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Quiz 6 – Algorithm Correctness

1. Consider the *reversal* of a finite sequence of arbitrary elements. For example, given the sequence:

$$(a_1, a_2, a_3, a_4, a_5)$$

… the reversal is $(a_5, a_4, a_3, a_2, a_1)$.

(a) Give the pseudocode for a recursive algorithm that reverses a sequence.

REVERSAL(A 🡐 $(a_1, a_2, \ldots, a_n)$)

Input: a non-empty, finite sequence of arbitrary elements

Output: the reversal of the sequence

if $n \leq 1$, then:

return a_1

else, do

return (a_n, REVERSAL($a_1, a_2, \ldots, a_{n-1}$))

(b) State a lemma that you would need to prove in order to show the correctness of your algorithm.

Let A be a non-empty, finite sequence of arbitrary elements. If REVERSAL(A) returns B, then B ∈ A and, |A| = |B|, and $\forall a \in A(a_i = b_{n+1-i} \in B)$.

(c) Prove that your algorithm is correct.

Proof: Let |A| be denoted n.

***Basis Step*:**

Let n = 1. Then A = (a_1). Certainly $a_1 \in A$ and $a_{n+1-1} = a_1$. Thus, REVERSAL correctly returns a_1.

***Inductive Hypothesis*:**

Suppose, for all integers $1 \leq n \leq k$, if A is a finite, non-empty sequence of arbitrary elements and REVERSAL(A) returns B, then B ∈ A and, |A| = |B|, and $\forall a \in A(a_i = b_{n+1-i} \in B)$.

***Inductive Step*:**

Let n = k+1. Since $k \geq 1$, $n \geq 2$. Let the $(a_2, \ldots, a_n)$ be denoted A'. Then REVERSAL recurses on A', which is of size $\leq k$.

By the hypothesis, REVERSAL obtains some value such that B ∈ A' and, |A'| = |B|, and $\forall a \in A'(a_i = b_{n+1-i} \in B)$.

If a_1 is then added on, then REVERSAL will return a_1, as stated in the basis step, at the end of the sequence to be returned. Thus, REVERSAL correctly returns the reversal of its input.

In all possible cases, REVERSAL returns B such that B is the reverse of A.

Therefore, by the PMI, REVERSAL is correct.