EE128 Lab6 Writeup

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Contents

1 Purpose

The objective of this lab is to design a full-state feedback controller using the Linear Quadratic Regulator (LQR) design technique and to understand the effect of varying the penalty matrices P and Q in the cost functional on the performance of the closed-loop system.

2 Prelab

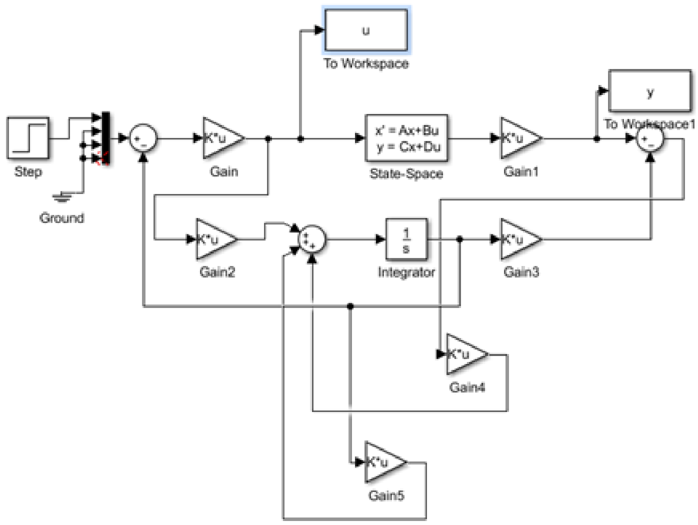

Figure 1: Prelab Simulink Model

2.1 Common Weight

By running the program shown in the appendix, we found that when q1,q3 and r are all 1,

$$K = (-20.0000 - 29.2131 - 154.3927 - 20.2569) \tag{1}$$

When the poles are -10 + 15j, -10 - 15j, -12 + 17j, -12 - 17j,

$$L = \begin{pmatrix} 36.6007 & 19.7269 \\ 58.9363 & -62.1016 \\ -46.0538 & -25.0957 \\ 351.1008 & 336.1232 \end{pmatrix} \tag{2}$$

Here is the plot of y and u:

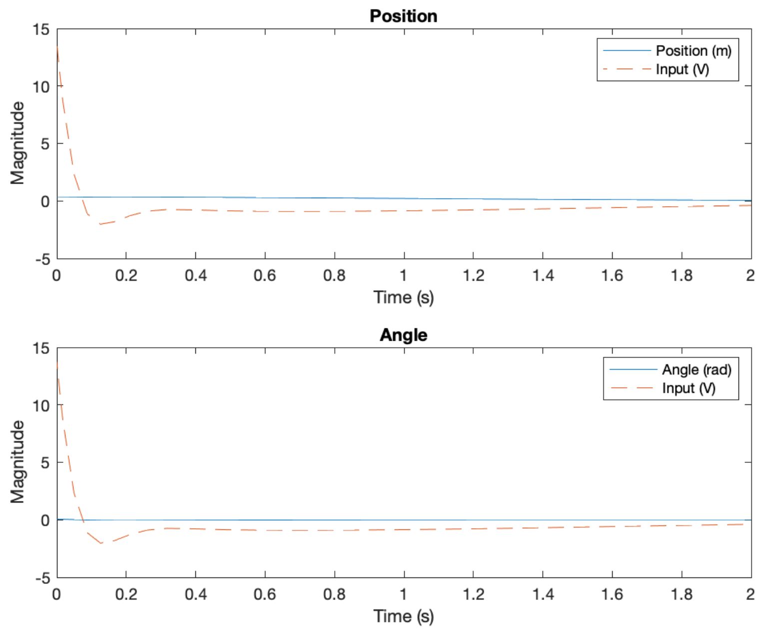

Figure 2: Plots of u and y for Normal Weights

2.2 System with large q3

q_3=100
P = [-10 + 15j, -10 - 15j, -12 + 17j, -12 - 17j]
K=[-20 -73.7 -1230.8 -5.71]

$$L = \begin{pmatrix} 101 & 68 \\ -5992 & -3318 \\ -214 & -149 \\ 14329 & 7837 \end{pmatrix}$$

umax=67.54V
positionmax=0.3254m
anglemax=0.05rad

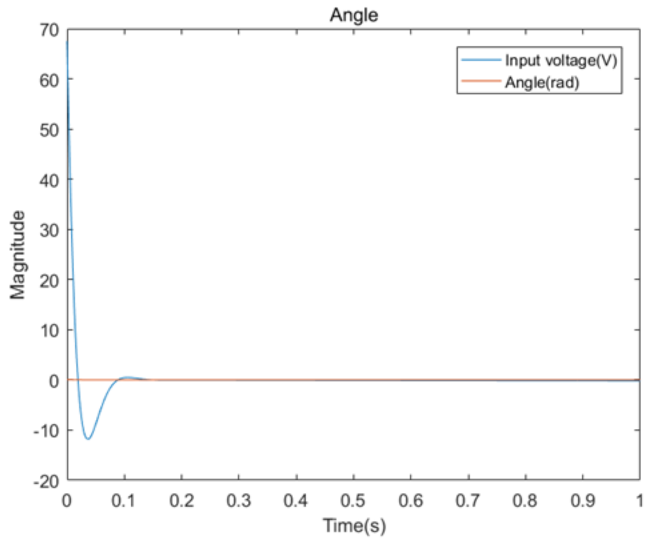

Figure 4: Angle with Large q3

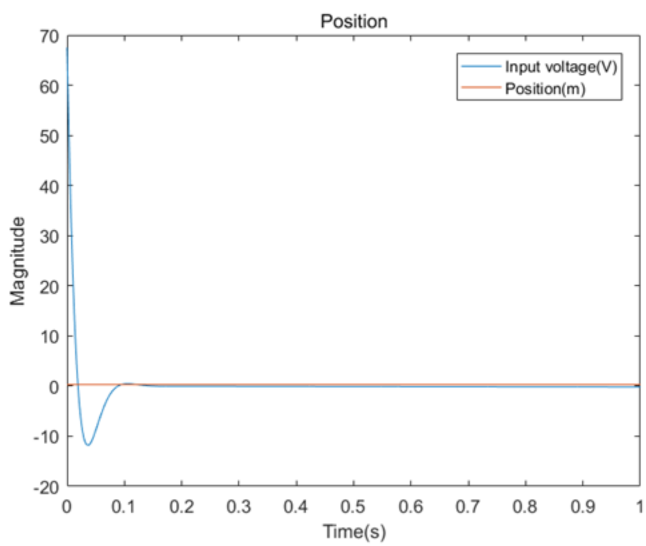

Figure 3: Position with Large q3

2.3 System with small q3

q3=0.01
P = [-10 + 15j, -10 - 15j, -12 + 17j, -12 - 17j]
K=[-20.0000 -20.0650 -62.3671 -12.7582]

$$L = \begin{pmatrix} 30.8650 & 12.6025 \\ 291.7367 & 30.6064 \\ -32.7290 & -7.3195 \\ -147.8192 & 117.7669 \end{pmatrix}$$

Umax=9.12V
Positionmax=0.3718m
Anglemax=0.05rad

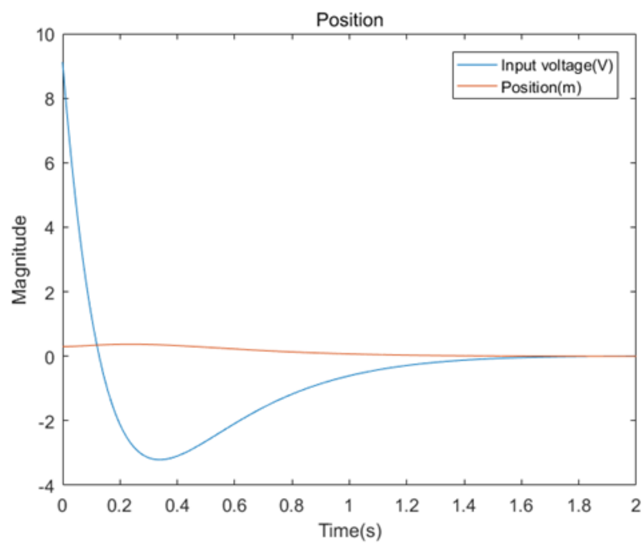

Figure 5: Position with small q3

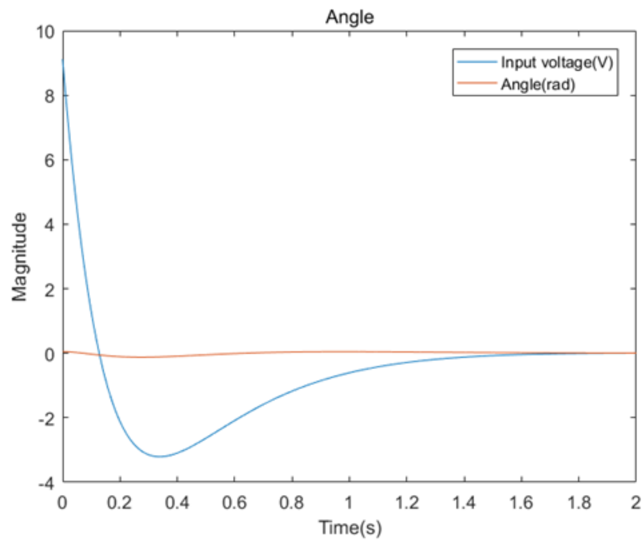

Figure 6: Angle with small q3

From above, we know that as q3 increases, the maximum input voltage will increase, the steady state error of the output and the voltage will decrease, the Tr of the input and output will decrease.

2.4 System with large r

r=100
P = [-10 + 15j, -10 - 15j, -12 + 17j, -12 - 17j]
K=[-2.0000 -10.1648 -38.7031 -7.5765]

$$L = \begin{pmatrix} 26.3701 & 10.0525 \\ 333.1714 & 35.8402 \\ -17.5498 & 0.0558 \\ -73.0016 & 156.9593 \end{pmatrix}$$

Positionmax=0.3495m
Umax=2.54V
Anglemax=0.05rad

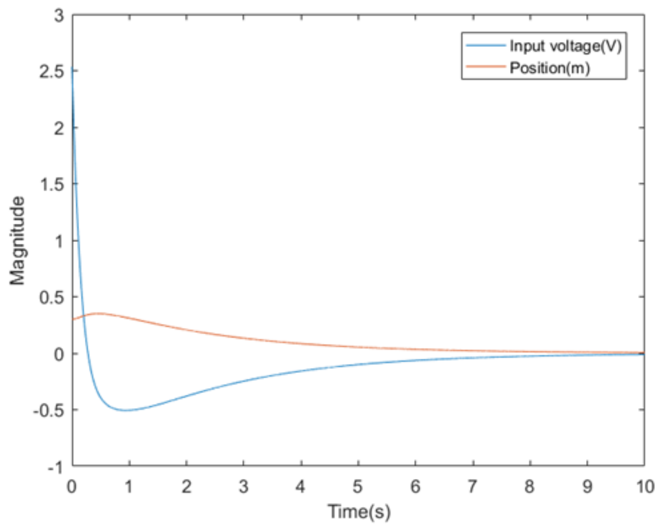

Figure 7: Position with large r

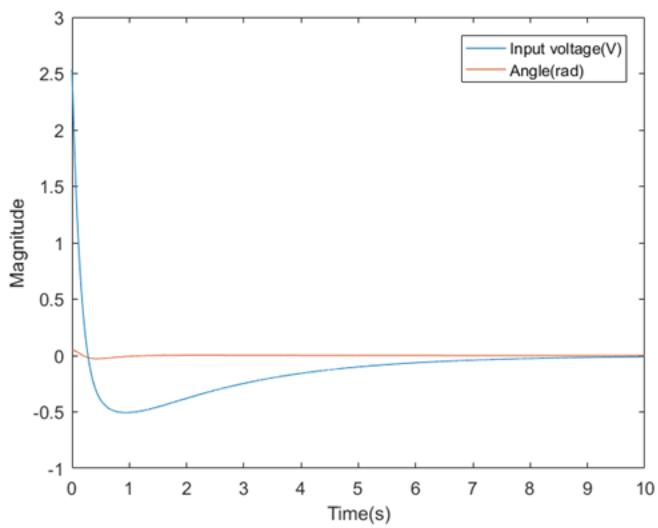

Figure 8: Angel with large r

2.5 System with small r

r=0.01
P = [-10 + 15j, -10 - 15j, -12 + 17j, -12 - 17j]

K=[-200 -236.9 -1346.8 -129.2]

$$L = \begin{pmatrix} 119 & 81 \\ -6974 & -4027 \\ -238 & -169 \\ 16148 & 9301 \end{pmatrix}$$

Positionmax=0.3464m
Anglemax=0.05rad
Umax=127.3403V

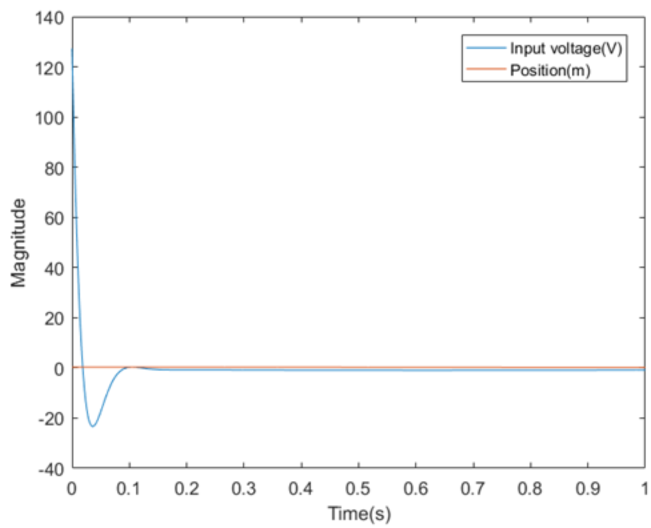

Figure 9: Position with small r

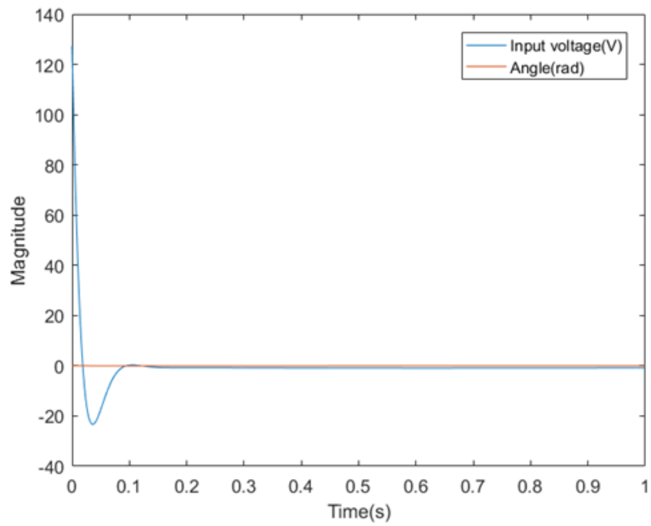

Figure 10: Angle with small r

From above, we know that as r increases, the maximum input voltage decreases, the sse of the output has almost no change, and the Tr of the position increases.

2.6 Question

Q:You will observe that the position x will first increase before converging to zero. What is the physical reason for this behavior?

A:It is because of the gravity that drag the stick down. Since the angle is not zero, there is a horizonal force that enlarges the position and needs to be overcome by the system.

3 Lab Results and Analysis

For the following four graphs, the reference signal is $[0.3, 0, 0, 0]^T$

3.1 Nominal Weights

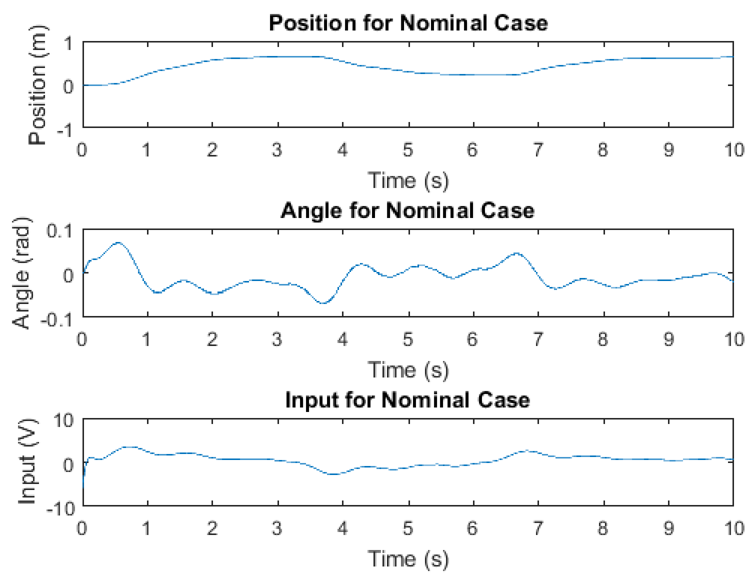

Figure 11: Output and Input Plot for Nominal Weights

This case worked with some oscillations in both cart position and angle, but input seems to get dangerously close to saturating. Oddly in the prelab we didn't see oscillations, but we did in experimentation. We also saw the voltage exceed 6V in the prelab, but it doesn't appear that the voltage hit 6V and saturated in the simulation.

3.2 Higher Relative weight q1

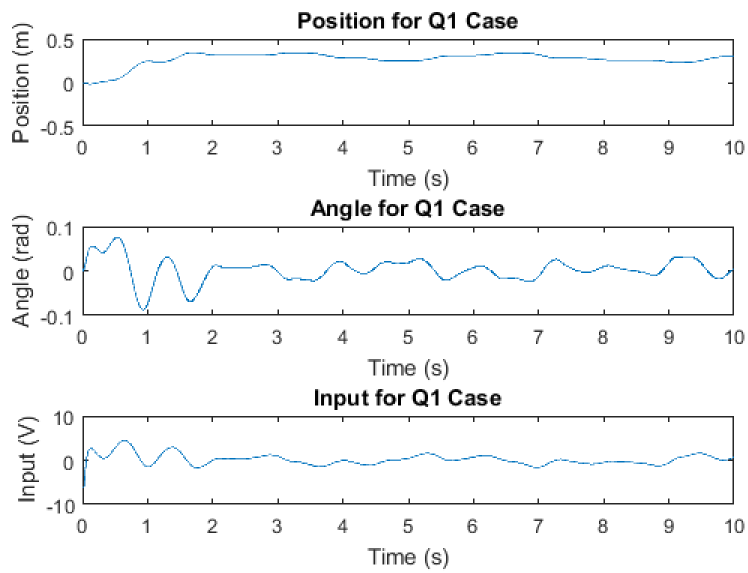

Figure 12: Output and Input Plot for a Relatively Higher q1 Weight

With the higher q1, we see less oscillation in cart position, but more oscillations in angle at first. The input is also pretty close to saturating at 6V. The higher weight q1 contributed to the lessening in position oscillation.

3.3 Higher Relative Weight q3

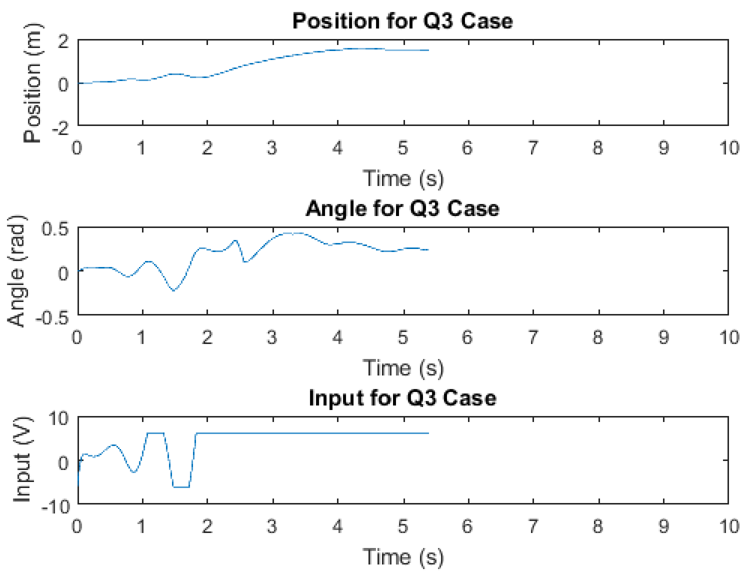

Figure 13: Output and Input Plot for a Relatively Higher q3 Weight

With the higher q3, we initially saw less oscillation in angle at first, but we see the input saturate after one second. The cart slammed into the end of the rail when the input saturated, so we had to stop the trial early. This matches the prelab trial, where the $umax = 67.54V$.

3.4 Higher Relative Weight r

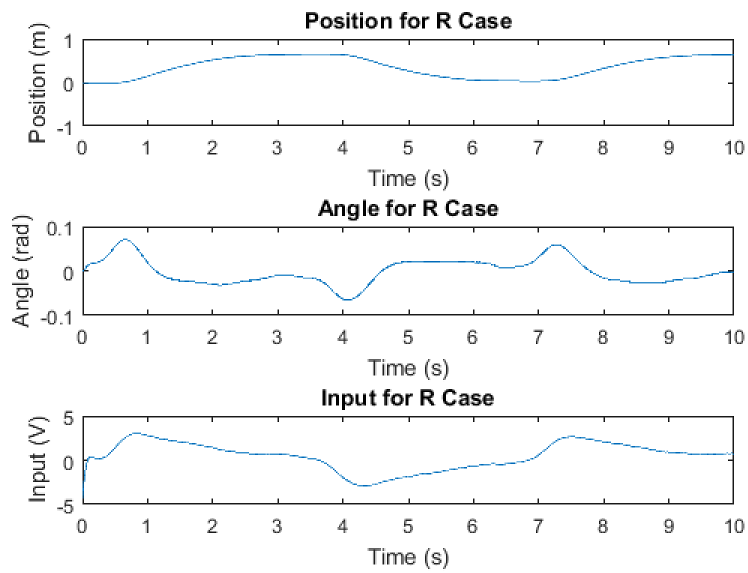

Figure 14: Output and Input Plot for a Relatively Higher r Weight

With the higher r, we see the input stay between 5 and -5 volts, just as it did in the prelab. These inputs are smaller than the nominal case. We see a larger position and angle deviation, similar to the prelab, but for some reason we see oscillations occurring, possibly due to some error in the Simulink input.

3.5 Sinusoidal Input

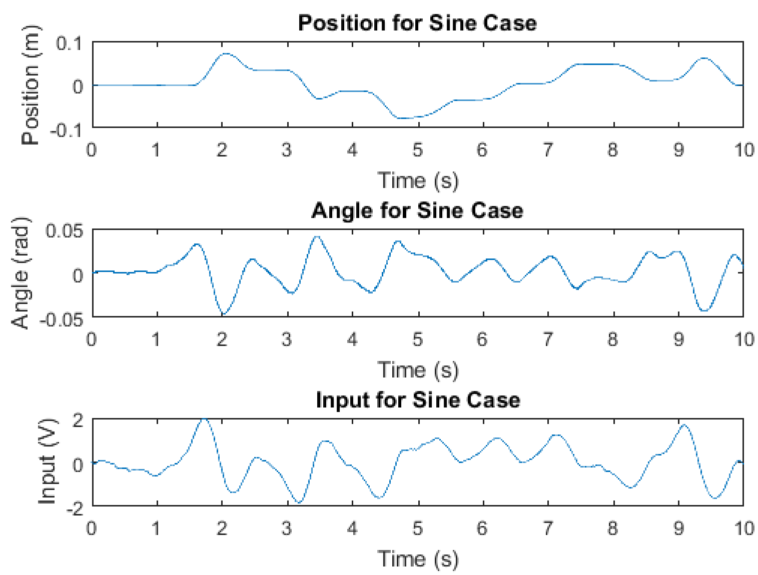

Figure 15: Position and Input Plot for Sinusoidal Input with Custom Weights

We set $q1 = 5$, $q3 = 1$, and $r = 2$. // // Compared to lab 6b, the position doesn't drop as low, but the angle seems to vary more, which may be able to be remedied with different rates. The input, while not tracked in lab 6b, is not close to saturating. I'd say this performance is better because while the deviation in angle is slightly larger, the finite track length constraint and input saturation are more important conditions to weight here.

Appendices

A Code for Prelab

```
a22 = -6.81
a23 = -1.5
a42 = 15.47
a43 = 25.67

b2 = 1.52
b4 = -3.46
```

```
A = [0 1 0 0; 0 a22 a23 0; 0 0 0 1; 0 a42 a43 0]

B = [0; b2; 0 ; b4]

C = [1 0 0 0; 0 0 1 0]
D = [0; 0]

sys = ss(A, B, C, D)
q1 = 1
q3 = 1
r = 1

q12=q1/0.09
q32=q3/(0.05)^2
r2=r/36
Q=diag([q12 0 q32 0])
R=r2
K=lqr(sys, Q, R);
%K=[-20.0000  -29.2131 -154.3927  -20.2569]

P = [-10 + 15j, -10 - 15j, -12 + 17j, -12 - 17j];
L = place((A-B*K)', C', P)';
%L =

  %36.6007   19.7269
  %58.9363  -62.1016
  %-46.0538  -25.0957
  %351.1008  336.1232


sim("prelab_model")

clf

figure 1
plot(tout, y(:, 1), tout, u)
legend("Position (m)", "Input voltage(V)")
title("Position")
xlabel("Time (s)")
ylabel("Magnitude")
xlim([0 2])


figure 2
plot(tout, y(:, 2), tout, u)
```

```
legend("Angle (rad)", "Input voltage(V)")
title("Angle")
xlabel("Time (s)")
ylabel("Magnitude")
xlim([0 2])


umax = abs(max(u));
max_pos = abs(max(x(:, 1)));
max_angle = abs(max(x(:, 2)));
```

B Lab Setup

```
a22 = -6.81;
a23 = -1.5;
a42 = 15.47;
a43 = 25.67;

b2 = 1.52;
b4 = -3.46;

A = [0 1 0 0; 0 a22 a23 0; 0 0 0 1; 0 a42 a43 0];

B = [0; b2; 0 ; b4];

C = [1 0 0 0; 0 0 1 0];
D = [0; 0];

sys = ss(A, B, C, D);

q1 = 10; %5
q3 = 1; %2
r = 1; %

q1_bar = q1/(0.3^2);
q3_bar = q3/(0.05^2);
r_bar = r/36;

Q = diag([q1_bar 0 q3_bar 0]);
R = r_bar;
N = 0;

K = lqr(sys, Q, R, N);
```

```
obs_poles = [-10 + 15j, -10 - 15j, -12 + 17j, -12 - 17j];
L = place((A-B*K)', C', obs_poles)';


Al = A - L*C;
Bl = [B eye(4)];
Cl = eye(4);
Dl = zeros([4 5]);
```

C Plotting Code

```
clf;

subplot(3, 1, 1);
plot(tout, y(:, 1), '-');
title('Position for Sine Case')
xlabel('Time (s)')
ylabel('Position (m)')
xlim([0 10])

subplot(3, 1, 2);
plot(tout, y(:, 2), '-');
title('Angle for Sine Case')
xlabel('Time (s)')
ylabel('Angle (rad)')
xlim([0 10])

subplot(3, 1, 3);
plot(tout, u, '-');
title('Input for Sine Case')
xlabel('Time (s)')
ylabel('Input (V)')
xlim([0 10])
```