

# Model fit

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This document describes the epidemiological model, data, and fitting procedure used to derive estimates of transmission parameters for a baseline model used to estimate the effect of agrochemical pollution on schistosomiasis transmission

## Initial exploration

First estimate  $R_0$  over a range of potential values of  $\beta$  and  $\lambda$  to get a general idea of ranges to include in fitting procedure

```
test_betas <- seq(0.05,1.5,0.05)*init_pars["beta"]
test_lambdas <- seq(0.05,1.5,0.05)*init_pars["lambda"]

test_trans_pars <- expand.grid(beta = test_betas,
                              lambda = test_lambdas)

test_pars <- init_pars
test_pars["mu_P"] <- 0.9 # Kill predators for pred free estimation

test_r0_beta_lambda <- function(beta, lambda){
  test_pars["beta"] <- beta
  test_pars["lambda"] <- lambda

  r0.Ag(parameters = test_pars)[3]
}

test_trans_pars$r0 <- mapply(test_r0_beta_lambda,test_trans_pars$beta, test_trans_pars$lambda)

#Plot heatmap
test_trans_pars %>%
  ggplot(aes(x = beta, y = lambda, fill = r0, z = r0)) +
  geom_tile() +
  geom_contour(breaks = c(1), col = "white") +
  scale_fill_viridis(option = "magma", direction = 1) +
  labs(y = expression(lambda),
       x = expression(beta)) +
  theme_bw() +
  theme(axis.ticks=element_blank(),
        axis.text=element_text(size=12),
        axis.title=element_text(size=14),
        legend.title=element_text(size=15),
        legend.text=element_text(size=12)) +
  guides(fill=guide_legend(title=expression(R[0]),
                           reverse = T)) +
  annotate(geom = "point", x = init_pars["beta"], y = init_pars["lambda"], col = "white")
```

We'll stick with the  $\beta$  estimate from previous work as we don't have sufficient snail data to fit to. This leaves us with the task of fitting worm burden model outputs to human infection data using the snail to man transmission parameter,  $\lambda$ . In previous work, we tried fitting with a single parameter and were unable to adequately fit the model due to highly seasonal transmission. We therefore divided transmission into low and high seasons and fit the model assuming a  $\lambda_{lo}$  and  $\lambda_{hi}$  were operative during low and high transmission seasons, respectively. We'll try both of those methods here as well as fitting a seasonal model that incorporates seasonality in the snail population.

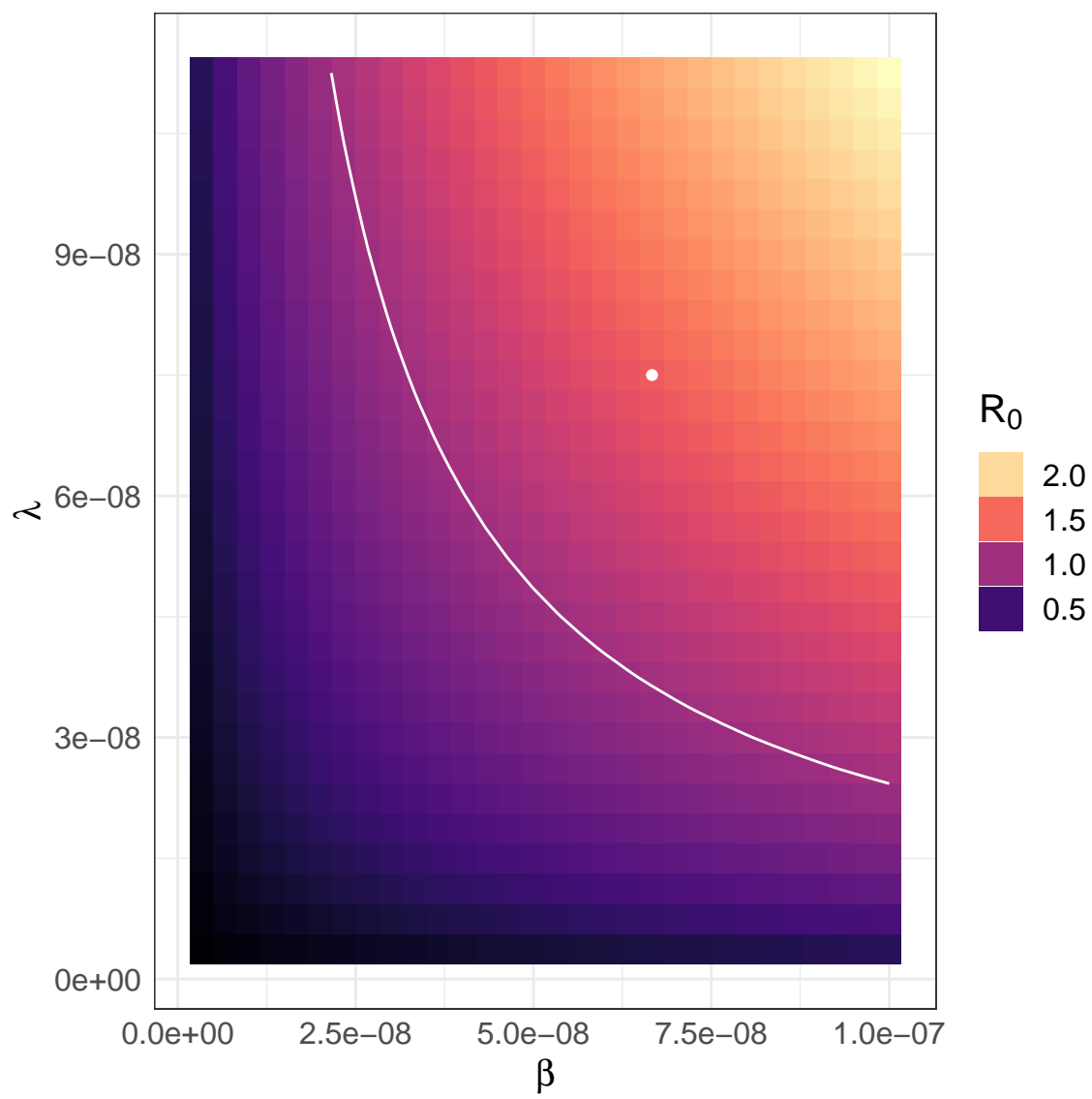


Figure 1: Line indicates  $R_0=1$  and dot indicates coordinates (and  $R_0$  value) of initial parameter set

## Gather epi data to fit to

Data comes from community in Senegal where prawn intervention was first piloted. This data comes from the control community where prawns were not introduced. Data and fitting routine are the same as reported in Arakala, Hoover, et al., but this model contains additional parameters in order to accomodate a range of potential agrochemical influences

```
N <- 129 #number of people in community who were treated
H <- 300

EggToWormConvert <- 3.6 #convert egg burden to worm burden based on estimate of eggs/mated female from Cheever et al.
cvrg <- N/H #43% coverage estimated in this community 129 out of 300 people

init_pars["H"] <- H
init_pars["cvrg"] <- cvrg

#Baseline (Feb 2012) estimates
W_baseline<-6.5/EggToWormConvert
W_baseline_k<-0.08
W_baseline_sd<-sqrt( (W_baseline)+((W_baseline^2)/W_baseline_k ) )
W_baseline_se<-W_baseline_sd/sqrt(N)

#5 month (July 2012) epi data point estimates
W_5month_field_mean<-1.5/EggToWormConvert
W_5month_field_k<-0.02
W_5month_field_sd<-sqrt( (W_5month_field_mean)+((W_5month_field_mean^2)/W_5month_field_k ) )
W_5month_field_se<-W_5month_field_sd/sqrt(N)

#Feb 13 epi data estimates (in the middle of high transmission season)
W_Feb13_field_mean<-161/EggToWormConvert
W_Feb13_field_k<-0.21
W_Feb13_field_sd<-sqrt( (W_Feb13_field_mean)+((W_Feb13_field_mean^2)/W_Feb13_field_k ) )
W_Feb13_field_se<-W_Feb13_field_sd/sqrt(N)

#Sept 13 epi data estimates (in the middle of high transmission season)
W_Sep13_field_mean<-17.6/EggToWormConvert
W_Sep13_field_k<-0.29
W_Sep13_field_sd<-sqrt( (W_Sep13_field_mean)+((W_Sep13_field_mean^2)/W_Sep13_field_k ) )
W_Sep13_field_se<-W_Sep13_field_sd/sqrt(N)

#Timepoints of epi datapoint collection
timepoints<-c(1, 156, 375, 594)
traj_time <- c(1:(max(timepoints) +2)) #time vector to run over

#Estimates of measured worm burden at epi timepoints
measured_ws <- c(W_baseline, W_5month_field_mean, W_Feb13_field_mean, W_Sep13_field_mean)
w_errors <- c(W_baseline_se, W_5month_field_se, W_Feb13_field_se, W_Sep13_field_se)
measured_ks <- c(W_baseline_k, W_5month_field_k, W_Feb13_field_k, W_Sep13_field_k)

epi_plot <- as.data.frame(cbind("time" = timepoints, "Worm_burden" = measured_ws, "se" = w_errors))

epi_plot %>%
  ggplot(aes(x = timepoints, y = measured_ws)) + geom_point() + theme_bw() + ylim(0,60) +
  geom_errorbar(ymin = measured_ws - w_errors, ymax = measured_ws + w_errors, width = 5)

#timepoints at which transmission shifts from low to high
seasons <- c(1, 134, 263, 502)

#Events data frame containing timepoints at which MDA occurs
mdas <- data.frame(var = rep('Wt.cvrg', 4),
  time = c(2, 22, 157, 376),
```

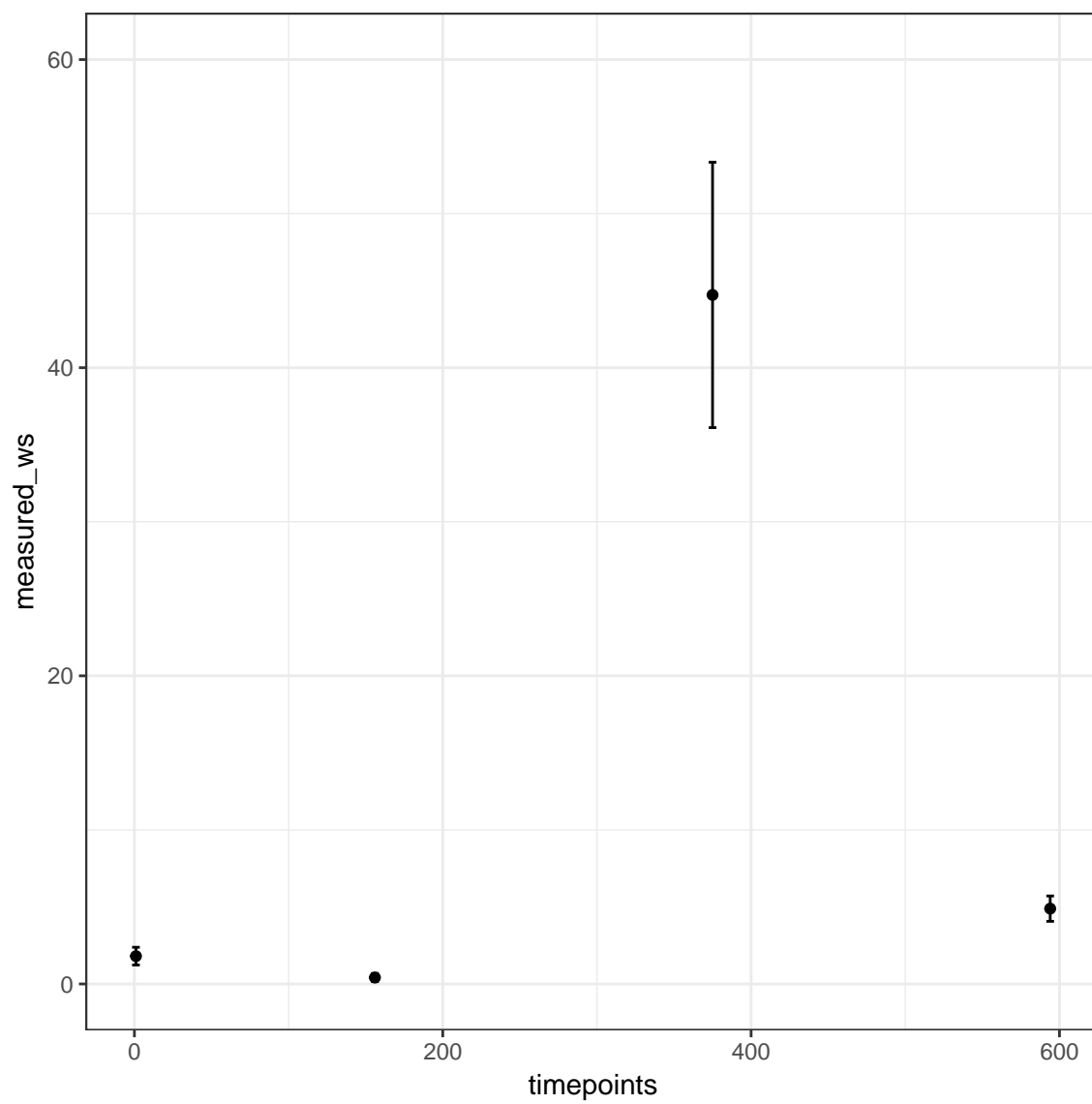


Figure 2: Estimates of mean worm burden in Senegal community to fit model to

```
value = rep(0.06, 4),      #94% efficacy
method = rep('mult', 4))
```

## Fit model to epi data

### Single transmission parameter

```
#Somewhat arbitrary starting conditions to initiate the model
```

```
cvrg <- init_pars["cvrg"]
```

```
base_mda.start = c(S = 40*area,
  E = 0,
  I = 0,
  Wt = 2*cvrg,
  Wu = 2*(1-cvrg),
  P = 0)
```

```
params <- init_pars
params["mu_P"] <- 0.9 # Kill predators for pred free estimation
```

```
#Function to simulate transmission, given single transmission parameter and return negative log likelihood from
```

```
Tx_simulate_1par<-function(lambda){
```

```
  params["lambda"] <- lambda
```

```
  eqbm_vals <- runsteady(y = base_mda.start, func = agrochem_mda_mod, parms = params)$y
```

```
  traj <- data.frame(ode(eqbm_vals, traj_time, agrochem_mda_mod,
    params, events = list(data = mdas)))
```

```
  modeled <- traj$Wt[timepoints + 2] * 0.5 * mapply(phi_Wk, traj$Wt[timepoints + 2], measured_ks)
```

```
  point_LL <- mapply(pointLogLike, measured_ws, w_errors, modeled)
```

```
  point_LL[!is.finite(point_LL)] <- NA
```

```
  logLike <- sum(point_LL, na.rm = TRUE)
```

```
  return(-logLike)
```

```
}
```

```
#Fit using Optimize for 1-dimensional optimization
```

```
op_min_1par <- optimize(Tx_simulate_1par, interval = c(init_pars["lambda"]/100, init_pars["lambda"]*10), tol =
```

```
#Extract best fitting parameter
```

```
bestLambda_1par <- op_min_1par$minimum
```

```
fit_pars_1par <- init_pars
```

```
fit_pars_1par["lambda"] <- bestLambda_1par
```

```
fit_pars_1par["mu_P"] <- 0.9 # Want to estimate predator-free R0
```

```
#simulate with best fitting parameter
```

```
fit_eq_vals_1par <- runsteady(y = base_mda.start, func = agrochem_mda_mod,
  parms = fit_pars_1par)$y
```

```
fit_traj_1par <- data.frame(ode(fit_eq_vals_1par, traj_time, agrochem_mda_mod,
  fit_pars_1par, events = list(data = mdas)))
```

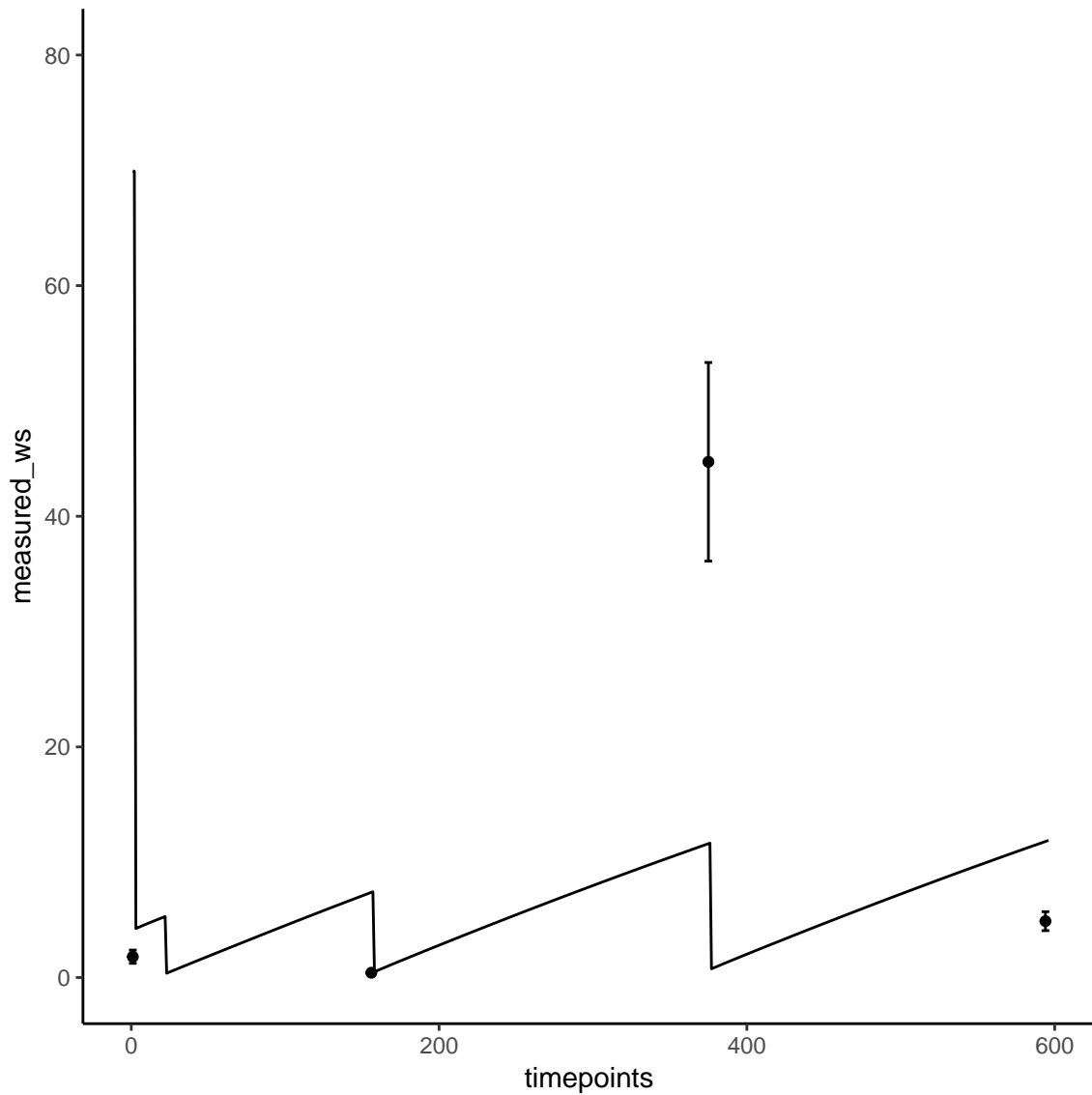


Figure 3: Single transmission parameter model fit to epi data resulting from best fit parameter set

```

epi_plot %>%
  ggplot(aes(x = timepoints, y = measured_ws)) +
  geom_point() +
  theme_classic() +
  ylim(0,80) +
  geom_errorbar(ymin = measured_ws - w_errors, ymax = measured_ws + w_errors, width = 5) +
  geom_line(data = fit_traj_1par, aes(x = time, y = Wt.cvrg))

```

So the main problem is the high transmission point, but we kind of knew that already. I think this works well, especially since it doesn't require summarizing the two seasonal transmission parameters into a single  $\lambda$  estimate to use in  $R_0$  estimates. Let's next estimate the 95%CI from the profile likelihood as before and then get the range of  $R_0$  resulting from this estimate

```

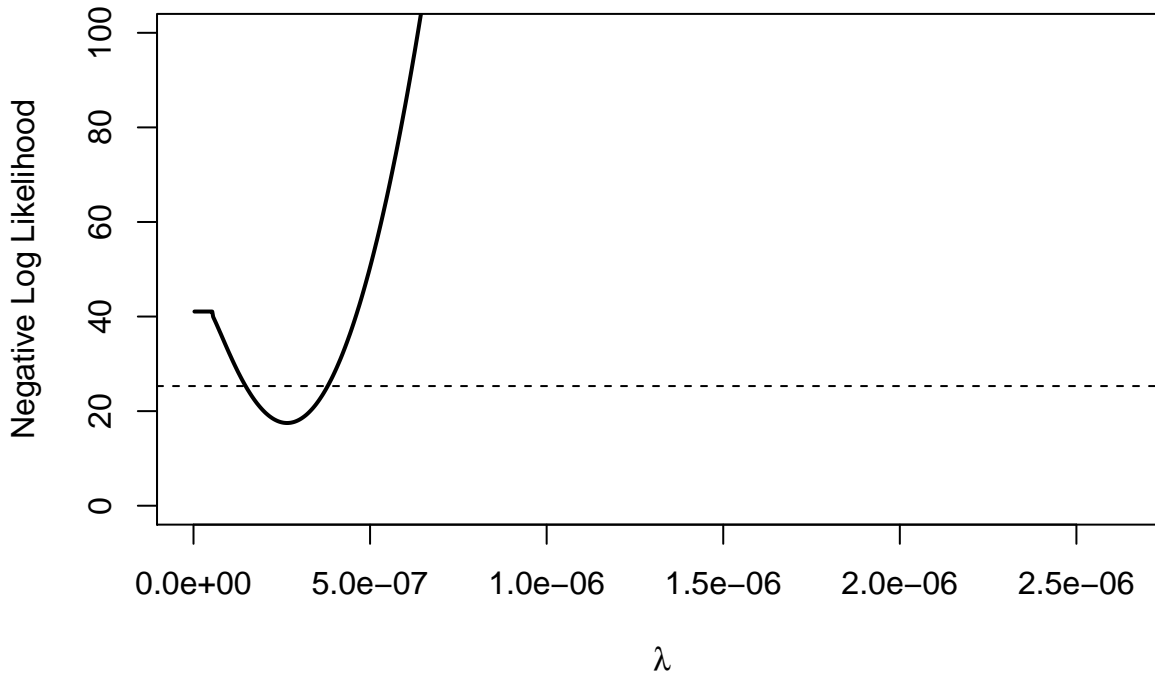
#Get 95%CI boundary
boundary_1par <- op_min_1par$objective + 7.815 #chi-square with 3 degree of freedom, 95% CI

#Grid search for 95%CI
lambda_1par <- seq(bestLambda_1par/100, bestLambda_1par*10, length.out = 1000)

negLL_1par <- sapply(lambda_1par, Tx_simulate_1par)

```

```
#plot begLL by lambda and include cutoff line that represents profile likelihood boundary
plot(lambda_1par, negLL_1par, type = "l", lwd = 2, ylim = c(0,100),
      xlab = expression(lambda), ylab = "Negative Log Likelihood")
abline(h = boundary_1par, lty = 2)
```



```
#generate matrix of parameters within 95%Ci, their fit, and an associated weight based on their fit, then save
fit_1par_mat <- data.frame(cbind(lambda_1par, negLL_1par)) %>%
  filter(negLL_1par <= boundary_1par) %>%
  mutate(weight = 1/negLL_1par)

save(fit_1par_mat, file = "../Models/trans_pars_fit.Rdata")
```

```
#Create parameter sets with these estimates to get R0 range
#Point estimate
point_r0_noPred <- r0.Ag(parameters = fit_pars_1par)[3]

#lower limit
fit_pars_1par_lo <- fit_pars_1par
fit_pars_1par_lo["lambda"] <- min(fit_1par_mat$lambda_1par)
lo_r0_noPred <- r0.Ag(parameters = fit_pars_1par_lo)[3]

#Upper limit
fit_pars_1par_hi <- fit_pars_1par
fit_pars_1par_hi["lambda"] <- max(fit_1par_mat$lambda_1par)
hi_r0_noPred <- r0.Ag(parameters = fit_pars_1par_hi)[3]

#Estimates with predators at reasonable wildlife density
#Point estimate
fit_pars_1par_pred <- fit_pars_1par
fit_pars_1par_pred["mu_P"] <- init_pars["mu_P"]
point_r0_Pred <- r0.Ag(parameters = fit_pars_1par_pred)[3]
```

```

#lower limit
fit_pars_1par_lo_pred <- fit_pars_1par_lo
fit_pars_1par_lo_pred["mu_P"] <- init_pars["mu_P"]
lo_r0_Pred <- r0.Ag(parameters = fit_pars_1par_lo_pred)[3]

#Upper limit
fit_pars_1par_hi_pred <- fit_pars_1par_hi
fit_pars_1par_hi_pred["mu_P"] <- init_pars["mu_P"]
hi_r0_Pred <- r0.Ag(parameters = fit_pars_1par_hi_pred)[3]

```

Without predators,  $R_0 = 2.7$  (95%CI: 2.02 - 3.23). With predators at natural densities,  $R_0 = 2.05$  (95%CI: 1.53 - 2.45).

```

#simulate with best fitting parameter
fit_eq_vals_1par <- runsteady(y = base_mda.start, func = agrochem_mda_mod,
                             parms = fit_pars_1par)$y

fit_traj_1par <- data.frame(ode(fit_eq_vals_1par, traj_time, agrochem_mda_mod,
                               fit_pars_1par, events = list(data = mdas)))

#simulate with best fitting parameter and pred introduction
base_mda.preds = c(S = 40*area,
                   E = 0,
                   I = 0,
                   Wt = 2*cvrg,
                   Wu = 2*(1-cvrg),
                   P = 0.2*area)

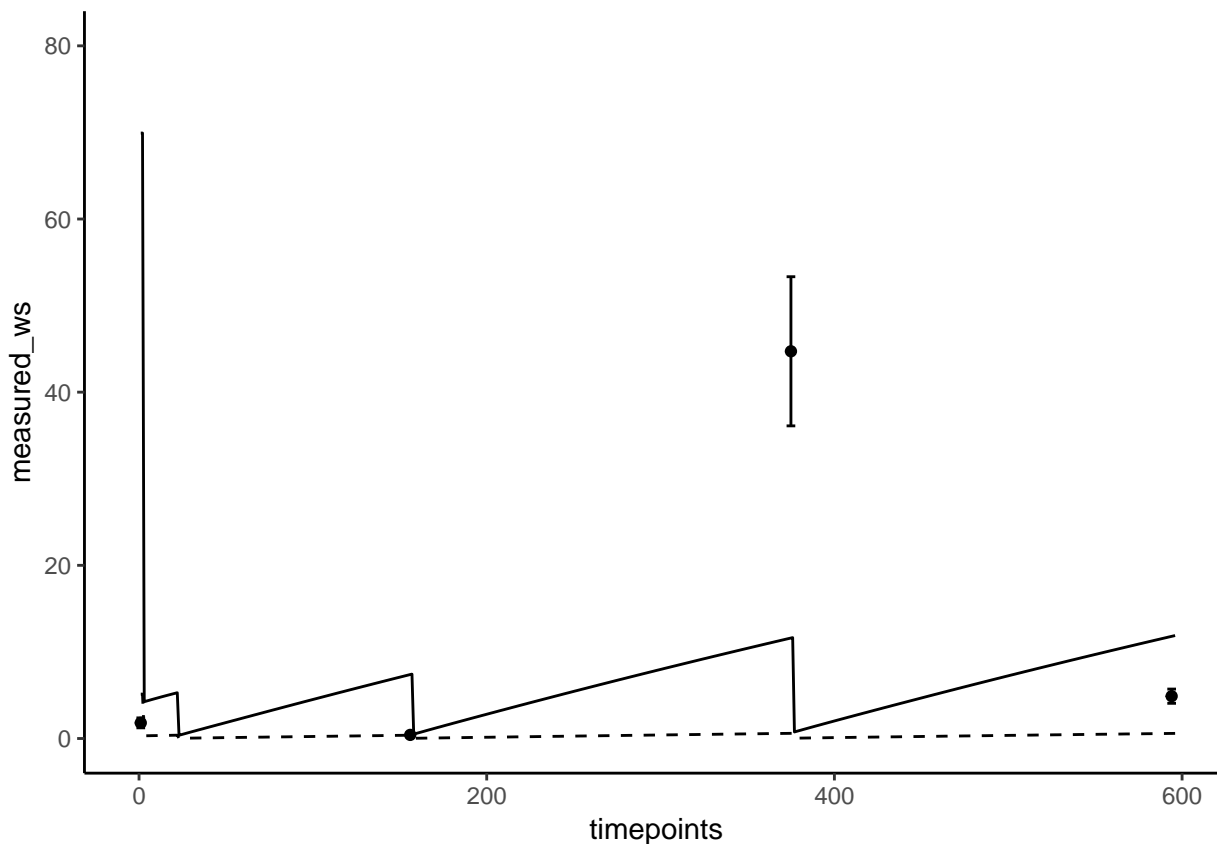
fit_eq_vals_1par_pred <- runsteady(y = base_mda.preds, func = agrochem_mda_mod,
                                   parms = fit_pars_1par_pred)$y

fit_traj_1par_pred <- data.frame(ode(fit_eq_vals_1par_pred, traj_time, agrochem_mda_mod,
                                    fit_pars_1par_pred, events = list(data = mdas)))

epi_plot %>%
  ggplot(aes(x = timepoints, y = measured_ws)) +
  geom_point() +
  theme_classic() +
  ylim(0,80) +
  geom_errorbar(ymin = measured_ws - w_errors, ymax = measured_ws + w_errors, width = 5) +
  geom_line(data = fit_traj_1par, aes(x = time, y = Wt.cvrg)) +
  geom_line(data = fit_traj_1par_pred, aes(x = time, y = Wt.cvrg), lty = 2)

```





## Low and high transmissison season parameters

```
#what's the negLL if we just guess 0 for all data points? (Used as a point of comparison)
base_fit_bad <- -sum(mapply(pointLogLike, measured_ws, w_errors, c(0,0,0,0)))
```

```
#Function to simulate transmission, given transmission parameters and return negative log likelihood from fit to
Tx_simulate<-function(lambdas){
```

```
  params["lambda1"]<-lambdas[1]
  params["lambda2"]<-lambdas[2]
  params["lambda"]<-( (375/594) * lambdas[1]) +( (219/594) * lambdas[2])

  eqbm_vals <- runsteady(y = base_mda.start, func = agrochem_mda_mod_fit, parms = params)$y

  traj <- data.frame(ode(eqbm_vals, traj_time, agrochem_mda_mod_fit,
                        params, events = list(data = mdas)))

  modeled <- traj$Wt[timepoints + 2] * 0.5 * mapply(phi_Wk, traj$Wt[timepoints + 2], measured_ks)

  point_LL <- mapply(pointLogLike, measured_ws, w_errors, modeled)

  point_LL[!is.finite(point_LL)] <- NA

  logLike <- sum(point_LL, na.rm = TRUE)

  return(-logLike)
}
```

```
#Fit using Optim
op_min <- optim(par=c(bestLambda_1par, bestLambda_1par*10),
```

```

Tx_simulate, method="Nelder-Mead")

#Extract best fitting parameters
bestLambda1<-op_min$par[1]
bestLambda2<-op_min$par[2]

fit_pars <- init_pars

fit_pars["lambda1"]<-bestLambda1
fit_pars["lambda2"]<-bestLambda2
fit_pars["lambda"] <- (375/594) * bestLambda1 + (219/594) * bestLambda2

#simulate with best fitting parameters
fit_eq_vals <- runsteady(y = base_mda.start, func = agrochem_mda_mod_fit, parms = fit_pars)$y

fit_traj <- data.frame(ode(fit_eq_vals, traj_time, agrochem_mda_mod_fit,
                          fit_pars, events = list(data = mdas)))

epi_plot %>%
  ggplot(aes(x = timepoints, y = measured_ws)) +
  geom_point() +
  theme_classic() +
  ylim(0,80) +
  geom_errorbar(ymin = measured_ws - w_errors, ymax = measured_ws + w_errors, width = 5) +
  geom_line(data = fit_traj, aes(x = time, y = Wt.cvrg))

```

Fit doesn't look too bad actually. Now find all parameter sets that lie within the 95% CI

```

#Range of low and high transmission season lambdas to test
lambda_lo<- seq(init_pars["lambda"]/100, init_pars["lambda"]*10, length.out = 50)
lambda_hi<- seq(init_pars["lambda"]/10, init_pars["lambda"]*100, length.out = 50)

lambda_tests <- expand.grid(lambda_lo = lambda_lo, lambda_hi = lambda_hi, negLL = 0)

params <- init_pars
params["lambda1"] <- lambda_lo[25]
params["lambda2"] <- lambda_hi[25]

#test all parameter combinations to find best starting place for optimization
lambda_tests$negLL <- apply(lambda_tests, 1, Tx_simulate)

#Value of negLL that serves as cutoff for 95%CI
boundary <- op_min$value + 7.815 #chi-square with 3 degree of freedom, 95% CI

#Keep obs within 95%CI
lambda_tests95ci <- lambda_tests %>%
  filter(negLL <= boundary)

#Plot heatmap with 95%CI delineated
lambda_tests %>%
  ggplot(aes(x = lambda_lo, y = lambda_hi, fill = negLL, z = negLL)) +
  geom_tile() +
  geom_contour(breaks = c(boundary), col = "black") +
  scale_fill_viridis(option = "magma", direction = -1) +
  labs(y = expression(lambda[hi]),
       x = expression(lambda[lo])) +
  theme_bw() +
  theme(axis.ticks=element_blank(),

```

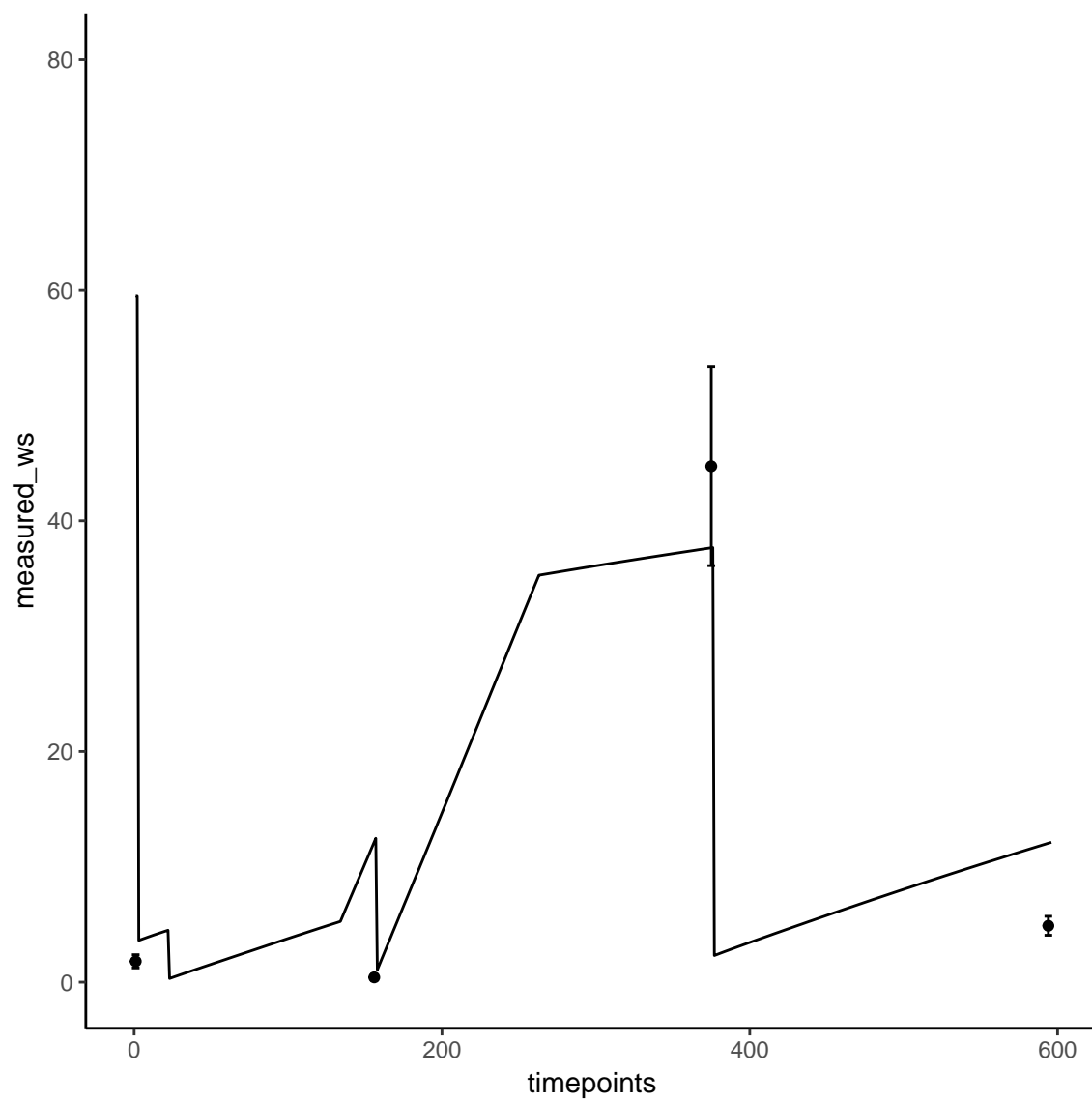


Figure 4: Two transmission parameter model fit to epi data resulting from best fit parameter set

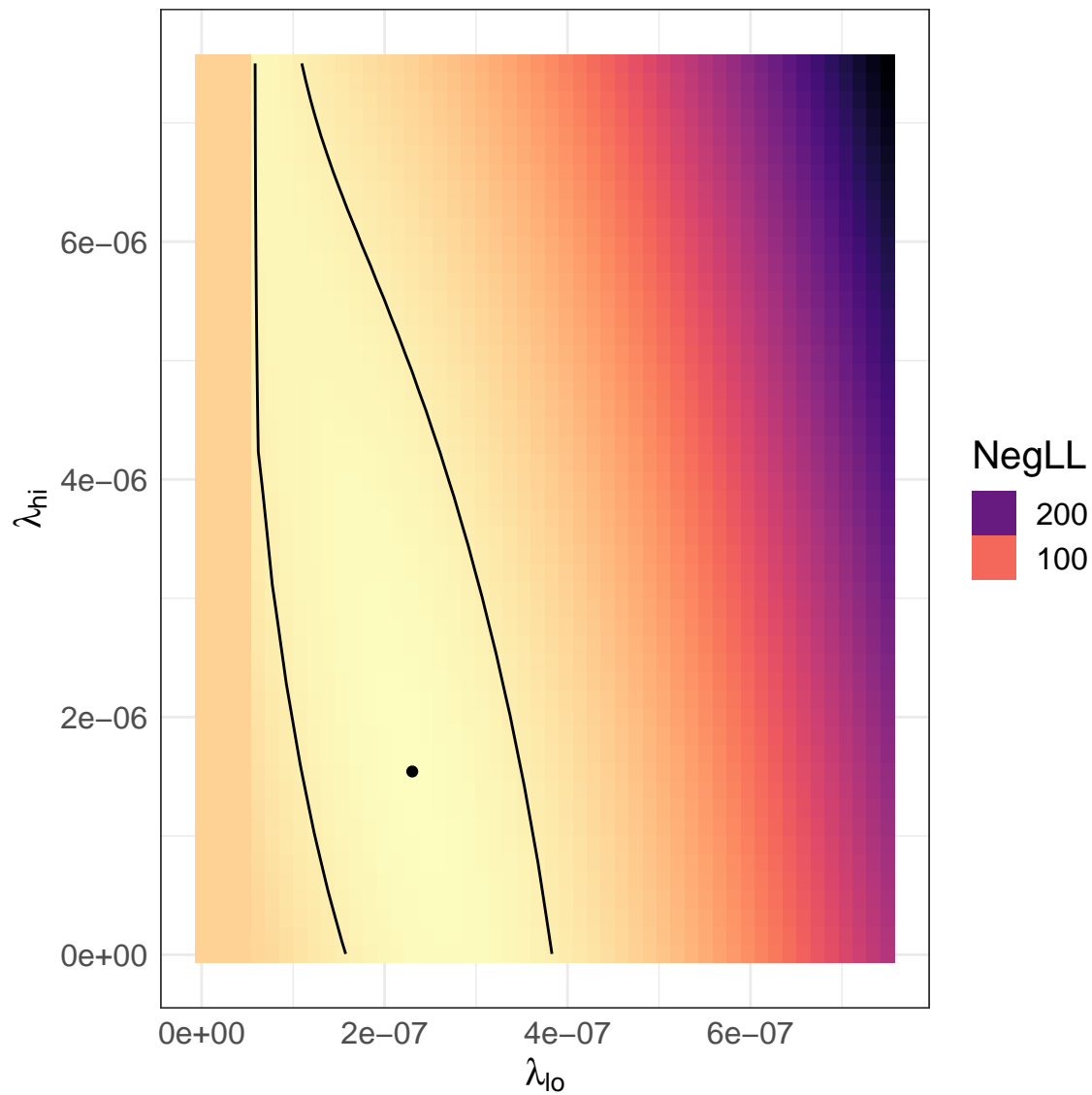


Figure 5: Heat map of transmission parameters and the negative log likelihood resulting from their fit to epi data. Black contour indicates 95% CI region and black point indicates coordinates of best fit parameters

```
axis.text=element_text(size=12),
axis.title=element_text(size=14),
legend.title=element_text(size=15),
legend.text=element_text(size=12)) +
guides(fill=guide_legend(title="NegLL",
reverse = T)) +
annotate(geom = "point", x = bestLambda1, y = bestLambda2, col = "black")
```

This kind of makes it look like the likelihood profile doesn't really have a bottom to it, i.e. given the right low transmission season parameter, high transmission season is unnecessary. That makes sense given the fitting above with one transmission parameter works fine, but let's check that out formally

```
Tx_simulate(c(bestLambda1, 0))
```

```
## [1] 18.6806
```

```
boundary
```

```
## [1] 24.76172
```

Indeed, high transmission parameter appears unnecessary. However it also appears that the range of Lambdas tested above doesn't encapsulate the full range of parameter values that could feasibly lie within the 95%CI, so let's expand the grid, but only in the dimensions that appear reasonable based on the above. In particular, it appears that we need to explore a broader range of  $\lambda_{hi}$  values, but can afford to restrict the  $\lambda_{lo}$  values a bit.

```
#Range of low and high transmission season lambdas to test
lambda_los2 <- seq(init_pars["lambda"]/100, init_pars["lambda"]*8, length.out = 30)
lambda_his2 <- seq(0, init_pars["lambda"]*150, length.out = 30)

lambda_tests2 <- expand.grid(lambda_los = lambda_los2, lambda_his = lambda_his2, negLL = 0)

params <- init_pars
params["lambda1"] <- lambda_los2[15]
params["lambda2"] <- lambda_his2[15]

#test all parameter combinations and get model fits
lambda_tests2$negLL <- apply(lambda_tests2, 1, Tx_simulate)

#Keep obs within 95%CI
lambda_tests2_95ci <- lambda_tests2 %>%
  filter(negLL <= boundary)

#Plot heatmap with 95%CI delineated
lambda_tests2 %>%
  ggplot(aes(x = lambda_los, y = lambda_his, fill = negLL, z = negLL)) +
    geom_tile() +
    geom_contour(breaks = c(boundary), col = "black") +
    scale_fill_viridis(option = "magma", direction = -1) +
    labs(y = expression(lambda[hi]),
         x = expression(lambda[lo])) +
    theme_bw() +
    theme(axis.ticks=element_blank(),
          axis.text=element_text(size=12),
          axis.title=element_text(size=14),
          legend.title=element_text(size=15),
          legend.text=element_text(size=12)) +
    guides(fill=guide_legend(title="NegLL",
                             reverse = T)) +
    annotate(geom = "point", x = bestLambda1, y = bestLambda2, col = "black")
```

So that appears to be true... Adding the second transmission parameter corresponding to high transmission season only improves the model fit marginally (negative log likelihood from 17.49 in the single parameter model as opposed to 16.95 in the model with two parameters). So definitely prefer the simpler, single transmission parameter at this point, but let's try the seasonal model as well

## Seasonality model

```
#Function to simulate seasonal transmission and return negative log likelihood from fit to epi data #####
Tx_simulate_seasonal<-function(lambda){

  params["lambda"] <- lambda

  eqbm_vals <- runsteady(y = base_mda.start, func = agrochem_mda_mod, parms = params)$y

  traj <- data.frame(ode(eqbm_vals, traj_time, agrochem_mda_seasonal_mod_fit,
                        params, events = list(data = mdas)))

  modeled <- traj$Wt[timepoints + 2] * 0.5 * mapply(phi_Wk, traj$Wt[timepoints + 2], measured_ks)
```

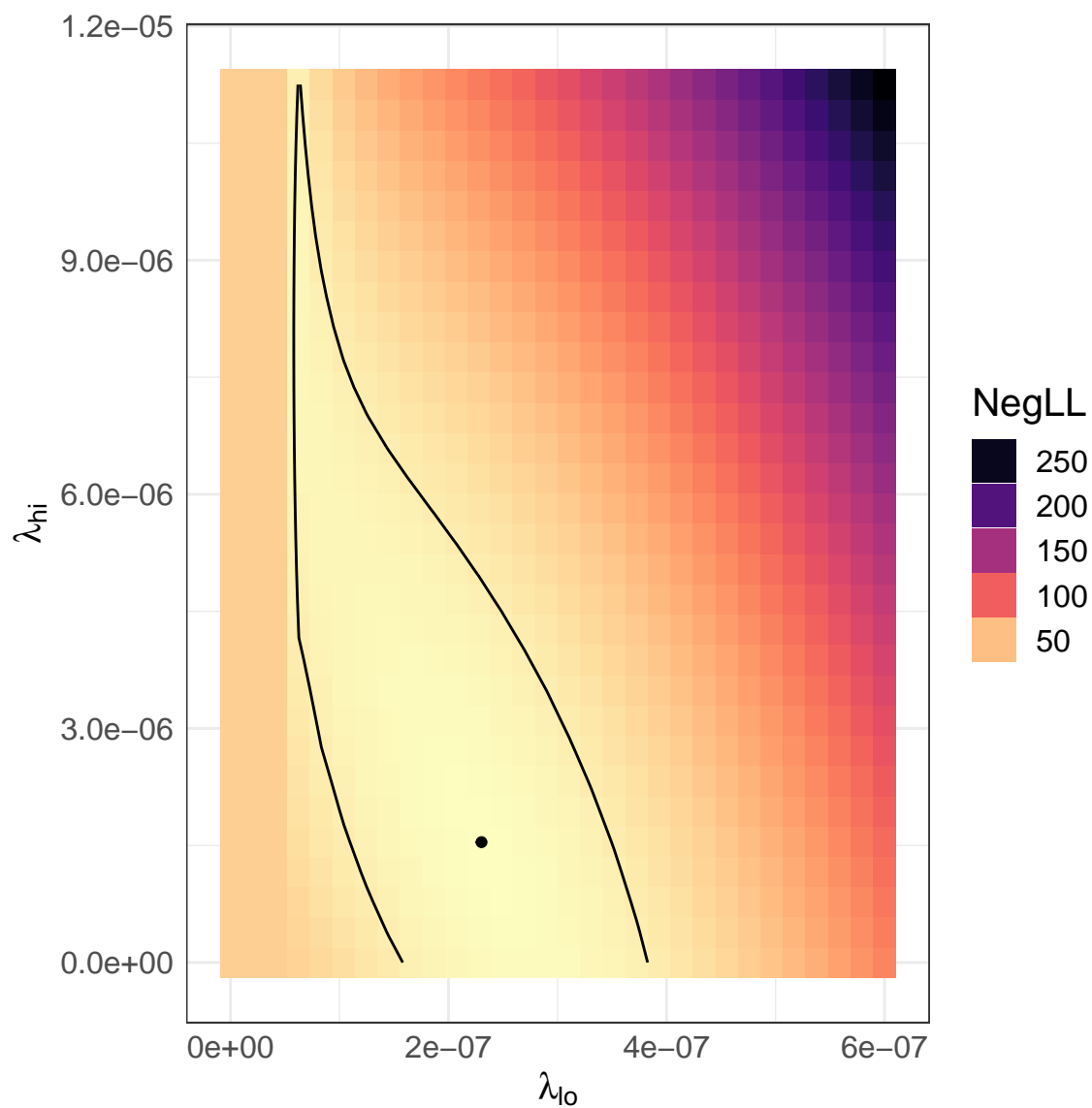


Figure 6: Heat map of expanded range of transmission parameters and the negative log likelihood resulting from their fit to epi data. Black contour indicates 95% CI region and black point indicates coordinates of best fit parameters

```

point_LL <- mapply(pointLogLike, measured_ws, w_errors, modeled)

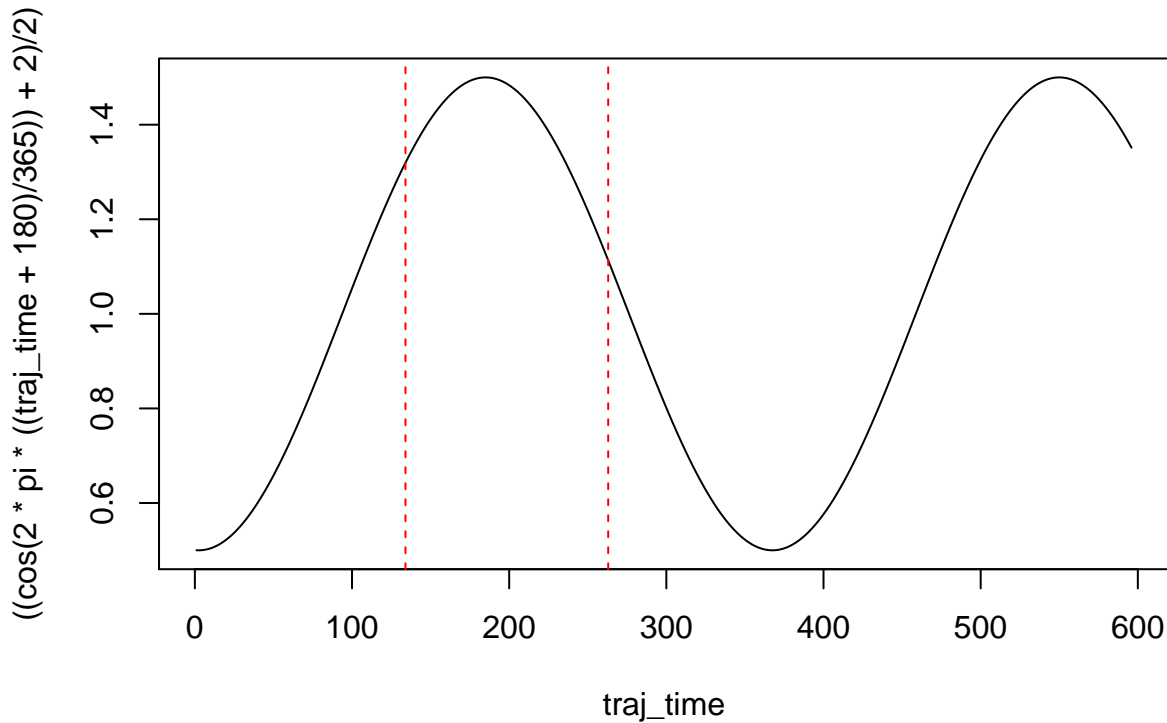
point_LL[!is.finite(point_LL)] <- NA

logLike <- sum(point_LL, na.rm = TRUE)

return(-logLike)
}

#First make sure that seasonality matches up with transmission seasons
plot(traj_time, ((cos(2*pi*((traj_time+180)/365)) + 2)/2), type = "l")
abline(v = c(seasons[c(2,3)]), lty = 2, col = 2)

```



```

#Fit using Optimize for 1-dimensional optimization
op_min_seasonal <- optimize(Tx_simulate_seasonal, interval = c(bestLambda_1par/100, bestLambda_1par*10))

#Extract best fitting parameter
bestLambda_seasonal <- op_min_seasonal$minimum

fit_pars_1par <- init_pars
fit_pars_1par["lambda"] <- bestLambda_1par
fit_pars_1par["mu_P"] <- 0.9 # Want to estimate predator-free R0

#simulate with best fitting parameter
fit_eq_vals_seasonal <- runsteady(y = base_mda.start, func = agrochem_mda_mod,
                                parms = fit_pars_1par)$y

fit_traj_seasonal <- data.frame(ode(fit_eq_vals_1par, traj_time, agrochem_mda_seasonal_mod_fit,
                                fit_pars_1par, events = list(data = mdas)))

epi_plot %>%
  ggplot(aes(x = timepoints, y = measured_ws)) +

```

```

geom_point() +
theme_classic() +
ylim(0,80) +
geom_errorbar(ymin = measured_ws - w_errors, ymax = measured_ws + w_errors, width = 5) +
geom_line(data = fit_traj_seasonal, aes(x = time, y = Wt.cvrg))

```

