LEMMA Model Descriptive

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Equations

$$
\begin{aligned}
\dot{S} &= -\beta S(I_R + I_H)/N \\
\dot{E} &= \beta S(I_R + I_H)/N - \sigma E \\
\dot{I_R} &= \sigma(1-\alpha)E - \gamma_R I_R \\
\dot{I_H} &= \sigma\alpha E - \gamma_H I_H \\
\dot{H} &= \gamma_H I_H - \psi H \\
\dot{DC} &= \psi H \\
\dot{R} &= \gamma_R I_R
\end{aligned}
$$

$\mathcal{R}_0$ formulation

For complex models such as this, best to use the next generation matrix method (see here) to estimate the basic reproduction number as a function of model equations. Briefly, the next generation matrix method constructs transmission, $\mathbf{T}$, and transition, $\mathbf{\Sigma}$, matrices from the linearized infection subsystem of a system of differential equations and $\mathcal{R}_0$ is the max eigenvalue of $-\mathbf{T}\mathbf{\Sigma}^{-1}$. User can select whether hospital population, H, is infectious(and therefore part of the infection subsystem) or not, so will construct estimates for case where H is and is not infectious.

Hospital population, H, is infectious

$$
\mathbf{T} = \begin{bmatrix} 0 & \beta & \beta & \beta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\mathbf{\Sigma} = \begin{bmatrix} -\sigma & 0 & 0 & 0 \\ \sigma(1-\alpha) & \gamma_R & 0 & 0 \\ \sigma\alpha & 0 & -\gamma_H & 0 \\ 0 & 0 & \gamma_H & -\psi \end{bmatrix}
$$

$$
\mathcal{R}_0 = \beta\sigma\frac{(1-\alpha)\gamma_H + \alpha\gamma_R}{\gamma_R\gamma_H\psi}
$$

Hospital population, H, is NOT infectious

$$
\mathbf{T} = \begin{bmatrix} 0 & \beta & \beta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\mathbf{\Sigma} = \begin{bmatrix} -\sigma & 0 & 0 \\ \sigma(1-\alpha) & \gamma_R & 0 \\ \sigma\alpha & 0 & -\gamma_H \end{bmatrix}
$$

$$
\mathcal{R}_0 = \beta\sigma\frac{(1-\alpha)\gamma_H + \alpha\gamma_R}{\gamma_R\gamma_H}
$$

This means that beta in the model should be estimated from input model parameters as:

$$
\beta = \frac{\mathcal{R}_0\gamma_R\gamma_H}{\sigma\big((1-\alpha)\gamma_H + \alpha\gamma_R\big)}
$$