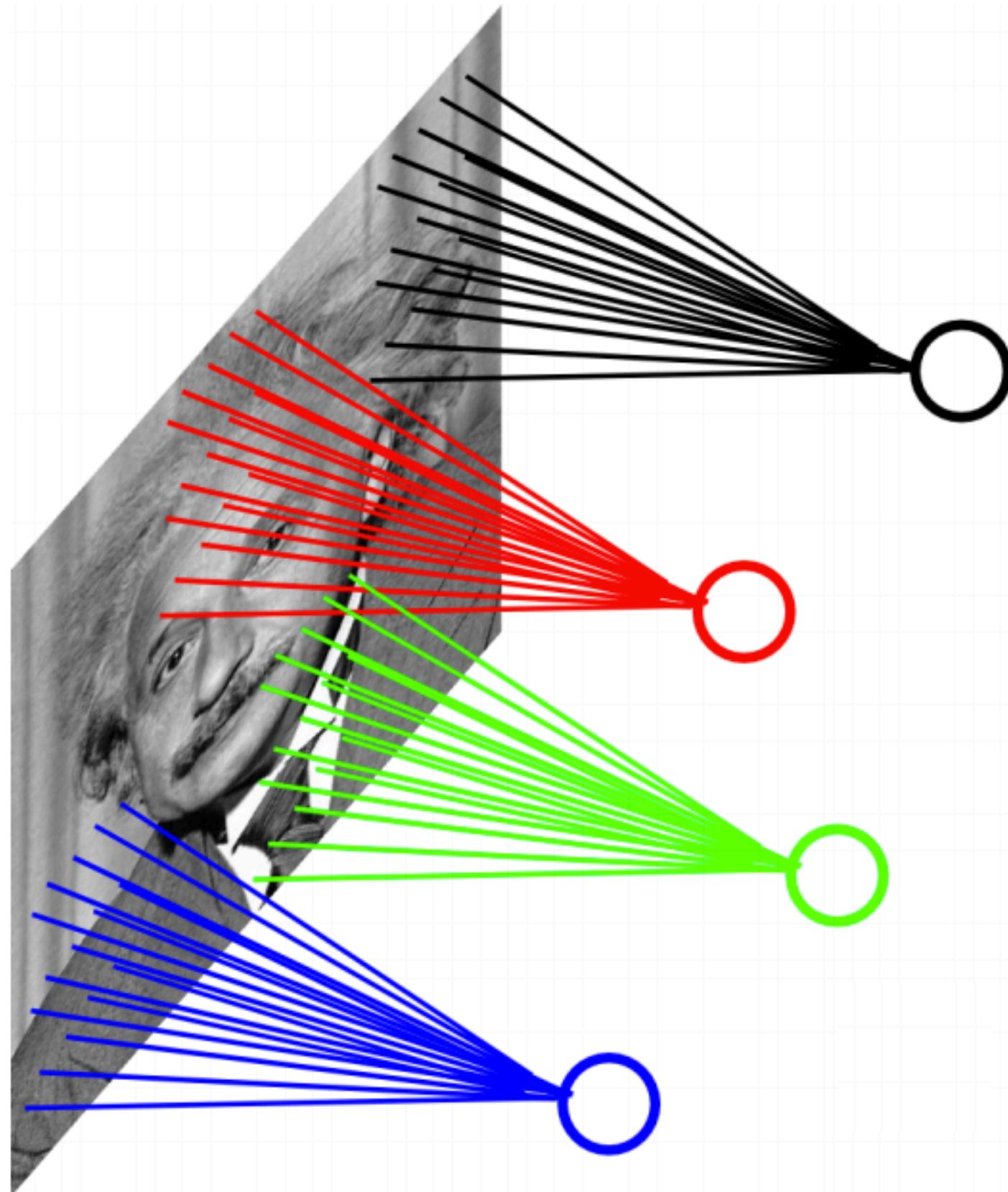
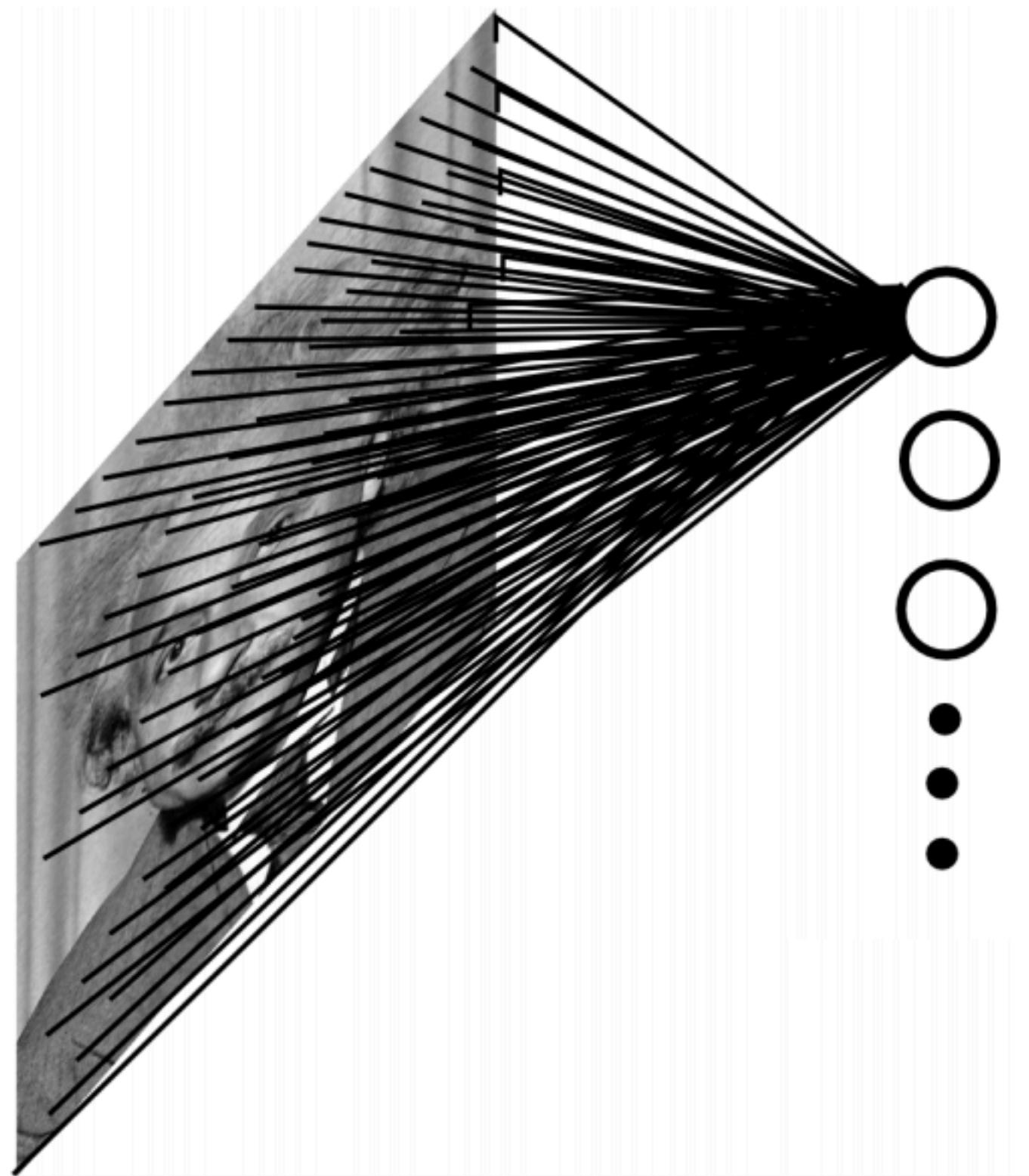
Local connectivity

- Each hidden unit is connected only to a sub-region of input
- It is connected to all channels (R, G, B)



Convolutional Neural Network

Local connectivity



Convolutional Neural Network

Parameter Sharing

- Making a reasonable assumption:
 - If one feature is useful to compute at some spatial position (x, y) ,
 - then it should also be useful to compute at a different position (x_2, y_2)
- Using the **convolution operator**

Convolutional Neural Network

Convolution

- The convolution of an image x with a kernel k is computed as

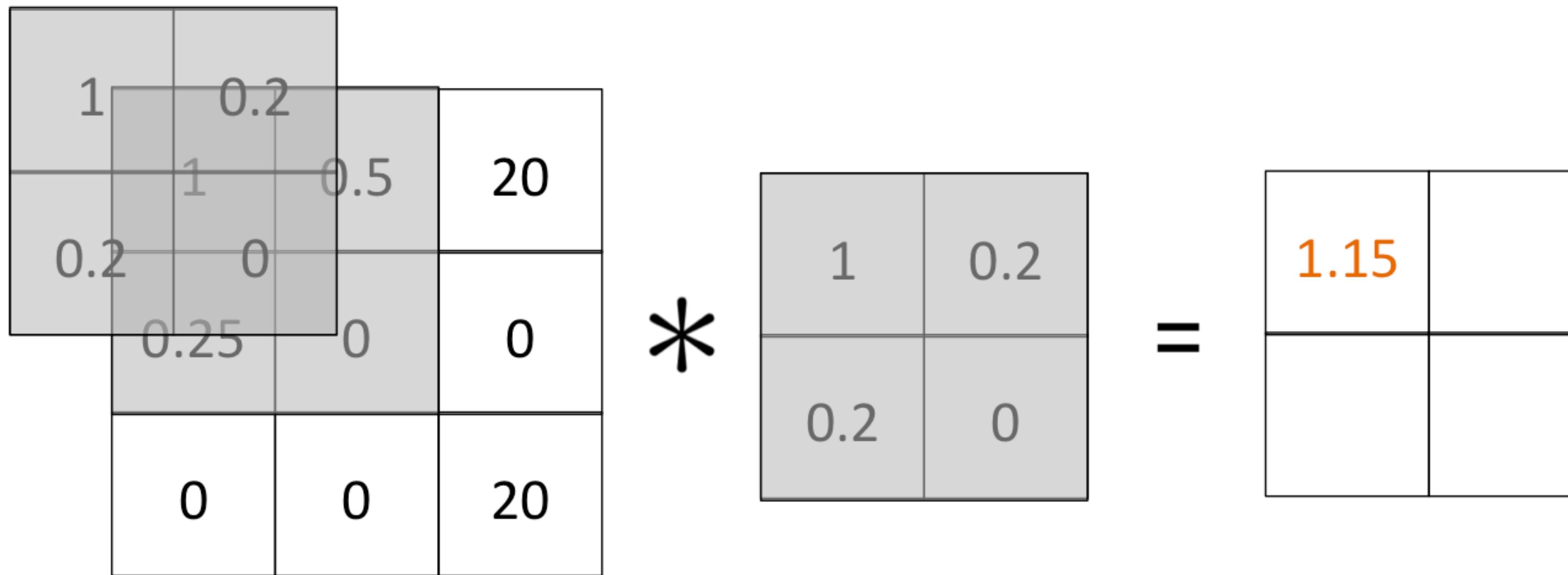
$$\bullet \quad (x * k)_{ij} = \sum_{pq} x_{i+p,j+q} k_{p,q}$$

$$\begin{array}{|c|c|c|} \hline 1 & 0.5 & 20 \\ \hline 0.25 & 0 & 0 \\ \hline 0 & 0 & 20 \\ \hline \end{array} * \begin{array}{|c|c|} \hline 1 & 0.5 \\ \hline 0.25 & 0 \\ \hline \end{array} = \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline & \\ \hline \end{array}$$

Convolutional Neural Network

Convolution

$$1*1 + 0.5*0.2 + 0.25*0.2 + 0*0 = 1.15$$



Convolutional Neural Network

Convolution

$$0.5*1 + 20*0.2 + 0*0.2 + 0*0 = 4.5$$

The diagram illustrates a convolution operation. On the left, a 3x3 input matrix is shown with values 1, 0.25, and 0 in the top row; 0.5, 0.2, and 0 in the middle row; and 20, 0, and 20 in the bottom row. A 2x2 kernel matrix is shown to its right, with values 1, 0.2 in the top row and 0.2, 0 in the bottom row. An asterisk (*) indicates the multiplication of the input and kernel matrices. An equals sign (=) follows the multiplication result, which is a 2x2 output matrix with values 1.15 and 4.5.

1	0.25	0	*	1.15	4.5
0.5	0.2	0	=		
20	0	20			

Convolutional Neural Network

Convolution

$$0.25*1 + 0*0.2 + 0*0.2 + 0*0 = 0.25$$

The diagram illustrates a convolution operation. On the left is a 3x3 input matrix with values: top row [1, 0.5, 20]; middle row [0.25, 0, 0]; bottom row [1, 0.2, 20]. A 2x2 kernel matrix is shown next to it, with values: top row [1, 0.2]; bottom row [0.2, 0]. An asterisk (*) indicates the multiplication of the input and kernel, followed by an equals sign (=). To the right is the resulting output matrix, which has values: top-left [1.15], top-right [4.5], bottom-left [0.25], and bottom-right [0]. The value 0.25 is highlighted in orange.

1	0.5	20
0.25	0	0
1	0.2	20

*

1	0.2
0.2	0

=

1.15	4.5
0.25	0

Convolutional Neural Network

Convolution

$$0*1 + 0*0.2 + 0*0.2 + 20*0 = 0$$

1	0.5	20
0.25	0 1 0.25	0 0.2 20 0
0	0 0.2	20 0

*

1	0.2
0.2	0

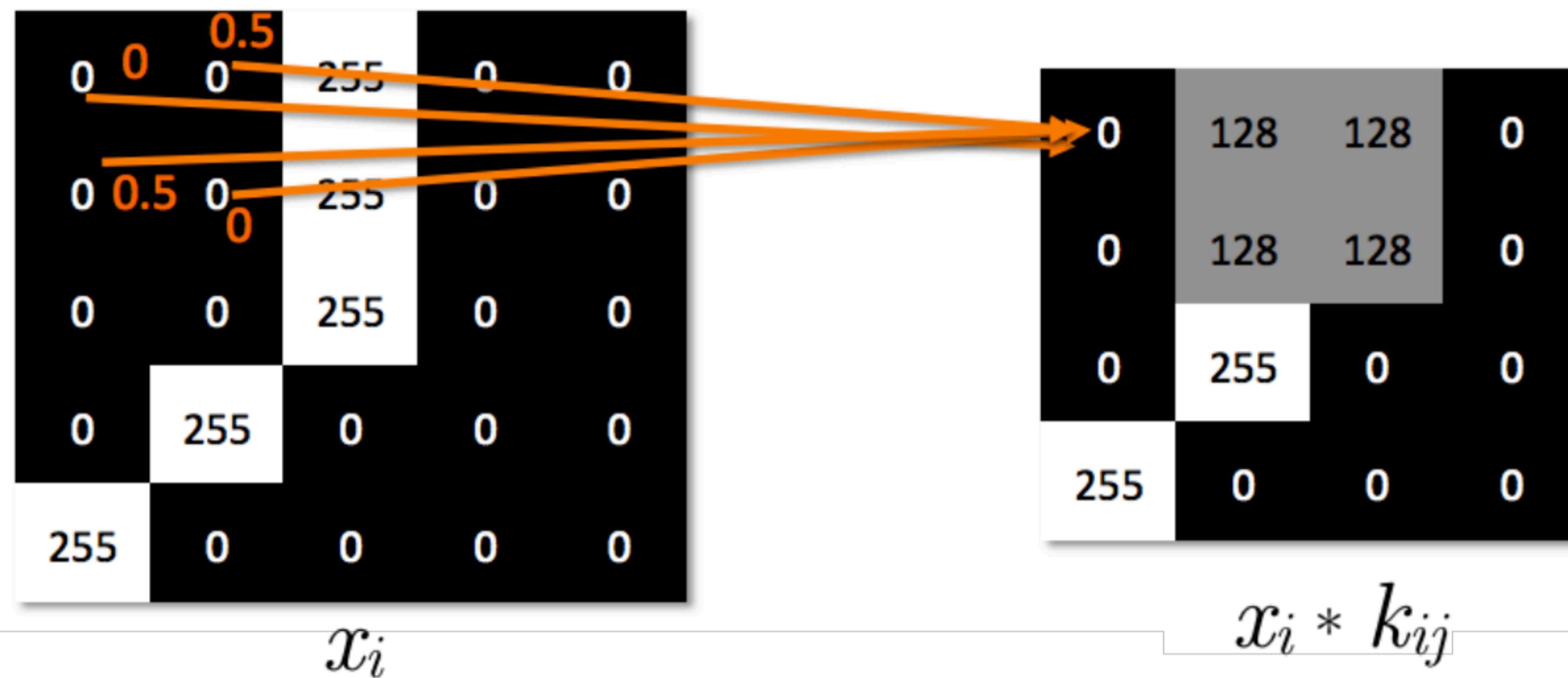
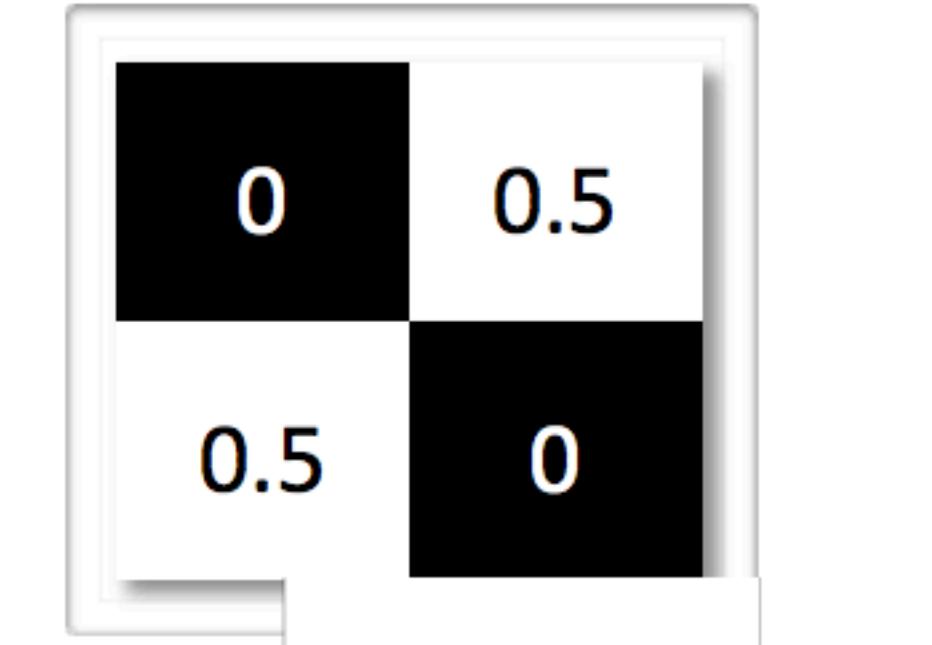
=

1.15	4.5
0.25	0

Convolutional Neural Network

Convolution

$$x * k_{ij}, \text{ where } W_{ij} = \tilde{W}_{ij}$$



Convolutional Neural Network

Convolution

- Element-wise activation function after convolution
 - \Rightarrow detector of a feature at any position in the image

$$x * k_{ij}, \text{ where } W_{ij} = \tilde{W}_{ij}$$

$$\begin{matrix} 0 & 0.5 \\ 0.5 & 0 \end{matrix}$$

$$\begin{matrix} 0 & 0 & 255 & 0 & 0 \\ 0 & 0 & 255 & 0 & 0 \\ 0 & 0 & 255 & 0 & 0 \\ 0 & 255 & 0 & 0 & 0 \\ 255 & 0 & 0 & 0 & 0 \end{matrix}$$

x_i

$$\begin{matrix} 0.02 & 0.19 & 0.19 & 0.02 \\ 0.02 & 0.19 & 0.19 & 0.02 \\ 0.02 & 0.75 & 0.02 & 0.02 \\ 0.75 & 0.02 & 0.02 & 0.02 \end{matrix}$$

$$\text{sigm}(0.02 x_i * k_{ij} - 4)$$

Convolutional Neural Network

Learned Kernels

- Example kernels learned by AlexNet

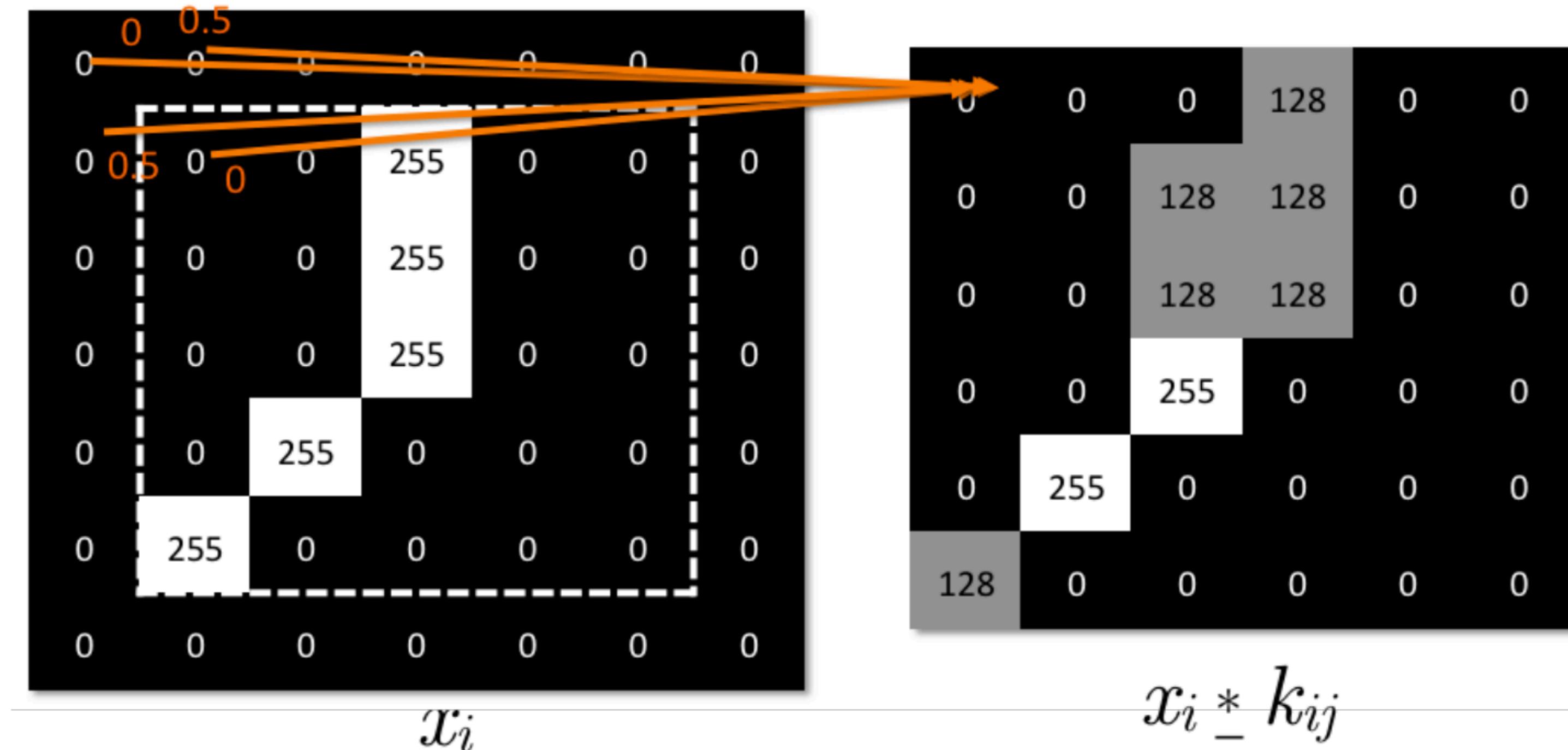


- Number of parameters:
 - Example: 200×200 image, 100 kernels, kernel size 10×10
 - $\Rightarrow 10 \times 10 \times 100 = 10K$ parameters

Convolutional Neural Network

Padding

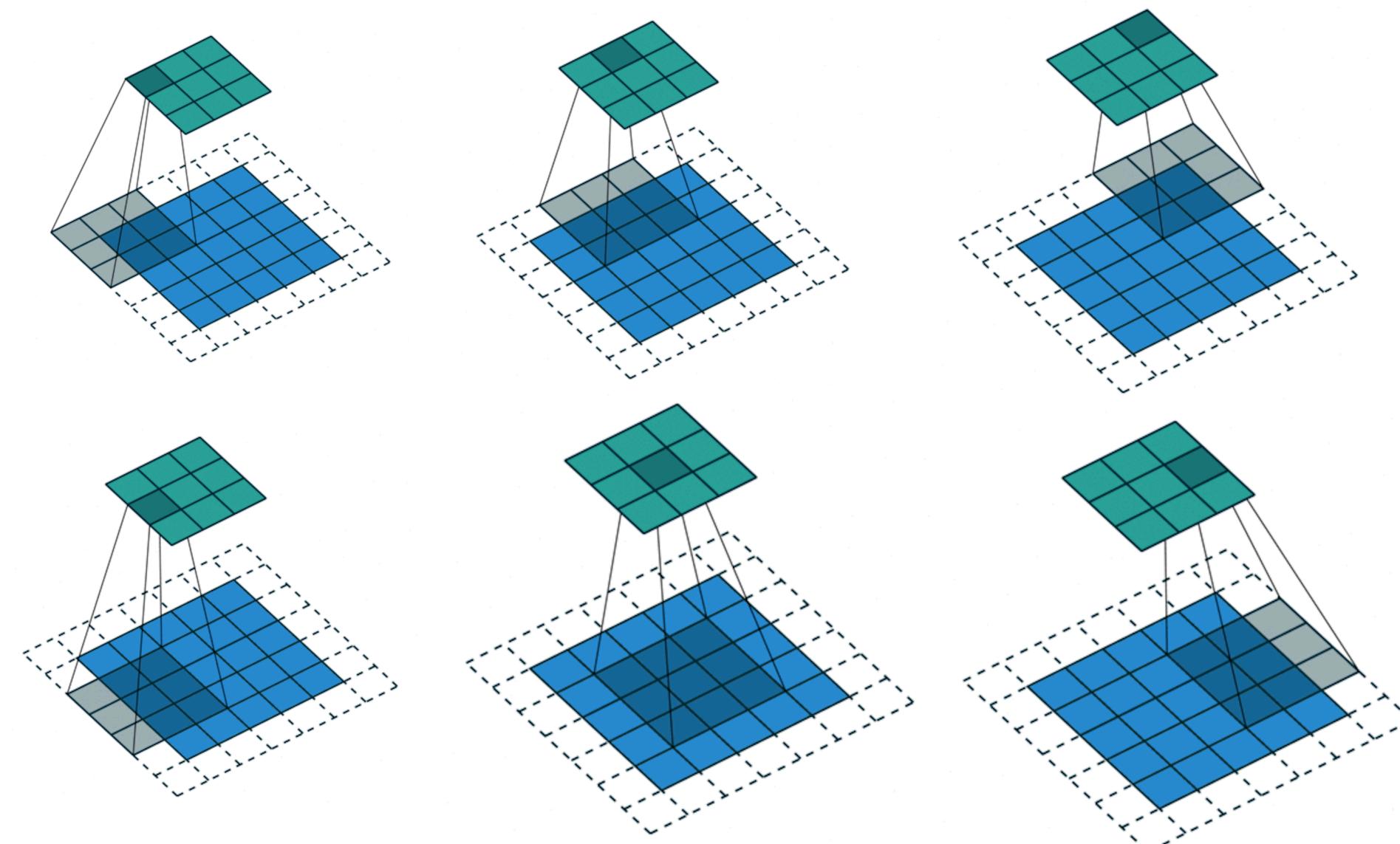
- Use **zero padding** to allow going over the boundary
 - Easier to control the size of output layer



Convolutional Neural Network

Strides

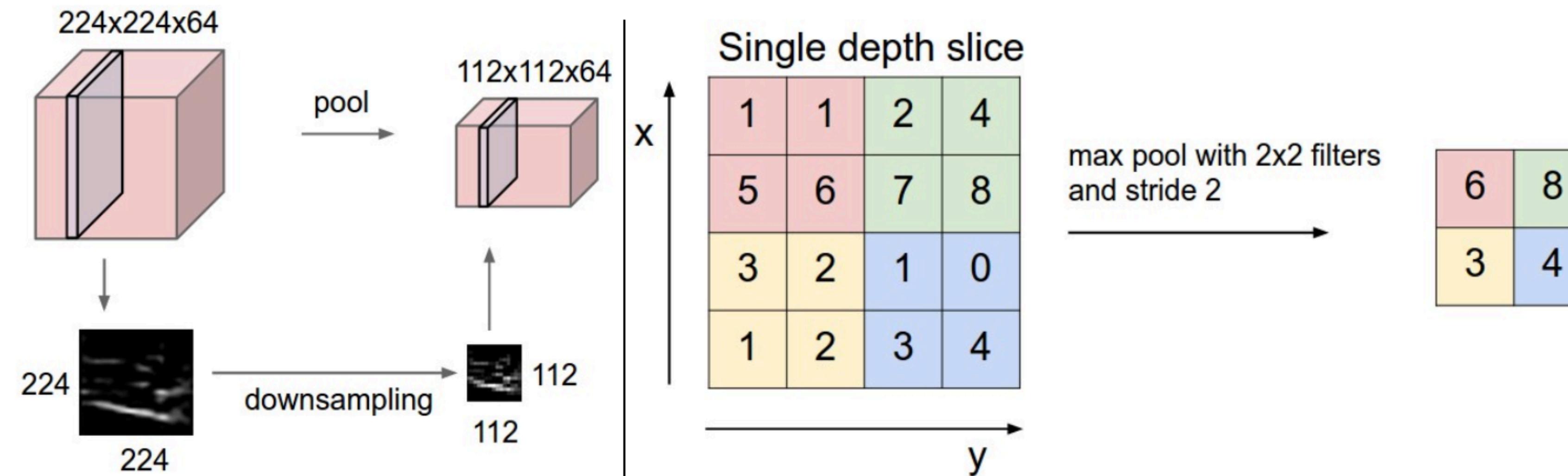
- Stride: The amount of movement between applications of the filter to the input image
- Stride (1,1): no stride



Convolutional Neural Network

Pooling

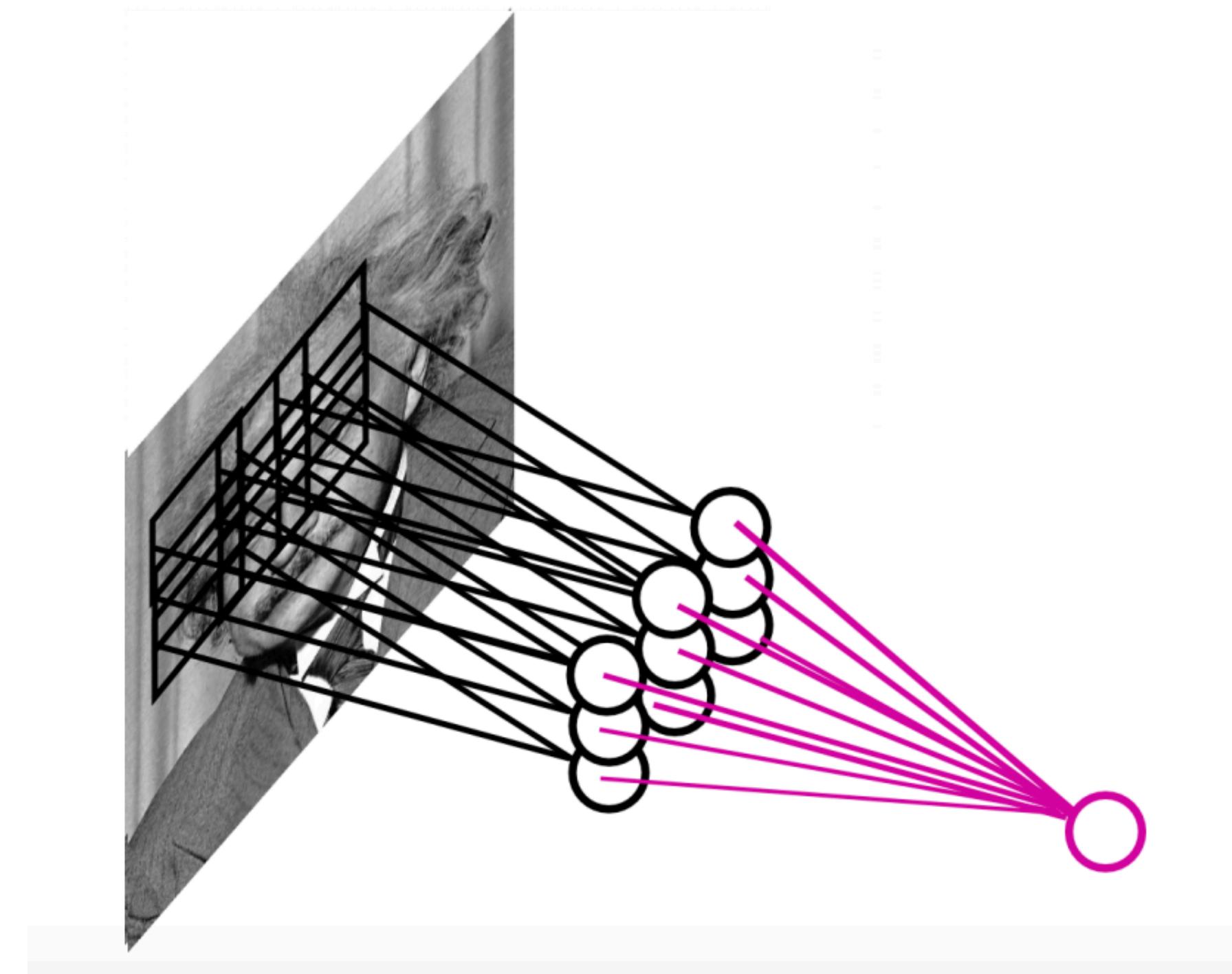
- It's common to insert a **pooling layer** in-between successive convolutional layers
- Reduce the size of presentation, down-sampling
- Example: **Max pooling**



Convolutional Neural Network

Pooling

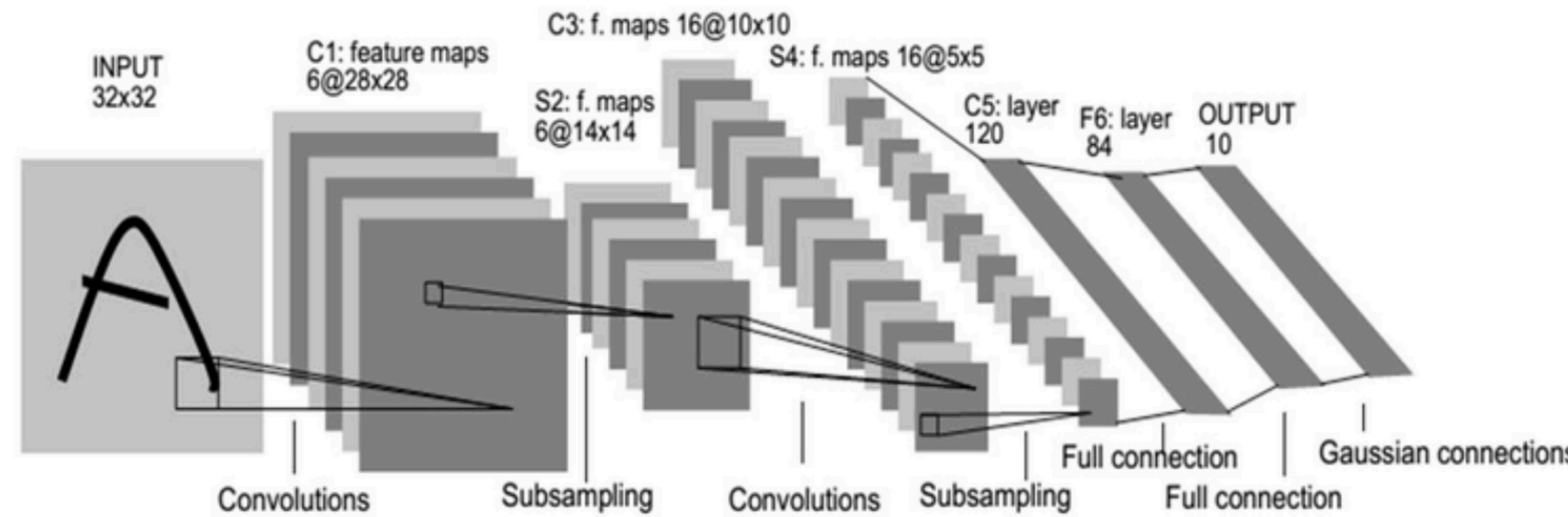
- By **pooling**, we gain robustness to the exact spatial location of features



Convolutional Neural Network

Example: LeNet5

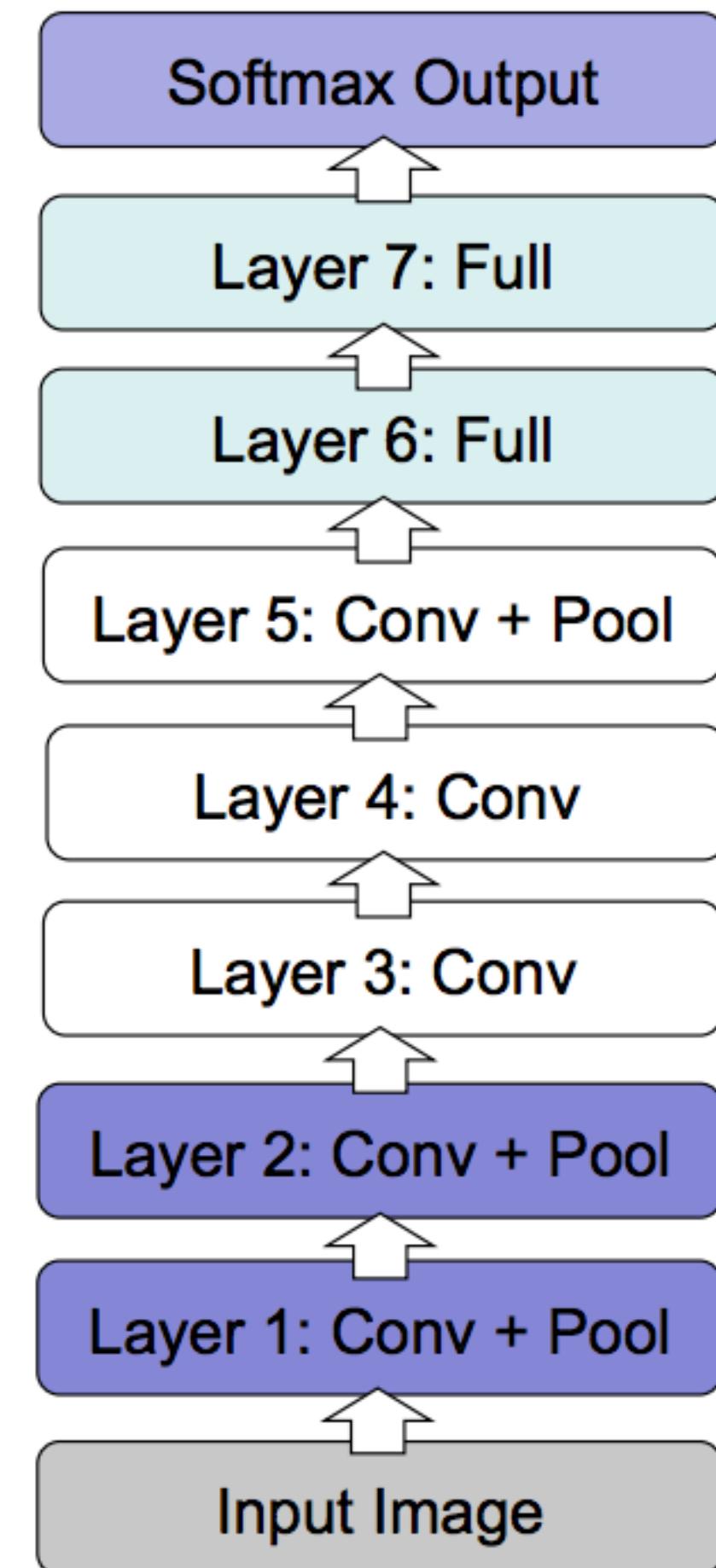
- Input: 32×32 images (MNIST)
- Convolution 1: 6 5×5 filters, stride 1
 - Output: 6 28×28 maps
- Pooling 1: 2×2 max pooling, stride 2
 - Output: 6 14×14 maps
- Convolution 2: 16 5×5 filters, stride 1
 - Output: 16 10×10 maps
- Pooling 2: 2×2 max pooling with stride 2
 - Output: 16 5×5 maps (total 400 values)
- 3 fully connected layers: 120 \Rightarrow 84 \Rightarrow 10 neurons



Convolutional Neural Network

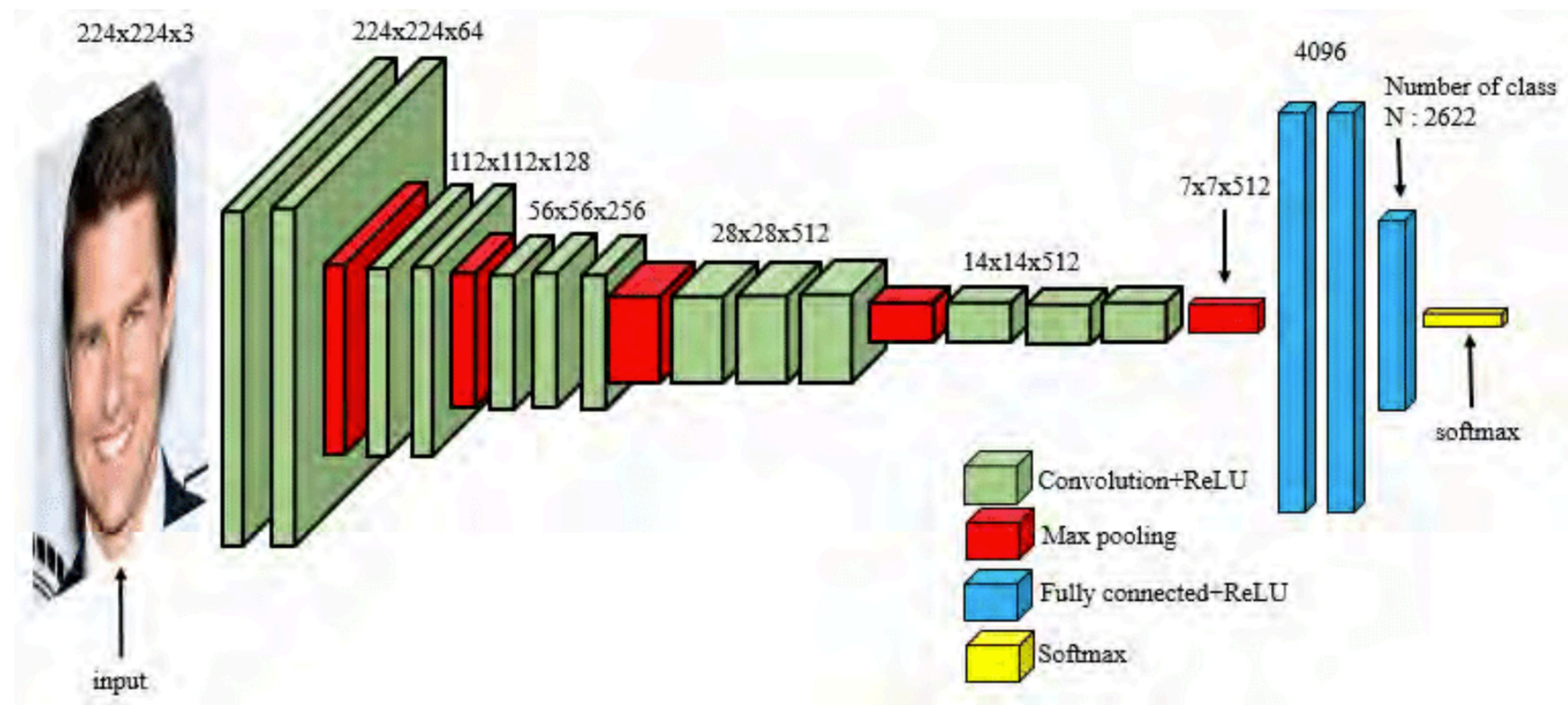
AlexNet

- 8 layers in total, about 60 million parameters and 650,000 neurons.
- Trained on ImageNet dataset
- 18.2% top-5 error
- “ImageNet Classification with Deep Convolutional Neural Networks”, by Krizhevsky, Sustskever and Hinton, NIPS 2012.



Convolutional Neural Network

VGG Network



Convolutional Neural Network

VGG Network

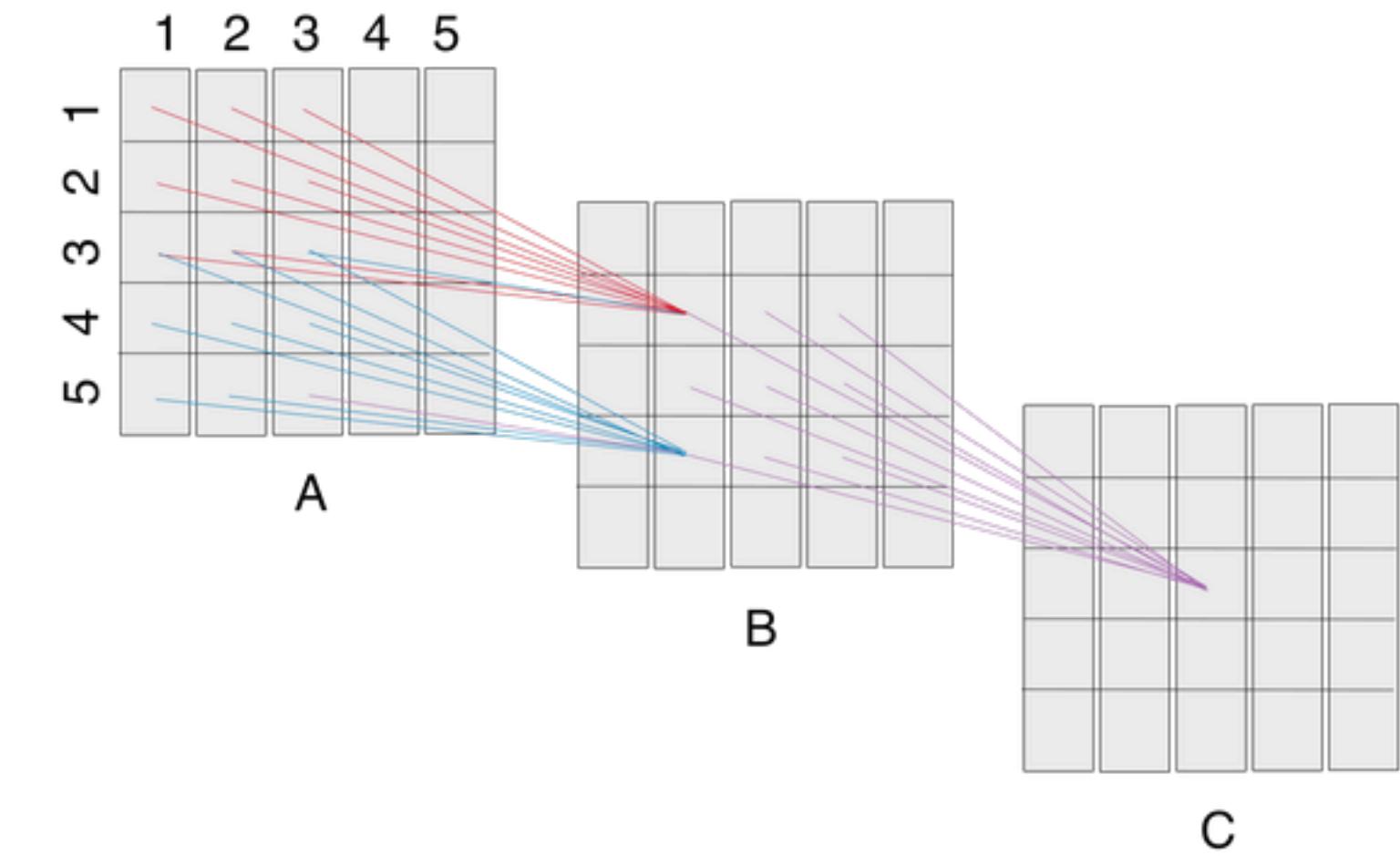
- Output provides an estimate of the conditional probability of each class

```
INPUT: [224x224x3]           memory: 224*224*3=150K  weights: 0
CONV3-64: [224x224x64]   memory: 224*224*64=3.2M  weights: (3*3*3)*64 = 1,728
CONV3-64: [224x224x64]   memory: 224*224*64=3.2M  weights: (3*3*64)*64 = 36,864
POOL2: [112x112x64]    memory: 112*112*64=800K  weights: 0
CONV3-128: [112x112x128]  memory: 112*112*128=1.6M  weights: (3*3*64)*128 = 73,728
CONV3-128: [112x112x128]  memory: 112*112*128=1.6M  weights: (3*3*128)*128 = 147,456
POOL2: [56x56x128]     memory: 56*56*128=400K  weights: 0
CONV3-256: [56x56x256]   memory: 56*56*256=800K  weights: (3*3*128)*256 = 294,912
CONV3-256: [56x56x256]   memory: 56*56*256=800K  weights: (3*3*256)*256 = 589,824
CONV3-256: [56x56x256]   memory: 56*56*256=800K  weights: (3*3*256)*256 = 589,824
POOL2: [28x28x256]     memory: 28*28*256=200K  weights: 0
CONV3-512: [28x28x512]   memory: 28*28*512=400K  weights: (3*3*256)*512 = 1,179,648
CONV3-512: [28x28x512]   memory: 28*28*512=400K  weights: (3*3*512)*512 = 2,359,296
CONV3-512: [28x28x512]   memory: 28*28*512=400K  weights: (3*3*512)*512 = 2,359,296
POOL2: [14x14x512]     memory: 14*14*512=100K  weights: 0
CONV3-512: [14x14x512]   memory: 14*14*512=100K  weights: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512]   memory: 14*14*512=100K  weights: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512]   memory: 14*14*512=100K  weights: (3*3*512)*512 = 2,359,296
POOL2: [7x7x512]       memory: 7*7*512=25K  weights: 0
FC: [1x1x4096]        memory: 4096  weights: 7*7*512*4096 = 102,760,448
FC: [1x1x4096]        memory: 4096  weights: 4096*4096 = 16,777,216
FC: [1x1x1000]        memory: 1000  weights: 4096*1000 = 4,096,000
```

Convolutional Neural Network

What do the filters learn?

- The **receptive field** of a neuron is the input region that can affect the neuron's output
- The receptive field for a first layer neuron is its neighbors (depending on kernel size) \Rightarrow capturing very local patterns
- For higher layer neurons, the receptive field can be much larger \Rightarrow capturing global patterns



Convolutional Neural Network

Training

- Training:
 - Apply SGD to minimize in-sample training error
 - Backpropagation can be extended to **convolutional layer** and **pooling layer** to compute gradient!
 - Millions of parameters \Rightarrow easy to overfit

Convolutional Neural Network

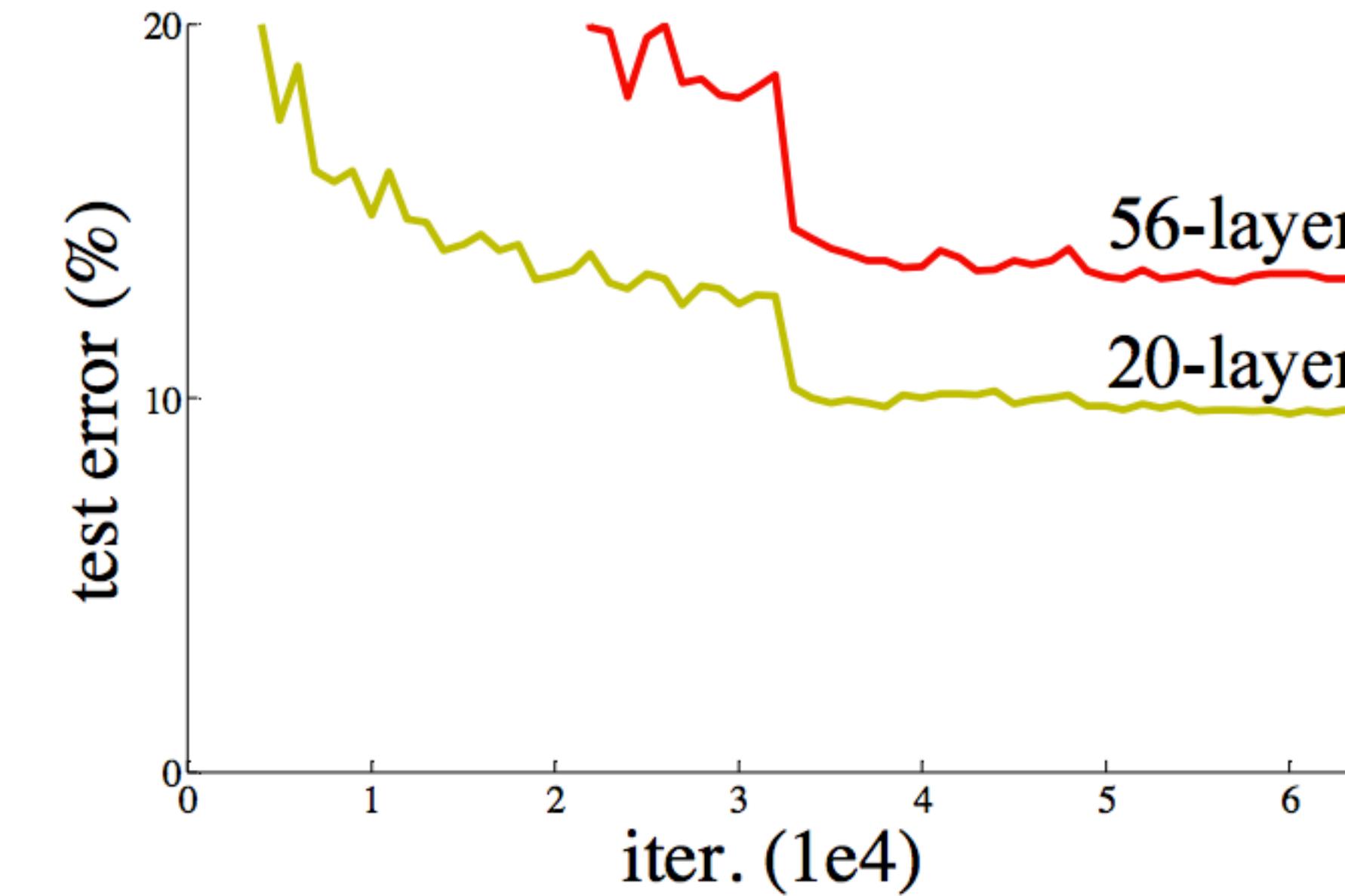
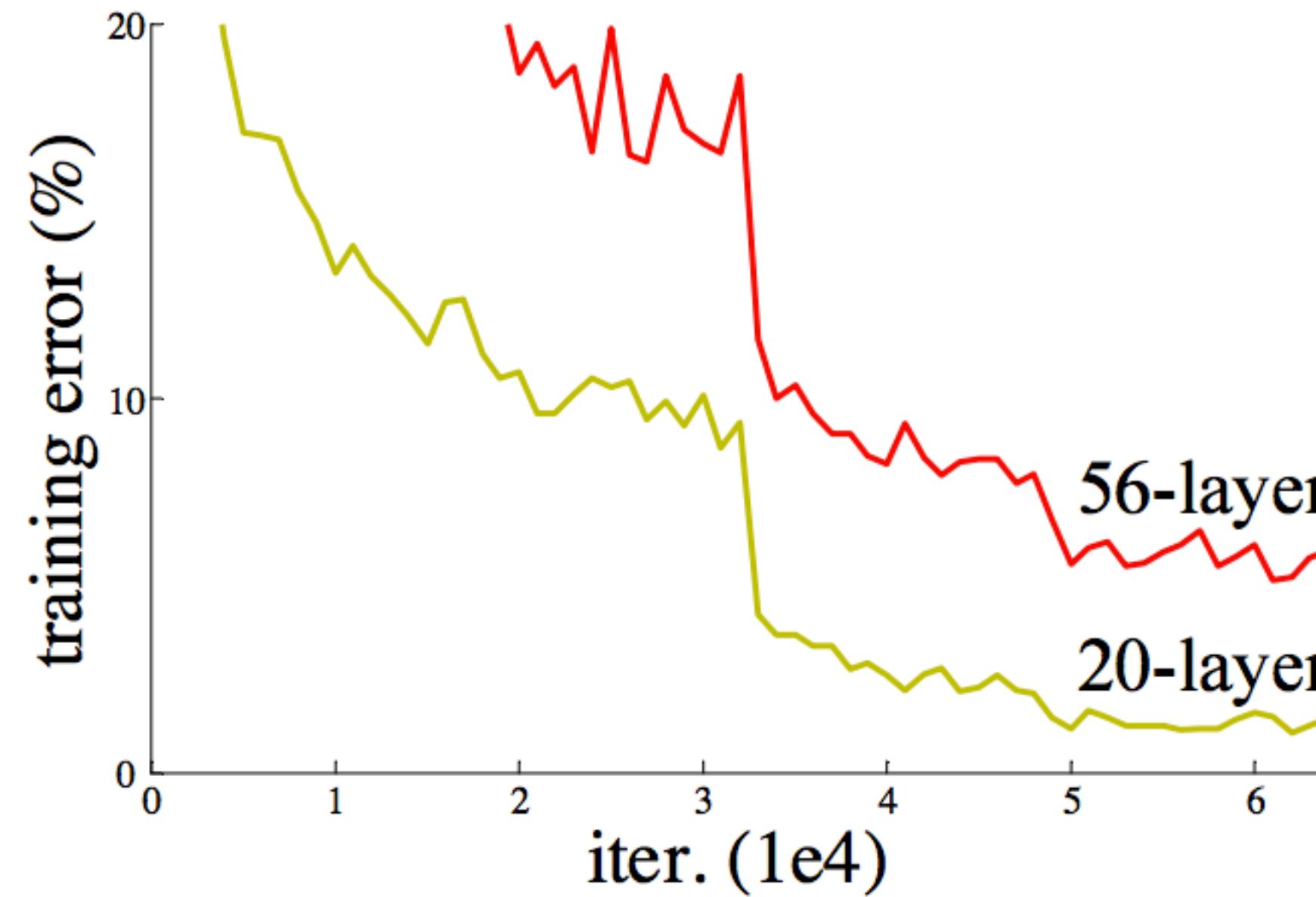
Revisit Alexnet

- Dropout: 0.5 (in FC layers)
- A lot of data augmentation
- Momentum SGD with batch size 128, momentum factor 0.9
- L2 weight decay (L2 regularization)
- Learning rate: 0.01, decreased by 10 every time when reaching a stable validation accuracy

Convolutional Neural Network

Residual Networks

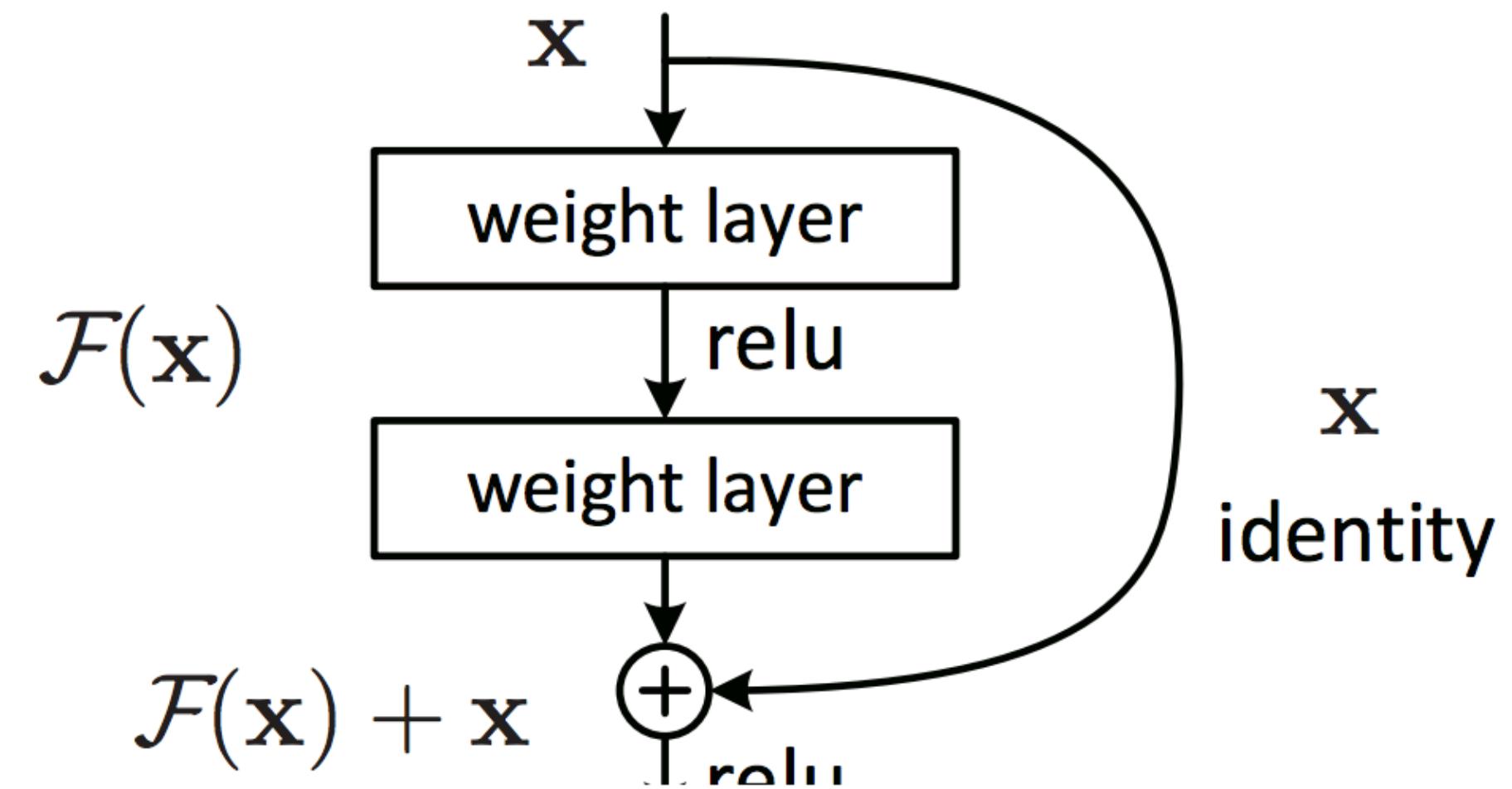
- Very deep convnets do not train well – **vanishing gradient problem**



Convolutional Neural Network

Residual Networks

- Key idea: introduce “pass through” into each layer



- Thus, only residual needs to be learned

Convolutional Neural Network Residual Networks

method	top-1 err.	top-5 err.
VGG [41] (ILSVRC'14)	-	8.43 [†]
GoogLeNet [44] (ILSVRC'14)	-	7.89
VGG [41] (v5)	24.4	7.1
PReLU-net [13]	21.59	5.71
BN-inception [16]	21.99	5.81
ResNet-34 B	21.84	5.71
ResNet-34 C	21.53	5.60
ResNet-50	20.74	5.25
ResNet-101	19.87	4.60
ResNet-152	19.38	4.49

Table 4. Error rates (%) of **single-model** results on the ImageNet validation set (except [†] reported on the test set).

