MA10174 - Semester 1, 2021/22

Problem Sheet 1

1. Write out the truth table for:

   a. $(P \Rightarrow Q) \vee (Q \Rightarrow P)$.

   b. $(P \wedge Q) \wedge (R \Rightarrow Q)$.

2. **Homework** Using the truth table show that the two statements

$$P \wedge Q \quad \text{and} \quad \neg(P \Rightarrow (\neg Q))$$

   are equivalent.

3. Determine whether the propositions P, Q, R, S are true or false knowing that

$$[(P \Leftrightarrow ((\neg Q) \vee R)] \Rightarrow [P \vee S]$$

   is false.

4. **Homework** Use De Morgan's law to write the negations of the statements in exercise 1.

   Hint. The statement $(P \Rightarrow Q)$ *is equivalent to the statement* $(\neg P \vee Q)$.

5. Write the negation of the following statement:

$$(\forall \varepsilon > 0)(\exists \delta > 0)(0 < |x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \varepsilon).$$

6. We know that:

   a. If Paul is a not student at the University of Bath or John is a student at the University of Bath, then John is a student at the University of Bristol.

   b. If Paul is a student at the University of Bath and John is not a student at the University of Bristol, then John is a student at the University of Bath.

   In which university is John studying?

7. Let $A = \{1, 2, 3, 4\}$. Determine the truth value of each one of the following statements:

   a. $\forall x \in A : x + 3 > 6$

   b. $\forall x \in A : x^2 - 10 \leq 8$

   c. **Homework** $\exists x \in A : 2x^2 + x = 15$

d. **Homework** $\exists x \in A : x^2 > 1 \Longrightarrow x + 2 = 0$

8. Let $A = \{2, 1, -1, -3\}$ and $B = \{1, 2, 3, 4\}$, and consider the following statements

   a. $(\exists x \in A)(\forall y \in B)(y^2 < x - 2 \Rightarrow 2y < 2 + x)$

   b. $(\forall x \in B)(\forall y \in A)(\neg(2x - y > 1) \vee (y^2 - x \text{ is a multiple of } 3))$

   Show that statement a is true and write its negation.

   Homework Show that statement b is false and write its negation.