

# Dedekind Cuts

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In this peer discussion, we try to answer very basic questions about the most basic structure known to us in Mathematics, the Real numbers. Are the real numbers actual quantities? What exactly do they aspire to describe? Is the Real line actually a line? Did high-school math lie to you?

Consider a sequence of rational numbers  $S_n = \sum_{j=1}^n \frac{1}{j!}$ . There is an argument to show that the series is bounded above, i.e. it never exceeds the value 3, no matter how many terms you include. It can also be shown that the quantity we reach after adding infinitely many terms in the series cannot be of the rational form. We need to introduce the Real numbers to assign a quantity to this series. Dedekind cuts give us an elegant and insightful process to construct the real numbers from the rationals but leave us with one doubt: are the real numbers actually quantities?