Claire Minahan

Quiz 3 – Sets

1. Using the notion "without loss of generality", prove the following statement:

   Let x and y be integers. If x and y have opposite parity, then $5x + 5y$ is odd.

"Without loss of generality", assume x is odd and y is even. Then, with k and m being some integers, $x = 2k + 1$ and $y = 2m$, then $5x + 5y = 5(2k + 1) + 5(2m) = 10k + 1 + 10m = 2(5k + 5m) + 1$. Let l be some integer where $l = 5k + 5m$ so the above is equal to $2l + 1$, which, by definition, is odd. Therefore, if x and y have opposite parity, then 5x + 5y is odd.

2. Determine whether each of the following statements is true or false:

   (a) $\{2\} \in \{x \in Z \mid x > 1\}$ **F**

   (b) $\{2\} \in \{\{\{2\}\}\}$ **F**

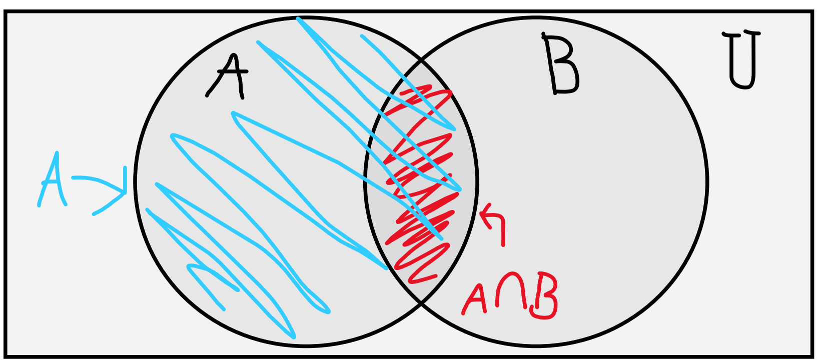
   (c) $\{0\} \in \{0\}$ **F**

   (d) $\{\emptyset\} \subseteq \{\emptyset\}$ **T**

3. Consider the following statement:

   Let A and B be sets. Then $(A \cap B) \subseteq A$.

   (a) Draw a Venn diagram to illustrate the above statement.



   (b) Prove the above statement.

Theorem: $(A \cap B) \subseteq A$.

Proof: Let $x \in A \cap B$. Because x is an element of the intersection of A and B, then, by definition, $x \in A \wedge x \in B$. So, x is an element of A, $x \in A$.

Therefore, $(A \cap B) \subseteq A$.