LEMMA Model Descriptive

Chris Hoover

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Equations

$$
\begin{aligned}
\dot{S} &= -\beta S(I_R + I_H)/N \\
\dot{E} &= \beta S(I_R + I_H)/N - \sigma E \\
\dot{I_R} &= \sigma(1-\alpha)E - \gamma_R I_R \\
\dot{I_H} &= \sigma\alpha E - \gamma_H I_H \\
\dot{H} &= \gamma_H I_H - \psi H \\
\dot{DC} &= \psi H \\
\dot{R} &= \gamma_R I_R
\end{aligned}
$$

$\mathcal{R}_0$ formulation

For complex models such as this, best to use the next generation matrix method (see here) to estimate the basic reproduction number as a function of model equations. Briefly, the next generation matrix method constructs transmission, $\mathbf{T}$, and transition, $\mathbf{\Sigma}$, matrices from the linearized infection subsystem of a system of differential equations and $\mathcal{R}_0$ is the max eigenvalue of $-\mathbf{T}\mathbf{\Sigma}^{-1}$. User can select whether hospital population, H, is infectious(and therefore part of the infection subsystem) or not, so will construct estimates for case where H is and is not infectious.

Hospital population, H, is infectious

$$
\mathbf{T} = \begin{bmatrix} 0 & \beta & \beta & \beta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\mathbf{\Sigma} = \begin{bmatrix} -\sigma & 0 & 0 & 0 \\ \sigma(1-\alpha) & -\gamma_R & 0 & 0 \\ \sigma\alpha & 0 & -\gamma_H & 0 \\ 0 & 0 & \gamma_H & -\psi \end{bmatrix}
$$

$$
-\mathbf{\Sigma^{-1}} = \begin{bmatrix} 1/\sigma & 0 & 0 & 0 \\ (1-\alpha)/\gamma_R & 1/\gamma_R & 0 & 0 \\ \alpha/\gamma_H & 0 & 1/\gamma_H & 0 \\ \alpha/\psi & 0 & 1/\psi & 1/\psi \end{bmatrix}
$$

$$
\mathcal{R}_0 = \frac{\beta(1-\alpha)}{\gamma_R} + \frac{\beta\alpha}{\gamma_H} + \frac{\beta\alpha}{\psi}
$$

Due to the two potential infection pathways (those who will recover on their own and those who will be hospitalized), can think of this as transmission driven by those that end up in the hospital, $\mathcal{R}_H = \beta\alpha(\gamma_H + \psi)/\gamma_H\psi$ and transmission driven by mild cases, $\mathcal{R}_R = \beta(1-\alpha)/\gamma_R$, so

$$
\mathcal{R}_0 = \mathcal{R}_R + \mathcal{R}_H
$$

$$\mathcal{R}_0 = \frac{\beta(1-\alpha)}{\gamma_R} + \frac{\beta\alpha(\gamma_H + \psi)}{\gamma_H \psi}$$

Given this, β in the model can be estimated from input model parameters as:

$$\beta = \frac{\mathcal{R}_0 \gamma_R \gamma_H \psi}{\big((1-\alpha)\gamma_H \psi + \alpha\gamma_R(\gamma_H + \psi)\big)}$$

Hospital population, H, is NOT infectious

$$\mathbf{T} = \begin{bmatrix} 0 & \beta & \beta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{\Sigma} = \begin{bmatrix} -\sigma & 0 & 0 \\ \sigma(1-\alpha) & -\gamma_R & 0 \\ \sigma\alpha & 0 & -\gamma_H \end{bmatrix}$$

$$-\mathbf{\Sigma^{-1}} = \begin{bmatrix} 1/\sigma & 0 & 0 \\ (1-\alpha)/\gamma_R & 1/\gamma_R & 0 \\ \alpha/\gamma_H & 0 & 1/\gamma_H \end{bmatrix}$$

$$\mathcal{R}_0 = \frac{\beta(1-\alpha)}{\gamma_R} + \frac{\beta\alpha}{\gamma_H}$$

$$\beta = \frac{\mathcal{R}_0 \gamma_R \gamma_H}{\big((1-\alpha)\gamma_H + \alpha\gamma_R\big)}$$