

# LEMMA Model Descriptive

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*5/11/2020*

## Equations

$$\begin{aligned}\dot{S} &= -\beta S(I_R + I_H)/N \\ \dot{E} &= \beta S(I_R + I_H)/N - \sigma E \\ \dot{I}_R &= \sigma(1 - \alpha)E - \gamma_R I_R \\ \dot{I}_H &= \sigma\alpha E - \gamma_H I_H \\ \dot{H} &= \gamma_H I_H - \psi H \\ \dot{D}C &= \psi H \\ \dot{R} &= \gamma_R I_R\end{aligned}$$

## $\mathcal{R}_0$ formulation

For complex models such as this, best to use the next generation matrix method (see here) to estimate the basic reproduction number as a function of model equations. Briefly, the next generation matrix method constructs transmission,  $\mathbf{T}$ , and transition,  $\mathbf{\Sigma}$ , matrices from the linearized infection subsystem of a system of differential equations and  $\mathcal{R}_0$  is the max eigenvalue of  $-\mathbf{T}\mathbf{\Sigma}^{-1}$ . User can select whether hospital population,  $H$ , is infectious (and therefore part of the infection subsystem) or not, so will construct estimates for case where  $H$  is and is not infectious.

### Hospital population, $H$ , is infectious

$$\mathbf{T} = \begin{bmatrix} 0 & \beta & \beta & \beta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \mathbf{\Sigma} = \begin{bmatrix} -\sigma & 0 & 0 & 0 \\ \sigma(1 - \alpha) & \gamma_R & 0 & 0 \\ \sigma\alpha & 0 & -\gamma_H & 0 \\ 0 & 0 & \gamma_H & -\psi \end{bmatrix}$$

$$\mathcal{R}_0 = \beta\sigma \frac{(1 - \alpha)\gamma_H + \alpha\gamma_R}{\gamma_R\gamma_H\psi}$$

### Hospital population, $H$ , is NOT infectious

$$\mathbf{T} = \begin{bmatrix} 0 & \beta & \beta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \mathbf{\Sigma} = \begin{bmatrix} -\sigma & 0 & 0 \\ \sigma(1 - \alpha) & \gamma_R & 0 \\ \sigma\alpha & 0 & -\gamma_H \end{bmatrix}$$

$$\mathcal{R}_0 = \beta\sigma \frac{(1 - \alpha)\gamma_H + \alpha\gamma_R}{\gamma_R\gamma_H}$$