Time Integration of Reacting Flows and Energy-Stable Hybrid Spatial Discretizations

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1 Time Integration of Reacting Flows

1.1 Motivation & Background

(Insert text here)

1.2 Governing Equations

Similarly to the local linear stability problem, we can make use of the Navier-Stokes equations, which can be written in vector form as

$$\frac{\partial Q}{\partial t} = R(Q) \tag{1}$$

where $R(Q)$ denotes the right-hand side. When linearizing this equation, we get the perturbation equation

$$\frac{\partial Q'}{\partial t} = L(\overline{Q})Q' \tag{2}$$

where $\overline{Q}$ denotes the mean quantity and $L(Q)$ denotes the linear operator. In order to solve this ODE, we assume a wave-like perturbation ansatz following

$$Q'(\mathbf{x}, t) = \hat{Q}(\mathbf{x})e^{\omega t} \tag{3}$$

where ω denotes the temporal wave number and $\mathbf{x} = [x, y]^T$. This can be rewritten as a generalized eigenvalue problem in the form of

$$L\hat{\mathbf{Q}} = \omega M \hat{\mathbf{Q}} \tag{4}$$

where ω the eigenvalue and $\hat{\mathbf{Q}}$ is the eigenvector in the $x - y-$ plane. A global mode solver that solves this system has been previously developed by Natarajan [**?**] and has been used within this study of a mechanically compliant panel under grazing compressible boundary layer flow. For this, it has been modified to account for a Kirchhoff-Love plate instead of a rigid wall. The solver uses an SBP-SAT discretization framework for the spatial derivatives and boundary conditions and solves the generalized eigenvalue problem employing a shift-invert Arnoldi method. It has been previously used and verified in both incompressible and compressible limit for various flows, including boundary layer flows. For more details about the solver and the implemented numerical schemes, the reader is referred to Natarajan [**?**]. The results obtained within this work have been gained for the inviscid governing equations neglecting viscous effects for the meanflow.

1.3 Numerical Methods

(Insert text here)

1.4 Validation

(Insert text here)

1.5 Results

(Insert text here)

1.6 Outlook

(Insert text here)

2 Energy-Stable Hybrid Spatial Discretizations

2.1 Motivation & Background

(Insert text here)

2.2 Governing Equations

Similarly to the local linear stability problem, we can make use of the Navier-Stokes equations, which can be written in vector form as

$$\frac{\partial Q}{\partial t} = R(Q) \tag{5}$$

where $R(Q)$ denotes the right-hand side. When linearizing this equation, we get the perturbation equation

$$\frac{\partial Q'}{\partial t} = L(\overline{Q})Q' \tag{6}$$

where $\overline{Q}$ denotes the mean quantity and $L(Q)$ denotes the linear operator. In order to solve this ODE, we assume a wave-like perturbation ansatz following

$$Q'(\mathbf{x}, t) = \hat{Q}(\mathbf{x})e^{\omega t} \tag{7}$$

where ω denotes the temporal wave number and $\mathbf{x} = [x, y]^T$. This can be rewritten as a generalized eigenvalue problem in the form of

$$L\hat{\mathbf{Q}} = \omega M\hat{\mathbf{Q}} \tag{8}$$

where ω the eigenvalue and $\hat{\mathbf{Q}}$ is the eigenvector in the $x - y-$ plane. A global mode solver that solves this system has been previously developed by Natarajan [?] and has been used within this study of a mechanically compliant panel under grazing compressible boundary layer flow. For this, it has been modified to account for a Kirchhoff-Love plate instead of a rigid wall. The solver uses an SBP-SAT discretization framework for the spatial derivatives and boundary conditions and solves the generalized eigenvalue problem employing a shift-invert Arnoldi method. It has been previously used and verified in both incompressible and compressible limit for various flows, including boundary layer flows. For more details about the solver and the implemented numerical schemes, the reader is referred to Natarajan [?]. The results obtained within this work have been gained for the inviscid governing equations neglecting viscous effects for the meanflow.

2.3 Numerical Methods

(Insert text here)

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(Insert text here)

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(Insert text here)