Algorithms Assignment 2

Clare Minnerath

19. e

20. c

22. Factorial limits use fact $n! \leq n^n$

1. n^n and $n^n + ln(n)$ First 3 obvious

2. $n!$

3. $10^n + n^{20}$
   $\lim\limits_{n\to\infty} \frac{10^n+n^{20}}{4^n} = \lim\limits_{n\to\infty} \frac{10}{4}^n + \frac{n^20}{4^n} = \infty + 0 = \infty$

4. 4^n
   $\lim\limits_{n\to\infty} \frac{e^n}{4^n} = \lim\limits_{n\to\infty} \frac{e}{4}^n = \infty$

5. e^n
   $\lim\limits_{n\to\infty} \frac{e^n}{(lgn)!} \leq \lim\limits_{n\to\infty} \frac{e^n}{(lgn)^{lgn}} = \lim\limits_{n\to\infty} \frac{nlge}{lgn\, lg(lgn)} = \lim\limits_{n\to\infty} nlge - lgn\, lg(lgn) = \infty$

6. $(lgn)!$

7. $n^{\frac{5}{2}}$
   $\lim\limits_{n\to\infty} \frac{n^{\frac{5}{2}}}{5^{lgn}} = \lim\limits_{n\to\infty} \frac{\frac{5}{2}lgn}{lg5\, lgn} = \lim\limits_{n\to\infty} (\frac{5}{2} - lg5)lgn = \infty$

8. 5^{lgn}

9. $5n^2 + 7n$

10. $nlgn$ and $log(n!)$ Hard to prove, just looked it up

11. $8n + 12$ Last 3 obvious

12. $n^{\frac{1}{2}}$

13. $lg^2 n$

27.

1. $\lim\limits_{n\to\infty} \frac{lgn}{n} = \lim\limits_{n\to\infty} \frac{1}{nln2} = 0$

   $\implies lgn \in O(n)$

2. $\lim\limits_{n\to\infty} \frac{n}{nlgn} = \lim\limits_{n\to\infty} \frac{1}{lgn} = 0$

   $\implies n \in O(nlgn)$

3. $\lim\limits_{n\to\infty} \frac{nlgn}{n^2} = \lim\limits_{n\to\infty} \frac{lgn}{n} = \lim\limits_{n\to\infty} \frac{1}{nln2} = 0$

   $\implies nlgn \in O(n^2)$

4. $\lim\limits_{n\to\infty} \frac{2^n}{5^{lgn}} = lg5 \lim\limits_{n\to\infty} \frac{nlg2}{lgn} = lg5 \lim\limits_{n\to\infty} n\, ln2 = \infty$

   $\implies 2^n \in \Omega(5^{lgn})$

5. $\lim\limits_{n\to\infty} \frac{lg^3 n}{\sqrt{n}} = 6 \lim\limits_{n\to\infty} \frac{lg(lgn)}{lgn} = 6\, ln2 \lim\limits_{n\to\infty} \frac{n\, ln2}{n\, lgn\, ln2} = 6\, ln2 \lim\limits_{n\to\infty} \frac{1}{lnn} = 0$

   $\implies lg^3 n \in \sqrt{n}$

34.

Basic operation: O(1) + $j = 2j$
T(n) = T(2^k)

$i = 2^k \implies$ basic op: 1
$i = 2^{k-1} \implies$ basic op: 2
$i = 2^{k-2} \implies$ basic op: 3
$i = 2^{k-3} \implies$ basic op: 4
.
.
.
$i = 1 \implies$ basic op: $lg(2^{k+1}) = lg(2n)$

$\implies T(n) = 1 + 2 + 3 + ... + lg(2n)$
$\implies T(n) \approx 2lg(2n)$

$2 \lim\limits_{n\to\infty} \frac{lg(2n)}{lgn} = 2 \lim\limits_{n\to\infty} \frac{nln2}{nln2} = 2$
$\implies T(n) \in \Theta(lgn)$

35b. Basic operation: addition in each loop
Input size: n
T(n) = n + n
$\lim\limits_{n\to\infty} \frac{n+n}{n} = \lim\limits_{n\to\infty} 1 + 1 = 2$
$\implies$ T(n) $\in \Theta(n)$

2. W(n)= $lg(700000000) + 1 = 30 + 1 = 31$

8. [123, 34, 189, 56, 150, 12, 9, 240]

[123, 34, 189, 56] [150, 12, 9, 240]

[123, 34] [189, 56] [150, 12] [9, 240]

[123] [34] [189] [56] [150] [12] [9] [240]

[34, 123] [56, 189] [12, 150] [9, 240]

[34, 56, 123, 189] [9, 12, 150, 240]

[9, 12, 34, 56, 123, 150, 189, 240]

3. .py file