Quiz 8 – Permutations and Combinations

1. Suppose there are 18 mathematics majors and 325 computer science majors at a university.
   (a) How many ways can 2 students be picked, a mathematician and a computer scientist?

   $18 * 325 = \underline{5850}$

   (b) How many ways can 1 student be picked, a mathematician or a computer scientist?

   $18 + 325 = \underline{343}$

2. A *DNA sequence* consists of the *bases* 'A', 'C', 'G', and 'T'.
   (a) How many DNA sequences of length 4 do not include the base 'T'?

   $3^4 = \underline{81}$

   (b) How many DNA sequences of length 4 include the subsequence "ACG"?

   $4 + 4 = \underline{8}$

   (c) How many DNA sequences of length 4 include exactly 3 of the 4 bases?

   $(3*3*2*1) * 4 = \underline{72}$

3. Suppose that each student at a university has one of 4 expected graduation years and one of 21 majors. How many students must be enrolled to guarantee 2 graduations in the same year and major?

   $(21*4) + 1 = \underline{85}$

4. Suppose that there are 25 students in a class, each either a freshman, a sophomore, or a junior. How many students must be in the same cohort?

   $\lceil n / k \rceil$, n = 25 and k = 3 → $\lceil 25 / 3 \rceil = \underline{9}$

5. The English alphabet consists of 21 consonants and 5 vowels.
   (a) How many strings of 6 lowercase letters contain exactly 2 vowels?

   $6nCr2 * 5^2 * 21^4 = 15 * 5^2 * 21^4 = \underline{72930375}$

   (b) How many strings of 6 lowercase letters contain at least 2 vowels?

   $(6nCr2)(5^2*21^4) + (6nCr3)(5^3*21^3) + (6nCr4)(5^4*21^2) + (6nCr5)(5^5*21) + 5^6$

   $= \underline{100626625}$