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Quiz 2 – Predicate Logic

1. Without using a truth table, show that $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$.

$$
\begin{aligned}
\boldsymbol{p \leftrightarrow q} &\equiv \mathbf{(p \wedge q) \vee (\neg p \wedge \neg q)} \\
&\equiv \mathbf{(p \rightarrow q) \wedge (q \rightarrow p)} \\
&\equiv \mathbf{(\neg p \vee q) \wedge (\neg q \vee p)} \\
&\equiv \mathbf{((\neg p \vee q) \wedge \neg q) \vee ((\neg p \vee q) \wedge p)} \\
&\equiv \mathbf{((\neg p \wedge \neg q) \vee (q \wedge \neg q)) \vee ((q \wedge p) \vee (p \wedge \neg p))} \\
&\equiv \mathbf{((\neg p \wedge \neg q) \vee F) \vee ((p \vee q) \vee F)} \\
&\equiv \mathbf{(\neg p \wedge \neg q) \vee (q \wedge p)} \\
&\equiv \mathbf{(p \wedge q) \vee (\neg p \wedge \neg q)} \qquad \textbf{:)}
\end{aligned}
$$

2. Show that $(p \wedge q) \rightarrow r \neq (p \rightarrow r) \wedge (q \rightarrow r)$.

p	q	r	$p \wedge q$	$(p \wedge q) \rightarrow r$	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
T	F	F	F	**T**	F	T	**F**

3. Let the domain for *n* and *m* consist of all integers.
Determine whether each of the following quantified predicates is true or false.
(a) $\forall n\ (n^2 \geq n)$ **T**
(b) $\forall n\ \exists m\ (n^2 < m)$ **T**
(c) $\exists n\ (n^2 = 2)$ **F**
(d) $\exists n\ \forall m\ (nm = m)$ **T**

4. Recall that integer *parity* is defined as follows:
Definition: Every integer is either even or odd. An integer n is even if there exists an integer k such that $n = 2k$. An integer is odd if there exists an integer k such that $n = 2k + 1$.
Consider the following statement:
Let x, y, and z be integers. If x + y + z is odd, then at least one of x, y, or z is odd.
(a) Which proof technique should be used to prove the above statement? Briefly explain your answer.

Contradiction, because to assume that at least one of x, y, or z is odd is to assume that they cannot all be even.

(b) Prove the above statement.

Theorem: If x + y + z is odd, then at least one of x, y, or z is odd.

Proof:

Suppose, for contradiction, that x + y + z is odd and x, y, and z are even by definition. Then, by definition, there exist some integers k, m, and w such that x = 2k, y = 2m, and z = 2w.
Then, x + y + z = 2k + 2m + 2w = 2(k + m + w) which, by definition, is even. This contradicts the assumption that x, y, and z are all even.

Therefore, by contradiction, if x + y + z is odd, then at least one of x, y, or z is odd.