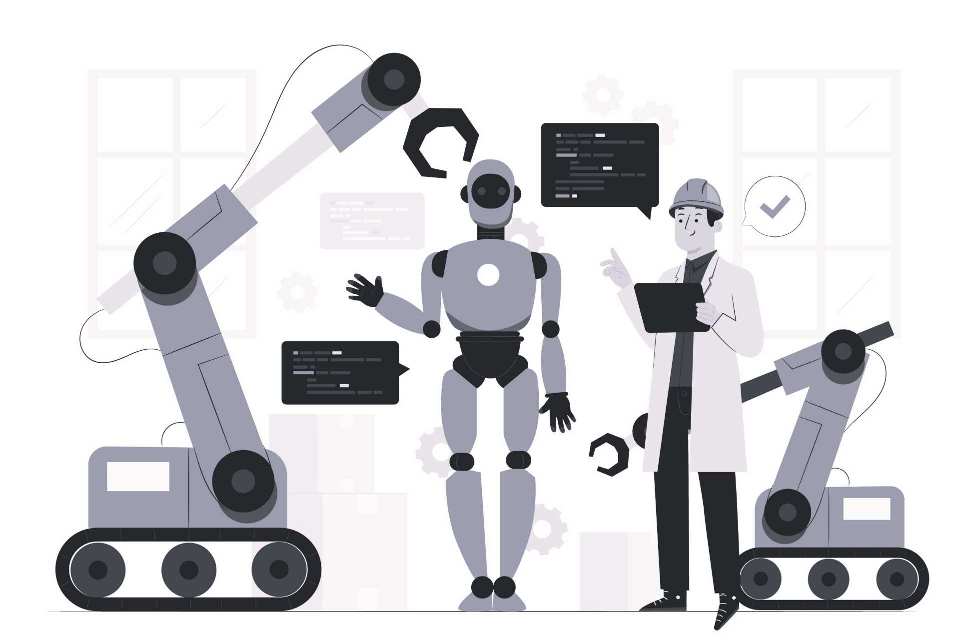
ROBOTICS

Lecture notes



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Chapter 1

Introduction

In this chapter we will see...

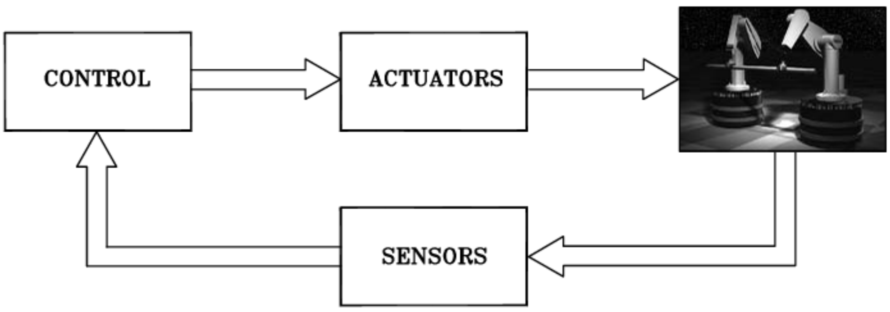


Figure 1.1: Components of a robotic system

1.1 Robot: a possible definition

What is robotics? Before answering this question, it makes sense to rise up another question: **what is a robot?** Among all the available definitions the most suitable is the one provided by Mike Brady:

A robot realizes the *intelligent* connection between **perception** and **action**.

With reference to such a definition we can dissect it finding useful insights. A *robotic system* is in the reality a complex system which is functionally represented by **multiple subsystems**. In particular you have a *mechanical systems* which is made up of two apparatus: **locomotion apparatus** and a **manipulation apparatus** by which the robot itself can carry out the task. Both these subsystems are provided with an *actuation system* which **animates** the mechanical part (this includes *servomotors,drives* and *transmissions*)[1] The **perception** part rely on sensors which can acquire data about either the internal status of the mechanical system (*proprioceptive sensors*, eg. position transducers) or the external status of the environments (they are called

[1]It is quite clear that the **action** part is linked to the actuation system

esteroceptive sensors, eg. cameras).
The connection between the the actuation and sensing part is provided by a *control system* which in an intelligent way can command the execution of some actions. It is remarkable that the control system exploits a mathematical model of the robot.
At this point, we can say that **Robotics** deals with

study and **design** of robots.

this is the reason why it results in an interdisciplinary subject involving *mechanics, control, computers* and *electronics*.
The term **robot** it is derived from the Slav term *robota* (executive labor), and it appeared several years before any robot could be built!

1.2 Robots classification

Nowadays, robots can be used in different contexts according to which they are split in:

- Industrial robots they are used in the industrial field in order to perform several tasks such as soldering, moving part, cutting parts and so on;
- Humanoid and biometric robots they are employed in order to interact with people or animal environments. Not rarely their mechanics is inspired by the real (natural) behaviour.
- Service robots are used in contexts where the actions to be carried out are dangerous for human beings. For example: think about manipulating radioactive matter.
- Exploration robots are used, for example, in the field of space missions to study the atmosphere and the surface of Mars.

Another available classification of robots is the one based on the type of mechanical structure. Is the robot base moving or is it fixed? In the first case we are talking about **mobile robots** (humanoid/biometric, service and exploration robots often fall in this category), in the latter case we refer them with the term of **robot manipulators** (the great majority of industrial robots).

1.2.1 Robot manipulator (Industrial robots)

We anticipate here that a *robot manipulator* is made up of a sequence of rigid bodies (links) connected by means of articulations (*joints*); the part of the manipulator that ensures mobility is the so-called **arm**, the part devoted to its agility (more technically, dexterity) is the **wrist** finally the part which perfors the action is the **end-effector**. Several types of robot manipulators are obtained changing such components.

1.2.2 Mobile robots

The main feature here is the presence of a *mobile base* which allows the robot to move into the environment it is placed. They are provided with a locomotion system that can be different (wheels, legs, wings...).

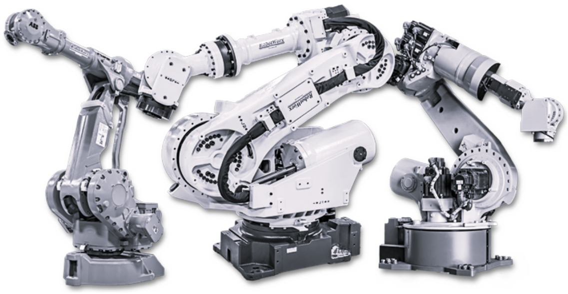


Figure 1.2: Examples of Industrial Robots

1.3 Robot modeling, planning and control

The totality of the tasks to be completed requires the execution of a *specific motion prescribed to the robot*. The correct execution of the motion is entrusted to the control systems which should provide actuators with **command** consitstent with the desired motion (motion control), this – in turn – rely on an accurate analysis on the features of (jointly) mechanical structrure, actuators and sensors. The final aim of such an analysis is the derivation of a mathematical model describing the I/O relationship for the robot components (modeling component). Relevant topics in *modeling, planning* and *control* are briefly resumed in the following.

1.3.1 Modelling

Obtaining a model for a robot (industrial or mobile) concerns essentially in studying both the **kinematics** and **dynamics**. The former is the description of the robot motion without taking into account forces and torques which causes it. The latter describes forces and torques which generates the motion.
A further distinction is made between between *kinematics* and *differential kinematics* is made. In the first case we want to find a relationship between the joints and end-effector positions. In the second case we want to study the relationship between the joint and end-effector motion by means of velocity. A very useful tool in this case is the *Jacobian* of the manipulator.
We conclude this discussion by saying that having a model for the robot is useful for the mechanical design of the structure, choice of actuators, determination of control strategies and for doing computer-based simulations.

1.3.2 Planning

Every robot performs a task, for example for a manipulator we have the necessity to describe in some way the motion at the joints or at the end-effector. In material-handling task for example it is sufficient to decide an initial position and and end position (*point-to-point* motion) in other situations we have to track a certain trajecotry (*path motion*). In all these cases the task is the **trajectory planning** that is the determination of timing laws for relevant variables. For a mobile robot, such a task is even more complicate since it requires to take into account the constraints imposed by the wheels, legs, wings...

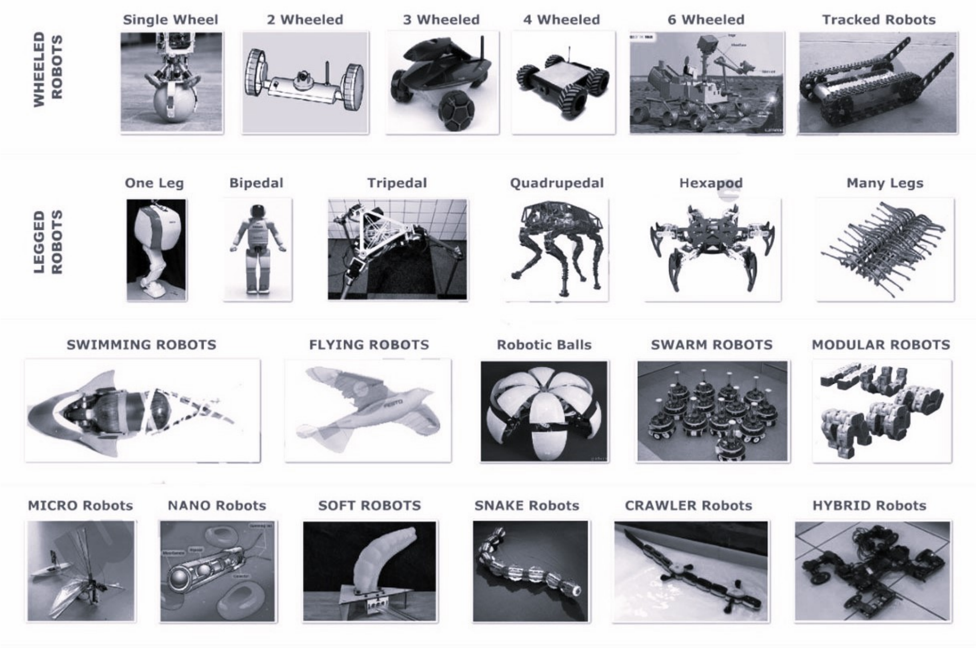
1.3.3 Control

The generated trajectories constitutes the reference signal for the *control system* of the mechanical structure. Techniques for controlling manipulators and mobile robots are very different how we will see this is due the presence or not of the interaction robot-environment and of the mobile base.

Chapter 2

Mobile robots

In this chapter we will see...



2.1 Introduction

Definition 2.1.1 (Mobile robot). A **mobile robot** is a structure capable of moving and act in *terrestrial*, *underwater* or *aerial* environments.

The characteristics of the environment are fundamental since planning and control are affected by it. Namely the environment can be totally structured, unstructured or partially structured. We refer the **structuredness** of the environment as the knowledge on geometric characteristics.

2.1.1 Autonomy

A mobile robot is equipped with a certain level of **autonomy** which makes the structure capable of moving independently from a human supervisor. For achieving it, it is required a computational part (CPU, Intelligence of the robot), some sensors and actuators and an energy source, which can be either generated on-board or provided by mean of an external source.

2.1.2 Locomotion

Another fundamental aspect is that, differently from manipulators, a mobile robot is equipped with a **locomotion apparatus** which drastically changes according to the environment they are acting in. A **terrestrial robot** can have wheels, legs or a biomimetic locomotion system; a **underwater robot** can provided with propellers or water jets; finally, an **aerial robots** can have rotating, fixed or flapping wings.

2.2 Types of wheels

In the following we are focusing our attention on the wheeled robots, as they represent the great majority of mobile robots used in applications. The basic mechanical structure of this robot is indeed the wheel. The table Table 2.1 shows a summary of the fundamental information about the most popular types of wheels.

2.3 Constraints and Kinematic Models

We have seen in the introduction that mobile and industrial robots have different characteristics, this results in different models, planning and control strategies. Since in general robots are made up of multiple rigid bodies (*multibody system*) linked together by joints, their motion is constrained. This holds in general, however also the types of constraints are different in the case of mobile robots and manipulators. This is because in the former case you have limitations in the **position** of the whole body, in the case of mobile robots, and in particular in terrestrial wheeled robots, you have instead limitations on **how the position can change** in magnitude, direction and side (this is nothing but the **velocity**). From now on we are focusing our attention on **modeling wheeled robots**.

2.3.1 Constraints and their classification

In order to properly describing and giving a classification of constraints, it is better recalling the concept of **generalized coordinates**.
Let the vector $\mathbf{q} \in \mathbb{R}^n$ be the *generalized coordinates* that describes the **configuration** of the robot (minimum number of variables needed to model the robot motion). For the moment, let us assume that the **configuration space** $\mathcal{C}$ coincides with $\mathbb{R}^n$. For example for a **unicycle** the generalized coordinates are in number of three:

$$\mathbf{q} = [x \quad y \quad \theta]^T \in \mathbb{R}^3 \tag{2.1}$$

where x and y are the position of the contact point between the (single) wheel and the plane on which the motion occurs, while θ is the angle of the wheel with respect to the horizontal axis. The **evolution in time** of the generalized coordinates vector *describe the motion* of the system.

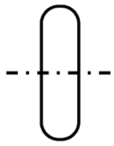
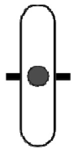
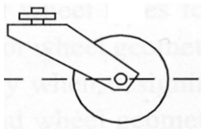
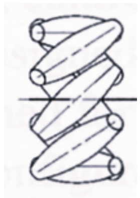
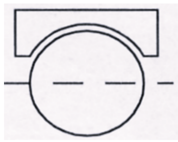
WHEEL TYPE	**DESCRIPTION**	**SYMBOL**
Simple non-steering wheel	They can rotate about an axis which passes through the center of the wheel itself, orthogonal to its plane. The orientation of the chassis with respect to the wheels is constant.	
Simple steering wheels	Has two axes of rotation, one that is orthogonal to the wheel plane, the other which is *vertical* and goes through the center of the wheel. This provides the wheel with the possibility of changing the orientation with respect to the chassis. Note that for both non-steering and simple steering wheels the component the velocity which orthogonal to the wheel plane is null since there is no sleeping. That is: $$v^{\perp}(t) = 0$$	
Castor wheel	It is a variant of the previous one in which the vertical axis does not pass through the center of the wheel from which it is **displaced** by a constant offset. This adds degrees of freedom to the vehicle on which they are mounted on. Such a type of wheels are often used for office chairs and supermarket carts.	
Omnidirectional or Swedish wheel	There is another type of non-conventional wheel that is the *mechanum* (or **swedish wheel**). It mounts some passive rollers whose rotation axis is inclined by 45 degrees with respect to the plane of the wheel itself. They are also called *omniwheels*.	
Spherical omniwheel	There is another type of omniwheel which is **spherical**. They can be either active or passive. Like in the case of swedish wheels, a vehicle equipped with four of them is called **omnidirectional**.	

Table 2.1: Types of wheels with description and symbols

Constraints can be found by mean of **equality** (in this case we refer them as *bilateral* constraints), by mean of **inequality** (we refer them as *unilateral* constraints). Furthermore, according to the fact they are or not time-variant, we can divide them in *rehonomic* (explicit dependence on time) and *scleronomic* (time-invariant constraints). In this course we will treat only **bilateral and scleronomic constraints** another term for indicating them is **holonomic** (or **integrable**) constraints.

Holonomic constraints

Such a type of constraint[1] can be expressed as:

$$h_i(\mathbf{q}) = 0, \quad i = 1, ..., k < n \tag{2.2}$$

A system whose motion is characterized only by holonomic constraints is called **holonomic system**. By using the *implicit function theorem* (or in Italian "Teorema del Dini"), we can reduce the dimension of the configuration space to $n - k$

Kinematic constraints

Such a type of constraints involve both generalized coordinates $\mathbf{q}$ and its derivative $\dot{\mathbf{q}}$. In the most general case they can be expressed as:

$$a_i(\mathbf{q}, \dot{\mathbf{q}}) = 0, \quad i = 1, ..., k < n \tag{2.3}$$

Kinematic constraints are limiting the set of generalized velocities that can be obtained by each configuration. In some cases they can be written in the so-called **Pfaffian form**, that is they can be written as a *linear combination of the generalized velocities* $\dot{\mathbf{q}}$:

$$\mathbf{a}_i^T(\mathbf{q})\dot{\mathbf{q}} = 0, \quad i = 1, ..., k < n \tag{2.4}$$

An example of kinematic constraint in Pfaffian form is:

$$3q_1\dot{q}_1 + 2\sin q_1\dot{q}_2 + \sin q_3\dot{q}_3 = 0$$

The reason for using such a notation is that we can immediately retrieve the expression of the associated function by doing the row-by-column product (standard inner product). Such k kinematic constraints can be written in compact form by introducing matrices

$$\mathbf{A}^T(\mathbf{q})\dot{\mathbf{q}} = \mathbf{0} \tag{2.5}$$

where $\mathbf{A}(\mathbf{q}) \in \mathbb{R}^{k,n}$. It is interesting to note that the presece of k holonomic constraints imply the presence of k kinematic constraints. This can be easily showed by computing the time derivative for the k holonomic constraints:

$$\frac{dh_i(\mathbf{q})}{dt} = \frac{dh_i(\mathbf{q})}{d\mathbf{q}} \cdot \frac{d\mathbf{q}}{dt} = \frac{dh_i(\mathbf{q})}{d\mathbf{q}} \cdot \dot{\mathbf{q}} = 0, \qquad i = 1, ..., k \tag{2.6}$$

where we have applied the fact that the constraints are holonomic while using the *Chain rule* for passing from the first to the second step. From the Equation (2.6) we have understood that:

$$\text{holonomic constraint} \Longrightarrow \text{kinematic constraints}$$

[1]We recall that for a multibody system described by using the Lagrangian approach, holonomic constraints are those allowing a reduction of the number of needed variables.

In general, we cannot say the inverse and in this case (since the step from the derivative to the primitive results in doing the integral) associated constraints are called **nonholonomic** (or **non-integrable**) ones. A system characterized by such constraints is called **nonholonomic system**. In presence of non-integrable constraints the dimension of the configuration space $\mathcal{C}$ cannot be reduced while the generalized velocities can be described over a subspace of dimension $n-k$ (how we are going to see in a minute).

Example of nonholonomic constraint

A unicycle rolls on a plane **without slippering**, we have already seen its generalized coordinates. For such a system we have the so called **pure rolling constraint**, this imply the velocity of the contact point not to have a non-zero component along the direction orthogonal to the wheels plane. By using simple trigonometric properties involving the inifinitesimal increment dx and dy, we can state that

$$\frac{dy}{dx} = \tan\theta \tag{2.7}$$

Can we obtain the Pfaffian form for such a constraint? The answer is YES. In fact by dividing and multiplying for an infinitesimal time increment dt we obtain:

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = \frac{\dot{y}}{\dot{x}} = \frac{\sin\theta}{\cos\theta} \iff \dot{y}\cos\theta = \dot{x}\sin\theta$$

Which is the same to say that

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0 \iff [\sin\theta \quad \cos\theta \quad 0]\dot{\mathbf{q}} = 0 \tag{2.8}$$

The constraint we have just derived can be demonstrated that is non-integrable and so nonholonomic, in fact we cannot reduce the dimension of the configuration space (in other words all of the generalized coordinates are needed to properly describe the unicycle motion). Note that, start from an initial state $\mathbf{q}_i$, you can bring the system to any final state $\mathbf{q}_f$, under the assumption of not to violating the pure rolling constraint[2].

2.3.2 Kinematic model

From the Equation (2.5), we can immediately see that the $n-k$ admissible generalized velocities belong to the null space[3] of $\mathbf{A}(\mathbf{q})^T$ that is

$$\dot{\mathbf{q}} \in \mathcal{N}(\mathbf{A}^\mathbf{T}(\mathbf{q})) \tag{2.11}$$

[2]We will see that in order to pass from an initial to a final state a *trajectory planning algorithm* must be used.

[3]Just for doing a brief recap. Given a matrix $A \in \mathbb{R}^{m,n}$ we can individuate the following sets (vector spaces):

- Null space that is the a subset of $\mathbb{R}^n$ defined as:

$$\mathcal{N}(A) = \{x \in \mathbb{R}^n : \; Ax = 0\} \tag{2.9}$$

- Range space that is a subset of $\mathbb{R}^m$ defined as:

$$\mathcal{R}(A) = \{y \in \mathbb{R}^m : \; y = Ax\} \tag{2.10}$$

  the dimension of such a vector space is called the rank of the matrix A (rank(A)) and it holds that its maximum value is the $\min(m,n)$.

Another important result is that

$$n = \dim\mathcal{N}(A) + \dim\mathcal{R}(A)$$

Knowing n (for us dimension of the configuration space) and the rank of A, we can find the dimension for the null space of A.

We know that this is a vector space and it has got a basis of $n-k$ elements which we can denote with $\{\mathbf{g}_i(\mathbf{q})\}_{i=1}^{n-k}$, we can group together such elements in a matrix $\mathbf{G}(\mathbf{q})$ so that the generalized velocities can be expressed as

$$\dot{\mathbf{q}} = \mathbf{G}(\mathbf{q})\mathbf{u} \tag{2.12}$$

this is nothing but the **kinematic model** of the constrained system (a system of ordinary differential equations[4]). The vector $\mathbf{q}$ is called the *state vector* while $\mathbf{u}$ is the **input vector**. Moreover the obtained system is said to be driftless since in absence of an input the generalized velocity is null. Not rarely the components u_i of $\mathbf{u}$ have a meaning related to the physics or the available control input.

In the following for better fixing the concepts we have just given, two example of kinematic models are given.

Kinematic model for the unicycle

Let us consider, a bit more in details, the unicycle system. It is noticeable that the line in which the motion does not occur is called **zero motion line**.
We have said that the generalized coordinated $\mathbf{q}$ are the ones in Equation (2.1). We have a single constraint that we have reduced in Pfaffian form. The next step *in order to obtain a kinematic model* is determining a base for the null space of the constraint matrix. One possible choice for vector fields $\mathbf{g_i}(\mathbf{q})$ is[5]:

$$\mathbf{g}_1(\mathbf{q}) = \begin{bmatrix}\cos\theta & \sin\theta & 0\end{bmatrix}^T, \quad \mathbf{g}_2(\mathbf{q}) = \begin{bmatrix}0 & 0 & 1\end{bmatrix}^T$$

Therefore, putting them together we obtain the matrix

$$\mathbf{G}(\mathbf{q}) = \begin{bmatrix}\cos\theta & 0\\ \sin\theta & 0\\ 0 & 1\end{bmatrix} \tag{2.13}$$

The kinematic model for the unicyle, then, can be written as:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\cos\theta\\ \sin\theta\\ 0\end{bmatrix} v + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}\omega \tag{2.14}$$

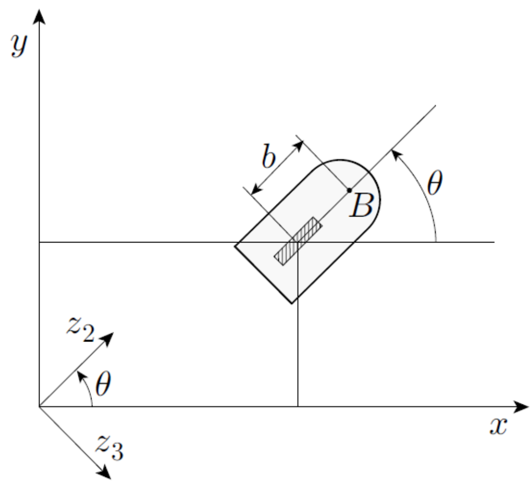


Figure 2.1: Choice of generalized coordinates for the unicycle

In such a context the elements of the input vector have a physical meaning, since v is the driving velocity, while ω is nothing but the steering velocity. Why are we interested in studying the kinematic model of a unicyle? Consider that without a human beings balancing the system this is also an unstable system! However, there are some robot (stable) robot structures that from a **kinematic point of view** are equivalent to unicycles. We are talking about *differential drive* and *synchro drive* vehicles.

[4]ODE

[5]Note that this is a vector field since both domain and codomain are vectors of suitable dimensions.

Differential drives: a stable structure for the unicycle model

A *differentially driven robot* has two independent wheels with different angular velocities ω_R and ω_L. Both wheels have a radius r and are constrained to be at a distance d. A third wheel is passive, in the sense that there is not a motor changing that velocity. By doing a proper choice for the control input, we can obtain a kinematic model that is totally equivalent to the one of a unicycle. In particular, from the left and right wheels angular velocities we can obtain:

$$v = \frac{r}{2}(\omega_R + \omega_L) \tag{2.15}$$

$$\omega = \frac{r}{d}(\omega_R - \omega_L) \tag{2.16}$$

Kinematic model for a bicycle

A **bicycle** is vehicle with a steered wheel and a fixed one, the distance between the wheels is fixed to be L. As usual we have to choose the (generalized) coordinates for such a vehicle a possible choice is the following:

- (x, y) being the contact point of the rear[6];
- θ is the angle between the rear wheel and the x axis (this is nothing but the orientation of the vehicle with respect to the x axis);
- ϕ is the steering angle of the front wheel.

Remark. It is interesting focus our attention on a point: why do not we introduce also the coordinates of the front wheel? Well, it is sufficient having a single contact point since, back and front wheel are at a fixed distance ℓ, and this is nothing but an *holonomic constraint* which allows us the shrinking of the configuration space dimension!

Going on into the discussion, there are essentially two *pure rolling constraints*, one for each wheel:

$$\dot{x}_f \sin(\theta + \phi) - \dot{y}_f \cos(\theta + \phi) = 0 \tag{2.17}$$

$$\dot{x} \sin(\theta) - \dot{y} \cos(\theta) = 0 \tag{2.18}$$

while indicating with (x_f, y_f) the coordinates of the front wheel center, while $(\theta + \phi)$ is its angle with respect to the fixed reference frame. We have just said that they are not strictly necessary, since they can be obtained starting from the coordinates of the other wheel in particular:

$$\begin{cases} x_f = x + \ell \cos\theta \\ y_f = y + \ell \sin\theta \end{cases} \tag{2.19}$$

using the trigonometry, the Equation (2.17) can be expressed as

$$\dot{x} \sin(\theta + \phi) - \dot{y} \cos(\theta + \phi) - \ell\dot{\theta} \cos(\phi) = 0 \tag{2.20}$$

where we have used that $\sin^2\theta + \cos^2\theta = 1$ and the trigonometric formulas for $\cos(\theta + \phi)$ and $\sin(\theta + \phi)$. The derived constraints can be put if Pfaffian form using the matrix

$$\mathbf{A}^T(\mathbf{q}) = \begin{bmatrix} \sin\theta & -\cos\theta & 0 & 0 \\ \sin(\theta + \phi) & -cos(\theta + \phi) & -\ell\cos\phi & 0 \end{bmatrix} \tag{2.21}$$

[6]back wheel

Our objective is obtaining a basis for the null space of such a matrix in order to derive the kinematic model. Since $rank(\mathbf{A^T}(\mathbf{q})) = 2$, the dimension of its null space is given by the difference between the dimension of the configuration space and such a rank, that is 2. A possible basis for such a null space is given by the column of

$$\mathbf{G}(\mathbf{q}) = \begin{bmatrix} \cos\theta\cos\phi & 0 \\ \sin\theta\cos\phi & 0 \\ \frac{\sin\theta}{\ell} & 0 \\ 0 & 1 \end{bmatrix} \tag{2.22}$$

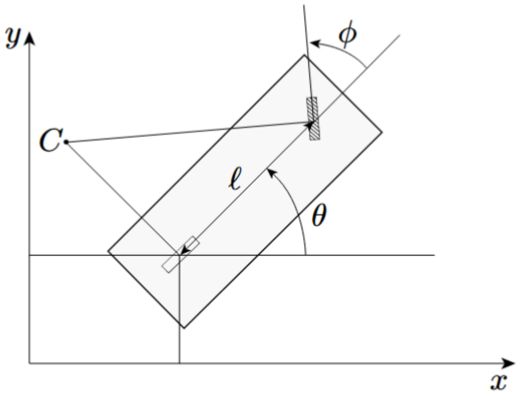


Figure 2.2: Possible choice for the generalized coordinates of a unicycle

The associated kinematic model is:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\phi \\ \sin\theta\cos\phi \\ \frac{\sin\theta}{\ell} \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \omega \tag{2.23}$$

the control input u_1 can be choosen according to the drive in particular $u_1 = v$ if the vehicle is front-drive, $u_1 = v/\cos\theta$ if the vehicle is back drive, since the first two equations must be equivalent to the ones of a unicyle. Just for a matter of notation/nomenclature the instersection point C between the two zero motion lines is called *instantaneous center of rotation*, depends only on $\mathbf{q}$ and it is interesting since each point of the chassis is moving instantaneously along along the circumference centered at C (see Figure 2.2).
Equivalent (stable) systems having the same kinematic model are the **tricyle** and the **automobile**.
An interesting point is that, if we assume that the steering velocity can be *manually controlled*, the number of state variables (generalized coordinates) for the bicycle is the same than the unicycle. In this way we can model both systems using the same kinematic model.

Remark. We remind that the kinematic model represent a system of ODE which can be solved using any integration method available in MATLAB. For example we can obtain the trajectory of the unicycle/bicyle model and the time law for θ by using the command `ode45`.

2.4 Sensors for mobile robots

2.4.1 Sensors classification

2.4.2 Sensors characteristics

2.4.3 Types of sensors

Chapter 3

Kinematics of manipulators