Εργασία $3^η$

VALIDITY CHECK OF GIANNAKIS' FORMULA!

Construct a real discrete signal $x[k], k = 1,2, \dots, N = 2048$, which is derived as the output of a MA-q process with coefficients of $[1.0, 0.93, 0.85, 0.72, 0.59, -0.10]$, driven by white non-Gaussian noise $v[k]$, which is derived from an exponential distribution with mean value of 1 (in Matlab, $v =$ `exprnd(1,[1,2048]);`). When you construct the signals $x[k]$ and $v[k]$ save them to use them throughout.

1. Justify the non-Gaussian character of input $v[k]$ by calculating its skewness γ_3^v using the following equation:

$$\gamma_3^v = \frac{\sum_{i=1}^{N}(v(i) - \hat{m}_v)^3}{(N-1)\hat{\sigma}_v^3},$$

where $\hat{m}_v$ and $\hat{\sigma}_v$ denote the estimated from the data mean and standard deviation, respectively.

2. Estimate and plot the 3^{rd}-order cumulants of $x[k]$, $c_3^x(\tau_1, \tau_2)$ using the indirect method with $K = 32$, $M = 64$, $L_3 = 20$ [that is $(-\tau_1{:}\,0{:}\,\tau_1) = (-20{:}\,0{:}\,20)$, $(-\tau_2{:}\,0{:}\,\tau_2) = (-20{:}\,0{:}\,20)$].

3. Use the estimated $c_3^x(\tau_1, \tau_2)$ to estimate the impulse response $\hat{h}[k]$ of the MA system using the Giannakis' formula, i.e.,

$$\hat{h}[k] = \frac{c_3^x(q,k)}{c_3^x(q,0)}, k = 0,1,2,\dots,q,$$
$$\hat{h}[k] = 0, k > q.$$

4. Estimate the impulse response of the MA system using the Giannakis' formula, yet considering:
   a. Sub-estimation of the order q, that is, $\hat{h}_{sub}[k]$: MA-q_{sub}, where $q_{sub} = q - 2$.
   b. Sup-estimation of the order q, that is, $\hat{h}_{sup}[k]$: MA-q_{sup}, where $q_{sup} = q + 3$.

5. Estimate the MA-q system output $x_{est}[k]$, using the convolution between the input $v[k]$ and the estimated impulse response from Step 3, i.e., $x_{est}[k] = v[k] * \hat{h}[k]$ and plot in the same figure the original $x[k]$ (blue color) and the estimated $x_{est}[k]$ (red color; keep only the first N samples). Find the normalized root mean square error (NRMSE) using the formula

$$NRMSE = \frac{RMSE}{\max(x[k]) - \min(x[k])}, \text{with}$$
$$RMSE = \sqrt{\frac{\sum_{k=1}^{N}(x_{est}[k] - x[k])^2}{N}}$$

Comment upon the findings.

6. Repeat Step 5, for the case of $x_{sub}[k] = v[k] * \hat{h}_{sub}[k]$ and $x_{sup}[k] = v[k] * \hat{h}_{sup}[k]$.
   Comment upon the findings and compare with the results of Step 5.
7. Consider that we add a noise source of white Gaussian noise at the output of the system, producing a variation in the signal-to-noise-ratio (SNR) of $[30{:}-5{:}-5]$dB, i.e., $y_i[k] = x[k] + n_i[k], i = 1{:}8$. Repeat Steps 2, 3 and 5, but instead of $x[k]$ use the noise contaminated output $y_i[k]$ for each ith given level of SNR (you can easily create the contaminate signal by using the awgn.m function of Matlab (`y = awgn(x,snr,'measured')` and simply changing each time the given SNR). Make a plot of the NRMSE error in the estimation of $y_i[k]$ versus the SNR range. Comment upon your results.

8. (optional) Instead of using just one realization of the input and output data of MA-q system, you could repeat the whole process 50-100 times and work with the mean values of your results (that is mean NRMSE) to increase the viability and generalization of your conclusions about the Giannakis' formula.