Modeling and control of cyber-physical systems
Project I

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In this project, we apply the mathematical models and algorithms discussed in class to estimate the state of a system, possibly in the presence of sensors attacks. In particular, we consider problems of target localization, in a two-dimensional indoor area.

The work is conceived for groups of 3-4 students. The choice of the programming language is free; we suggest MATLAB or Python.

Students are required to write a report ($\sim$ 4-5 pages) with the analysis of the obtained results.

Objectives

The goal of this activity is to learn to

1. implement algorithms for CPSs
2. enhance the algorithms to improve the performance (e.g., by a suitable tuning of the hyperparameters)
3. analyse the obtained results
4. write a technical report

Requirements

1. Implement the algorithms and solve the proposed problems
2. Write a report ($\sim$ 4-5 pages) with the analysis of the obtained results
3. Upload the report and the code in the delivery page of the course, at least one week before the oral examination

Task 1: Implementation of ISTA

Given $C \in \mathbb{R}^{q,p}$, a h-sparse $\widetilde{x} \in \mathbb{R}^p$, and noisy measurements $y = C\widetilde{x} + \eta$, where $\eta \in \mathbb{R}^q$ is the measurement noise, the LASSO problem is:

$$\min_{x \in \mathbb{R}^p} \frac{1}{2}\|Cx - y\|_2^2 + \lambda \|x\|_1$$

where $\lambda \in \mathbb{R}$ is a design hyperparameter.

Solve LASSO by implementing ISTA.

Algorithm 1 ISTA

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1: Initialization: x(0) = 0 ∈ R^p
2: for all k = 0, ..., T_max do
3:     x(k+1) = S_{τΛ} [x(k) + τ C^⊤ (y − C x(k))]
4: end for
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Given $x \in \mathbb{R}^p$, the shrinkage/thresholding operator $\mathbb{S}_\gamma : \mathbb{R}^p \mapsto \mathbb{R}^p$, with $\gamma = (\gamma_1, \ldots, \gamma_p)^\top \in \mathbb{R}^p_+$, is defined as

$$\mathbb{S}_{\gamma_i}(x_i) := \begin{cases} x_i - \gamma_i & \text{if } x_i > \gamma_i \\ x_i + \gamma_i & \text{if } x_i < -\gamma_i \\ 0 & \text{if } |x_i| \leq \gamma_i \end{cases}$$

for each $i = 1, \ldots, p$.

Suggested setting:

1. $\tau = \|C\|_2^{-2} - \epsilon$, $\epsilon = 10^{-8}$
2. $q = 10$, $p = 20$, $k = 2$, $\lambda = \frac{1}{100\tau}$
   ($\Rightarrow \Lambda = \lambda(1, \ldots, 1)^\top$, $\tau\Lambda = 10^{-2}(1, 1, \ldots, 1)^\top$).
3. Generate the components of C according to a standard normal distribution $\sim \mathcal{N}(0, 1)$ (randn)
4. Generate the support S of the true $\widetilde{x}$ with uniform distribution and the non-zero components $i \in S$ such that $\widetilde{x}_i \in [-2, -1] \cup [1, 2]$, with uniform distribution
5. Measurement noise $\eta \sim \mathcal{N}(0, \sigma^2)$, $\sigma = 10^{-2}$ ($\sigma *$randn)

6. Stop criterion: T_{max} = first step such that $\|x(T_{max}+1) - x(T_{max})\|_2 < \delta$, $\delta = 10^{-12}$.

Repeat the experiment for at least 20 runs and analyse the mean results, by considering the following points:

1. Support recovery rate: how many times the support of $\widetilde{x}$ is correctly estimated?

2. Can we obtain 100% of success in the support recovery by increasing q?

3. Convergence time: how many iterations are required (mean, min, max)?

4. Try different values for τ, by keeping $\tau\lambda$ constant

5. Try different values for λ, by keeping τ constant

Task 2: Secure estimation under sparse sensor attacks

Reminder: we can extend LASSO by a assigning a different ℓ_1 weight to each component, e.g.,

$$\min_{x \in \mathbb{R}^p} \frac{1}{2}\|Cx - y\|_2^2 + \sum_{i=1}^{p} \lambda_i |x_i|.$$

Reformulate the previous problem to estimate a (non-sparse) $\widetilde{x} \in \mathbb{R}^n$ under sparse sensor attacks, by using ISTA, given the measurements $y = C\widetilde{x} + a + \eta$, where $\eta \in \mathbb{R}^q$ is the measurement noise.

Suggested data and hyperparameters:

1. $n = 10$, $q = 20$, $h = 2$ sensor attacks

2. $C \sim \mathcal{N}(0, 1)$; $\widetilde{x} \sim \mathcal{N}(0, 1)$

3. Support of the attack vector a: uniform distribution

4. How to perform the attacks?

   - "Unaware" attack: $a_i \in [-2, -1] \cup [1, 2]$, uniformly distributed
   - "Aware" attack: the attacker takes the sensor measurement y_i and corrupts it of a quantity equal to $\frac{1}{2}y_i$

5. $\tau\Lambda = \tau\lambda(0, \ldots, 0, 1, \ldots, 1)^\top \in \mathbb{R}^{n+q}$, $\tau\lambda = 2 \times 10^{-3}$

6. Measurement noise $\eta = 0$ and $\eta \sim \mathcal{N}(0, \sigma^2)$, $\sigma = 10^{-2}$ (σ *randn)

Repeat the experiment for at least 20 runs and analyse the mean results, by considering the following points:

1. Rate of attack detection: how many times the support of a is correctly estimated, i.e., we identify the sensors under attack?

2. Estimation accuracy: is the estimation of $\widetilde{x}$ accurate? Compute $\|\widetilde{x} - \hat{x}\|_2^2$ where $\hat{x} = x(T_{max})$ is the obtained estimate.

Task 3: Target localization under sparse sensor attacks

We consider an indoor localization problem with an RSS fingerprinting setting. We propose a localization problem where 3 targets are deployed in a square room with $p = 100$ cells. A sensor network with $q = 25$ sensors is randomly deployed in the room, see the figure. The sensors measure the RSS from targets. We assume that few sensors are under attack. The goals are

1. Localize the targets, e.g., estimate which cells of the grid are occupied by a target

2. Find which sensors are under attack.

Optional task: compare the obtained results to the results of other methods (e.g., k-NN), in terms of estimation accuracy and computational complexity/run time.

The dictionary D and the run-time measurements y are given in file localization.mat.

To localize the targets and identify the sensors under attack, implement ISTA to solve the following weighted Lasso

$$\min_{x \in \mathbb{R}^p, a \in \mathbb{R}^q} \left\| (D, I) \begin{pmatrix} x \\ a \end{pmatrix} - y \right\|_2^2 + \lambda_1 \|x\|_1 + \lambda_2 \|a\|_1$$

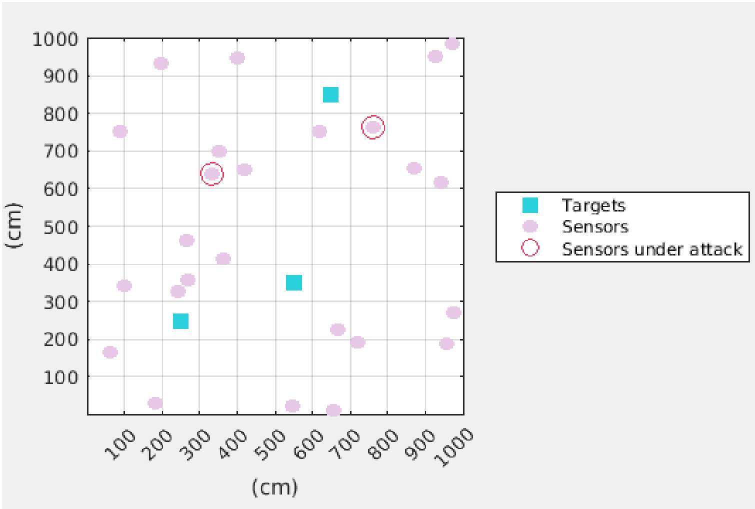
Hints

1. $\lambda_1 = 10$, $\lambda_2 = 20$

2. $G = [D, I] \rightarrow$ normalize G, e.g., by using normalize(G) in MATLAB

3. $\tau = \|G\|_2^{-2} - \epsilon$

[Solution: supp$(\widetilde{x}) = \{23, 36, 87\}$, supp$(\widetilde{a}) = \{12, 16\}$]



Task 4: Sparse observer

We consider the problem of Task 3 with moving targets and sparse sensor attacks. There are 3 targets, moving according to the dynamics

$$x(k+1) = Ax(k).$$

A is given in the file tracking_moving_targets.mat. Basically, at each time step, the targets move towards left of one cell. The given $Y \in \mathbb{R}^{q,50}$ are the measurements for 50 time instants, i.e., for $k = 0, \ldots, 49$.

Implement the sparse observer to track the moving targets and to identify which sensors are under attack.

Notation:

$$z(k) = \begin{pmatrix} x(k) \\ a(k) \end{pmatrix} \qquad \hat{z}(k) = \begin{pmatrix} \hat{x}(k) \\ \hat{a}(k) \end{pmatrix} \qquad \hat{z}^+(k) = \begin{pmatrix} \hat{x}^+(k) \\ \hat{a}^+(k) \end{pmatrix} \qquad G = \begin{pmatrix} D & I \end{pmatrix}$$

Hint: $\lambda = (10, \ldots, 10, 20, \ldots, 20)$

Algorithm 2 Sparse observer

for all $k = 0, \ldots, 49$ **do**
 $\hat{z}^+(k) = \mathbb{S}_{\tau\lambda}\left[\hat{z}(k) + \tau G^\top (y(k) - G\hat{z}(k))\right]$
 $\hat{x}(k+1) = A\hat{x}^+(k)$
 $\hat{a}(k+1) = \hat{a}^+(k)$
end for

Analyse the following points:

1. Does the sparse observer converge?

2. After how many iterations the sparse observer converges?

Optional tasks:

1. Implement an "aware" time-varying attack: the attacker attacks two sensors by taking their measurements $y_i(k)$ and by adding a quantity equal to $a_i(k) = \frac{1}{2} y_i(k)$

2. Implement a time-varying attack where also the sensors under attack change in time; for example, two sensors are under attack for $k = 0, \dots, 24$ and two other sensors are under attack for $k = 25, \dots, 49$.

3. Increase the number of sensors under attack. Which is the maximum number of attacks which allows us to perform a correct tracking of the targets?

Task 5: Distributed target localization under sparse sensor attacks

In this task, we retrieve the target localization problem in Task 3 and we solve it in-network, i.e., in a distributed way, through the distributed ISTA (DISTA). The aim is to localize 2 targets in the presence of sparse sensor attacks. The measurement model is $y = D\widetilde{x} + \eta + \widetilde{a} \in \mathbb{R}^q$, where $\eta \in \mathbb{R}^q$ is a meausurement noise and $\widetilde{a} \in \mathbb{R}^q$ is the attack vector.

We remark that the attacks consist in a physical tampering of the sensor measurements in the run-time phase. On the other hand, we assume there are no attacks on the communication links, which are reliable.

This marks a difference with respect to the centralized case, where a manipulation of $D_i\widetilde{x}$ can be done either on the sensor or in the transmission of the data to the fusion center.

We consider a distributed setting where each sensor node $i \in \{1, \dots, q\}$ knows D_i ($= i$th row of the dictionary D) and $y_i \in \mathbb{R}$, and it does not share them. In the file distributed_localization_data.mat, we provide the data y and D and 4 possible network topologies, described by 4 different stochastic matrices Q. Check whether these matrices solve the consensus problem, by analyzing their eigenvalues.

Repeat the task for each stochastic matrix. Hints:

1. $G = \begin{pmatrix} D & I \end{pmatrix}$ cannot be normalized, since it is not stored in a fusion center!

2. $\tau = 4 \times 10^{-7}$

3. $\lambda = (10, \ldots, 10, 0.1, \ldots, 0.1)$

4. Final refinement of the attacks: set to zero all the estimated attack components with magnitude < 0.002

5. In the implementation of DISTA, pay attention to the fact theat the estimate of each node is a vector.

Analysis:

1. Does DISTA reach a consensus?

2. Is the final estimation accurate?

Algorithm 3 DISTA

Initialization: for each node $i = 1, \ldots, q$, $z^{(i)}(0) \in \mathbb{R}^{n+q}$, e.g., $z^{(i)}(0) = 0$

for all $k = 1, \ldots, T$ **do**

    for all $i = 1, \ldots, q$ **do**

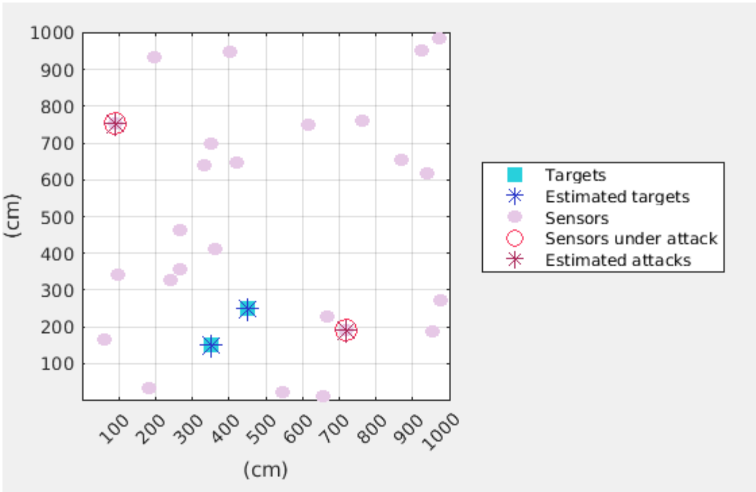
$$z^{(i)}(k+1) = \mathbb{S}_{\tau\lambda}\left[\sum_{j=1}^{q} Q_{i,j} z^{(j)}(k) + \tau G_i^T (y_i - G_i z^{(i)}(k))\right]$$

    end for

    Stop criterion: T = first time instant s.t. $\sum_{i=1}^{q} \|z^{(i)}(T+1) - z^{(i)}(T)\|_2^2 < \delta$, $\delta = 10^{-8}$.

end for

[Solution: supp($\widetilde{x}$) $= \{14, 25\}$, supp($\widetilde{a}$) $= \{8, 23\}$]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Estimated targets
Sensors
Sensors under attack
Estimated attacks