Decoding the Mysteries of Binary Computations and Musical Harmony in Ancient India

Dharmadhikari Ganesh Jayantrao

History of Mathematics in India,
IIT Gandhinagar

11-04-2023

Table of Contents

Introduction

- Who Invented Binary Number System?

Introduction

- Who Invented Binary Number System?
- Gottfried Wilhelm Leibniz is the inventor of the binary number system but what if I tell you that way before he invented it in 1695, some ancient Indian mathematicians were already using something equivalent to the binary system?

Introduction

- Who Invented Binary Number System?
- Gottfried Wilhelm Leibniz is the inventor of the binary number system but what if I tell you that way before he invented it in 1695, some ancient Indian mathematicians were already using something equivalent to the binary system?
- Pingala is credited with the first use of binary numbers, using light (laghu) and heavy (guru) syllables to describe the combinatorics of the Sanskrit meter. For this reason, Pingala is also sometimes credited with the first use of zero, as he used the Sanskrit word sunya to explicitly denote a number.

Introduction

- Who Invented Binary Number System?
- Gottfried Wilhelm Leibniz is the inventor of the binary number system but what if I tell you that way before he invented it in 1695, some ancient Indian mathematicians were already using something equivalent to the binary system?
- Pingala is credited with the first use of binary numbers, using light (laghu) and heavy (guru) syllables to describe the combinatorics of the Sanskrit meter. For this reason, Pingala is also sometimes credited with the first use of zero, as he used the Sanskrit word sunya to explicitly denote a number.
- Piṅgala's binary meter system begins with four light laghu syllables as the first pattern ("0000" in binary), three light laghus and one heavy guru as the second pattern ("0001" in binary), and so on, so that in general the pattern n of the nth syllable corresponds to the binary representation $n - 1$ (with increasing positional values).

Table of Contents

Chandaḥśāstra

- Chanda Sutra is also known as Chandahsastra or Pingala Sutra after its author Pingala.

Chandaḥśāstra

- Chanda Sutra is also known as Chandahsastra or Pingala Sutra after its author Pingala.
- The earliest Hindu treatise on prosody to survive

Chandaḥśāstra

- Chanda Sutra is also known as Chandahsastra or Pingala Sutra after its author Pingala.
- The earliest Hindu treatise on prosody to survive
- 8 books, with a total of 310 sutras. It is a collection of aphorisms focused mainly on the art of poetic meters and presents some mathematics mainly binary systems and Combinatorics in the service of music.

Chandaḥśāstra

- Chanda Sutra is also known as Chandahsastra or Pingala Sutra after its author Pingala.
- The earliest Hindu treatise on prosody to survive
- 8 books, with a total of 310 sutras. It is a collection of aphorisms focused mainly on the art of poetic meters and presents some mathematics mainly binary systems and Combinatorics in the service of music.
- Piṅgala's Chandaḥśāstra is the first book that uses a system equivalent to the modern binary number system but with few differences such as the numbers 0 and 1 were termed as Guru and Laghu which we will denote as l and g respectively in this entire text.

Chandaḥśāstra

- Chanda Sutra is also known as Chandahsastra or Pingala Sutra after its author Pingala.
- The earliest Hindu treatise on prosody to survive
- 8 books, with a total of 310 sutras. It is a collection of aphorisms focused mainly on the art of poetic meters and presents some mathematics mainly binary systems and Combinatorics in the service of music.
- Piṅgala's Chandaḥśāstra is the first book that uses a system equivalent to the modern binary number system but with few differences such as the numbers 0 and 1 were termed as Guru and Laghu which we will denote as l and g respectively in this entire text.
- In his Chandaḥśāstra (c.300 BCE), Piṅgala introduces some combinatorial tools called pratyayas which can be employed to study the various possible metres in Sanskrit prosody.

Table of Contents

Further Reading on Pratyaya

Following are some of the important texts which include a discussion of various pratyayas:

1. Piṅgala (c.300 BCE): Chandaḥśāstra
2. Bharata (c.100 BCE): Natyasastra
3. Brahmagupta (c.628 CE): Brahmasphutasiddhanta
4. Virahanka (c.650): Vrttajatisamuccaya
5. Mahavıra (c.850): Ganitasarasangraha
6. Halayudha (c.950): Mrtasanjıvanı Commentary on Piṅgala's Chandaḥśāstra
7. Kedarabhatta (c.1000): Vrttaratnakara
8. Yadavaprakasa (c.1000): Commentary on Piṅgala's Chandaḥśāstra
9. Hemacandra (c.1200): Chandonusasana
10. Prakrta-Paingala (c.1300)
11. Narayana Pandita (c.1350): Ganitakaumudı
12. Damodara (c.1500): Vanıbhusana
13. Narayanabhatta (c.1550): Narayanı Commentary on Vrttaratnakar

Varna-Vrtta

- Each 'chandas' is recognized by the number of 'aksharaas' or syllables present in each line of the poem. As an 'akshara' can be either a 'laghu' or a 'guru' (also known as laghuvu and guruvu in Telugu), the number of variations possible in each type of 'chandas' follows a 'binary system'.

Varna-Vrtta

- Each 'chandas' is recognized by the number of 'aksharaas' or syllables present in each line of the poem. As an 'akshara' can be either a 'laghu' or a 'guru' (also known as laghuvu and guruvu in Telugu), the number of variations possible in each type of 'chandas' follows a 'binary system'.
- The ganas formed by Laghu and Guru are called'Matra-Ganas' in Marathi and Kannada, and their variations are more commonly known as metres of poetry.

Varna-Vrtta

- Each 'chandas' is recognized by the number of 'aksharaas' or syllables present in each line of the poem. As an 'akshara' can be either a 'laghu' or a 'guru' (also known as laghuvu and guruvu in Telugu), the number of variations possible in each type of 'chandas' follows a 'binary system'.
- The ganas formed by Laghu and Guru are called'Matra-Ganas' in Marathi and Kannada, and their variations are more commonly known as metres of poetry.
- For instance the following verse of Kalidasa is in Sragdhara metre:

  म्रभ्नैर्यानां त्रयेण त्रिमुनियतियुता स्रग्धरा कीर्तितेयम्।

  Thus Sragdhara is characterised by the pattern: MaRaBhaNaYaYaYa, with a break (yati) after seven syllables each.
  GGGGLGG LLLLLLG GLGGLGG

There is the mnemonic attributed to Panini:

- yamatarajabhanasalagam
- ya-gana: ya-ma-ta = L-G-G
- ma-gana: ma-ta-ra = G-G-G
- ta-gana: ta-ra-ja = G-G-L
- ra-gana: ra-ja-bha = G-L-G
- ja-gana: ja-bha-na = L-G-L
- bha-gana: bha-na-sa = G-L-L
- na-gana: na-sa-la = L-L-L
- sa-gana: sa-la-ga = L-L-G If we replace G by 0 and L by 1, we obtain a binary sequence of length 10. 1 0 0 0 1 0 1 1 1 0 The above linear binary sequence generates all 8 binary sequences of length 3. We can remove the last pair 1, 0 and view the rest as a cyclic binary sequence of length eight. For example, ya is LGG = 100, ma is GGG = 000, and so on...

Table of Contents

Pratyayas in Piṅgala's Chandaḥśāstra

In chapter eight of Chandaḥśāstra, Piṅgala introduces the following six pratyayas:

1. *Prastāra:* A procedure by which all the possible metrical patterns with a given number of syllables are laid out sequentially as an array.
2. *Sankhya:* The process of finding total number of metrical patterns (or rows) in the prastāra.
3. *Nasta:* The process of finding for any row, with a given number, the corresponding metrical pattern in the prastāra.
4. *Uddista:* The process for finding, for any given metrical pattern, the corresponding row number in the prastāra.
5. *Lagakriya:* The process of finding the number of metrical forms with a given number of laghus (or gurus).
6. *Adhvayoga*: The process of finding the space occupied by the prastara.

Prastāra

द्विकौ ग्लौ। मिश्रौ च। पृथग्लामिश्राः। वसवस्त्रिकाः।

(छन्दःशास्त्रम् ८.२०-२३)

Dvikau glau | Miśrau ca | Pruthaglāmiśraḥ | Vasavastrikāḥ |

- Form a G, L pair. Write them one below the other.

Prastāra

द्विकौ ग्लौ। मिश्रौ च। पृथग्लामिश्राः। वसवस्त्रिकाः।

(छन्दःशास्त्रम् ८.२०-२३)

Dvikau glau | Miśrau ca | Pruthaglāmiśraḥ | Vasavastrikāḥ |

- Form a G, L pair. Write them one below the other.
- Insert on the right Gs and Ls.

Prastāra

द्विकौ ग्लौ। मिश्रौ च। पृथग्लामिश्राः। वसवस्त्रिकाः।

(छन्दःशास्त्रम् ८.२०-२३)

Dvikau glau | Miśrau ca | Pruthaglāmiśraḥ | Vasavastrikāḥ |

- Form a G, L pair. Write them one below the other.
- Insert on the right Gs and Ls.
- (Repeating the process) We have eight (vasavah) metric forms in the 3-syllable-prastāra.

Prastāra

द्विकौ ग्लौ। मिश्रौ च। पृथग्लामिश्राः। वसवस्त्रिकाः।

(छन्दःशास्त्रम् ८.२०-२३)

Dvikau glau | Miśrau ca | Pruthaglāmiśraḥ | Vasavastrikāḥ |

- Form a G, L pair. Write them one below the other.
- Insert on the right Gs and Ls.
- (Repeating the process) We have eight (vasavah) metric forms in the 3-syllable-prastāra.
- Therefore, following this procedure single syllable, prastāra will be:

1	G	0
2	L	1

Table 1: Comparison between Single syllable prastāra and modern binary numbers

2-syllable Prastāra

Similarly, two-syllable Prastara will be:

Serial no.	**prastāra**	**binary representation**	**mirror image of binary representation**
1	GG	00	00
2	LG	10	01
3	GL	01	10
4	LL	11	11

Table 2: Comparison between two syllable prastāra and modern binary numbers

Remark

We can say that the prastāra starts from Sr. no. 1 whereas the numbers we deal with for binary representation start from 0. Please note this difference as it will come in handy when we learn more about the methods of conversion from binary to decimal and decimal to binary in the following sections of the text.

Example of Prastāra

Example 1

The following are five successive rows in 4-syllable prastāra:

G	G	G	L
L	G	G	L
G	L	G	L
L	L	G	L
G	G	L	L

If we set G=0 and L=1, then we see that each metric pattern is the mirror reflection of the binary representation of the associated "row-number-1". Using a note from last section we can easily find the integer representation for the above binary mirror images obtained.

Serial no.	prastāra	binary representation	mirror image	Integer equivalent
1	GGGG	0000	0000	0
2	LGGG	1000	0001	1
3	GLGG	0100	0010	2
4	LLGG	1100	0011	3
5	GGLG	0010	0100	4
6	LGLG	1010	0101	5
7	GLLG	0110	0110	6
8	LLLG	1110	0111	7
9	GGGL	0001	1000	8
10	LGGL	1001	1001	9
11	GLGL	0101	1010	10
12	LLGL	1101	1011	11
13	GGLL	0011	1100	12
14	LGLL	1011	1101	13
15	GLLL	0111	1110	14
16	LLLL	1111	1111	15

Table 3: Comparison between two syllable prastāra and modern binary numbers

Sāṅkhya

Dvirardhe | Rupe ṣūnyam | Dwih ṣūnye | Tāvadardhe Tadgunitam | (Chandahṣastram 8.28-31)
The number of metres of n-syllables is $S_n = 2^n$.
Piṅgala gives an optimal algorithm for finding 2^n by means of multiplication and squaring operations that are much less than n in number. In terms of binary computation, sāṅkhya gives us the total possible number of n-bit binary numbers. (Syllables in ancient texts can be replaced by the term bit in modern binary computations.)

Algorithm explained in sāṅkhya:

- Halve the number and mark "2"

Sāṅkhya

Dvirardhe | Rupe ṣūnyam | Dwih ṣūnye | Tāvadardhe Tadgunitam | (Chandahṣastram 8.28-31)
The number of metres of n-syllables is $S_n = 2^n$.
Piṅgala gives an optimal algorithm for finding 2^n by means of multiplication and squaring operations that are much less than n in number. In terms of binary computation, sāṅkhya gives us the total possible number of n-bit binary numbers. (Syllables in ancient texts can be replaced by the term bit in modern binary computations.)

Algorithm explained in sāṅkhya:

- Halve the number and mark "2"
- If the number cannot be halved deduct one and mark "0"

Sāṅkhya

Dvirardhe | Rupe ṣūnyam | Dwih ṣūnye | Tāvadardhe Tadgunitam | (Chandahṣastram 8.28-31)
The number of metres of n-syllables is $S_n = 2^n$.
Piṅgala gives an optimal algorithm for finding 2^n by means of multiplication and squaring operations that are much less than n in number. In terms of binary computation, sāṅkhya gives us the total possible number of n-bit binary numbers. (Syllables in ancient texts can be replaced by the term bit in modern binary computations.)

Algorithm explained in sāṅkhya:

- Halve the number and mark "2"
- If the number cannot be halved deduct one and mark "0"
- Proceed till you reach zero. Start with 1 and scan the sequence of marks from the end

Sāṅkhya

Dvirardhe | Rupe ṣūnyam | Dwih ṣūnye | Tāvadardhe Tadgunitam | (Chandahṣastram 8.28-31)
The number of metres of n-syllables is $S_n = 2^n$.
Piṅgala gives an optimal algorithm for finding 2^n by means of multiplication and squaring operations that are much less than n in number. In terms of binary computation, sāṅkhya gives us the total possible number of n-bit binary numbers. (Syllables in ancient texts can be replaced by the term bit in modern binary computations.)

Algorithm explained in sāṅkhya:

- Halve the number and mark "2"
- If the number cannot be halved deduct one and mark "0"
- Proceed till you reach zero. Start with 1 and scan the sequence of marks from the end
- If "0", multiply by 2

Sāṅkhya

Dvirardhe | Rupe ṣūnyam | Dwih ṣūnye | Tāvadardhe Tadgunitam | (Chandahṣastram 8.28-31)
The number of metres of n-syllables is $S_n = 2^n$.
Piṅgala gives an optimal algorithm for finding 2^n by means of multiplication and squaring operations that are much less than n in number. In terms of binary computation, sāṅkhya gives us the total possible number of n-bit binary numbers. (Syllables in ancient texts can be replaced by the term bit in modern binary computations.)

Algorithm explained in sāṅkhya:

- Halve the number and mark "2"
- If the number cannot be halved deduct one and mark "0"
- Proceed till you reach zero. Start with 1 and scan the sequence of marks from the end
- If "0", multiply by 2
- If "2", square.

Algorithm explained in sāṅkhya:

Mathematically for 'n' bits (i.e. syllables) this algorithm can be understood in the following way:

- Assign ' n ' to a variable, say ' n_1 '
- Start with $s = 1$. If ' n_1 ' is divisible by 2, $n_1 = \frac{n_1}{2}$ & $s = s^2$
- Else $n_1 = n_1 - 1$ & $s = 2 * s$
- Stop when $n_1 = 0$.
- E.g. $\mathrm{n} = 7$,

(step-1) $n_1 = 7, \ \mathrm{s} = 1.$
(step-2) $n_1 = 6, \ \mathrm{s} = 2.$
(step-3) $n_1 = 3, \ \mathrm{s} = 4$
(step-4) $n_1 = 2, \ \mathrm{s} = 8$
(step-5) $n_1 = 1, \ \mathrm{s} = 64$
(step-6) $n_1 = 0, \ \mathrm{s} = 128.$

Example on sāṅkhya

Example 2

Six-syllable metres ($n = 6$)

- $\frac{6}{2} = 3$ and mark "2"
- 3 cannot be halved. 3-1=2 and mark "0"
- $\frac{2}{2} = 1$ and mark "2"
- $1 - 1 = 0$ and mark " 0 ".
- Sequence 2, 0, 2, 0 yields

$$1 \times 2, (1 \times 2)^2, (1 \times 2)^2 \times 2, \left((1 \times 2)^2 \times 2\right)^2 = 2^6$$

Naturally, Piṅgala's algorithm became the standard method for computing powers in Indian mathematics.

Sutras from Sāṅkhya

The next sutra of Piṅgala gives the sum of all the sankhyas S_r for $r = 1, 2, ...n$.
Dvirdyūnam tadantānam |
$S_1 + S_2 + S_3 + ... + S_n = 2S_n - 1$
Then the next sutra is as follows:
Pare Pūrnam |
$S_{n+1} = 2S_n$.
Together, the two sutras imply:
$S_n = 2^n$ and
$1 + 2 + 2^2 + ... + 2^n = 2^{n+1} - 1$.
This clearly is the formula for the sum of a geometric series.
There are more such applications of Sankhya in Algebra but our main focus in this text is binary computations so we will move to next Pratyaya i.e. Naśtam.

Naśṭam

लर्धे। सैके ग्। (छन्दःशास्त्रम् ८.२४-२५)

Larghe | Saike g |

- To find the metric pattern in a row of the prastāra, start with the row number.
- Halve it (if possible) and write an L.
- If it cannot be halved, add one and halve and write a G.
- Proceed till all the syllables of the metre are found.

Find the 7^{th} metrical form in a 5-syllable prastāra.

Remark

This is the same as the modern way of decimal to binary conversion.

Naśtam Example

Example 3

Find the 7th metrical form in a 4-syllable prastāra

- $\dfrac{(7+1)}{2} = 4$ Hence G,
- $\dfrac{4}{2} = 2$ Hence GL,
- $\dfrac{2}{2} = 1$ Hence GLL,
- $\dfrac{(1+1)}{2} = 1$ Hence GLLG.

If we set $G = 0$ and $L = 1$, we can see that Piṅgala's naṣta process leads to the desired metric form via the binary expansion

$$7 = 0 + 1.2 + 1.2^2 + 0.2^3$$

Uddiṣṭa

pratilomaguNam dviH-lārdyam |
tatāH-gi-ekaM jahyāt |
To find the row number(decimal number) of a given metrical pattern:

- Start with number 1
- Scan the pattern from the right beginning with the first L from the right
- Double it when an L is encountered
- Double and reduce by 1 when a G is encountered.

Example 4

The same algorithm can be understood with the example GLLGG,

- Start with GL**L**GG, $s = 1$
- Go to the first L from the right. Double s; i.e.GLLGG, $s = 2$
- Whenever a subsequent L is encountered s is doubled, GLLGG, $s = 4$.
- When a G is encountered $s = 2\,\mathrm{s} - 1$, GLLGG, $s = 7$.

Uddiṣṭa Example

Example 5

To find the row number of the pattern GLLG in a 4-syllable prastāra:

- Start with 1.
- Skip the G and go to L. So we get $1 \times 2 = 2$
- Then we find L. So we get $2 \times 2 = 4$
- Finally we have G. We get $4 \times 2 - 1 = 7$

Remark

In examples 4 and 5 we got the same serial number because g represents 0 and gllgg is 00110 after mirroring and gllg is 0110 after mirroring now $0110 = 00110$ because 0 in the left has no place value hence the serial number we got was the same in both the cases.

Another Method

uddiṣṭam dvigunānādyān upari aṅkān samālikhet |
laghusthā ye tu tatra aṅkāstaiH saikaiH mishritaiH bhavet |
This was given by Kedar Bhatt and his version of Uddiṣṭa is bit different than Pingala. It means - "To get the row number corresponding to the given laghu guru combination, starting from the first, write double (the previous one) on the top of each laghu-guru. Then all the numbers on top of Laghu are added with 1. (Since the starting number is not mentioned, by default, we start with 1)."

- Place 1 on top of the leftmost syllable of the given metrical pattern.
- Double it at each step while moving right.
- Sum the numbers above L and add 1 to get the row number.

Another Method

Example 6

To find the row number of the pattern GLLG

$$\begin{array}{cccc} 1 & 2 & 2^2 & 2^3 \\ G & L & L & G \end{array}$$

Row-Number $= 0.1 + 1.2 + 1.2^2 + 0.2^3 + 1 = 7$

Both the naṣṭa and uddiṣṭa processes of Piṅgala are essentially based on the fact that every natural number has a unique binary representation: It can be uniquely represented as a sum of the different sankhya S_n or the powers 2^n.

How does it work?

Let us consider any decimal number. The highest digit allowed in a decimal system is 9. The number 995 can be expressed as

$$999 - 4 = 9 * 10^2 + 9 * 10^1 + (9 - 4) \tag{1}$$

The way Piṅgala's Naṣṭa process multiplies with powers of 2 here for decimal representation we can multiply the number by 10. So, first, we take 9 then multiply it by 10 and get 90 and add 9 to it and get 99 the first two digits of our number now this can happen if we just add(/place) 9 next to the earlier 9 because each digit here has place value 10 so placing a digit means multiplying by 10. now, place another 9 after the earlier formed 99 to get 999. but we need 995 so reduce 4. This is just like in equation 1.
Now, another 9 is added at the end to make 9959, then it can be expressed as
$9959 = 995 * 10 + 9$

$$\rightarrow 9959 = \left(9 * 10^2 + 9 * 10^1 + (9 - 4)\right) * 10 + 9$$

Thus, whenever we encounter a digit 5 in decimal, it can be written as $(9-4)$ and similarly if we encounter a digit 7 in decimal, it can be written as $(9-2)$ raised to the appropriate power of 10.
Similarly, in binary, whenever we encounter a 0, it can be expressed as one less than 1 raised to the appropriate power of 2; for e.g., $10 = 1 * 2^1 + (1-1) * 2^0$
Now, if we add a 1 and make the number 101, we multiply 10 by 2 and add a 1.
i.e., $101 = (1 * 2^1 + (1-1) * 2^0) * 2 + 1$.
Thus, starting from s=1, whenever a 1 is encountered, s is doubled and whenever a 0 is encountered, s is doubled and reduced by 1.
Alternate rationale: Let L denote the number 2 and G denote the number 1. Then LL (read from right to left will denote the number 4 which is equivalent to doubling L by 2. GL denotes 3 which is 1 less than 4. Hence whenever a G is encountered, one needs to double the previous number and reduce it by 1.

Lagakriya

परे पूर्णमिति। (छन्दःशास्त्रम् ८.३४)

Pare Punamiti |

Piṅgala's sutra on lagakriya process is too brief. Halāyudha, the tenth-century commentator explains it as giving the basic rule for the construction of a table of numbers which he refers to as the Meru-prastāra:

The number of metrical forms with r gurus (or laghus) in the prastāra of metres of n-syllables is the binomial coefficient ${}^{n}C_{r}$.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

Lagakriya: How does it work?

- Lagakriya is the process of finding the number of n-bit binary numbers with a given number of 0s and 1s (Gurus and Laghus).
- The number can be represented using a Meru-prastāra (Pascal's triangle).
- From top to bottom, each row of Pascal's triangle represents the values of n i.e., the number of bits starting with 1.
- From left to right, the columns denote the number of n-bit binary having 'n' 1s, 'n-1' 1s,. . . , 0 1s.
- Each element in a row is the sum of two elements above.

Example 7

There are six 4-bit numbers with 2 Laghus and 2 Gurus (two 0s and two 1s).
Such numbers will have either an L or a G at the end (LLGG and its variations) & (GGLL and variations).
These can be derived either by adding a G to a number with 2Ls and 1G or by adding an L to a number with 2Gs and 1L.
Therefore, the number of 4-bit integers with 2L and 2Gs will be the sum of the number of 3-bit integers with 1L and 2Gs plus the number of 3-bit integers with 1G and 2Ls.

Remark

Adhvayoga is Pratyaya in Piṅgala's Chandaḥśāstra which does not lie in the scope of this text because it is not related to binary computations but if you are interested you can refer to our booklet on Combinatorics as well as the references mentioned in the reference section to know more about it.

Table of Contents

Examples from modern poems

Example 8

Kshaṇāt rusate kshaṇāt hasate, Naśib fasave aseca asate.
This Marathi poem written by Aruṇ Savant is in Vṛtta Jaloddhatagati (a name given for the poems of form lglllg lglllg in Marathi) Ksh is laghu, ṇā is guru, t is laghu, ru is laghu, sa is laghu and te is guru. The above verse has Prastāra: lglllg (twice per line) which essentially means 101110 and has mirror image 011101 which is a binary representation as per modern system and its decimal representation is 29.
Now let's calculate using uddiṣṭa: We start with s=1.
Go to the first l from the right and double the s; i.e. 2
Repeat this subsequently, i.e, $2 \times 2 \implies 4 \times 2 = 8$,
Then we have g which means multiply by 2 and subtract 1, i.e., $8 \times 2 - 1 = 15$.
Now, for the next l, we double it i.e. $15 \times 2 = 30$. This means the prastāra lglllg comes in the 30th row serially for six syllable prastāras.
As discussed earlier, the serial numbers in prastāra start from 1 whereas binary numbers start from 0, therefore, we have to reduce 1 at the end i.e. $30 - 1 = 29$ to get the decimal conversion.

Example 9

premāta jīva hotā pratyeka āksharāchyā
vairīca prema hotā ghālū kunā ukhānī

These verses are from a poem written by Arcana Murugakar which is written in Vṛtta Ānandakanda (Just another name for poems of the form gglg lgg gglg lgg). Just like in example 8 we can figure out pre is guru, mā is guru, ta is laghu, jī is guru, va is laghu, ho is guru, tā is guru; pra is guru, tye is guru, ka is laghu, ā is guru, ksha is laghu, rā is guru and chyā guru. Similarly, for the next line vai is g, rī is g, ca is l, pre is g, ma is l, ho is g, tā is g; ghā is g, lū is g, ku is l, nā is g, u is l, khā is g and nī is g. Forming the seven-syllable prastāra gglg lgg.

Remark

The format of the verse in example 9 is same as the popular Hindi retro song 'Ai dil mujhe batā de, tu kispe ā gayā hai' sung by Mohammad Rafi. The key point is that if the prastāra of different poems are the same then the same tune fits them all. From this example, you can see the formats i.e. Prastāras in which poems are written (you can have a general idea about prastāra in many Indian songs and try to use uddiṣṭa to convert them from binary to decimal number.) This exercise is fruitful to understand how Mathematics is directly or indirectly used in Poetry writing and eventually in music composition!

References Primary

[1] Chhandasutras and Binary Notation.

[2] Chhandasutras: An Introduction

[3] Computing Science in ancient India; T. R. N. Rao, Subhash Kak

[4] The Concept of Sunya

[5] Combinatorial Methods in Indian Music: Pratyayas in Sangitaratnakara of Sarngadeva

[6] Chandah-sutra of Pingalanaga(c. 300 BC), Ed. with commentary Mrtasanjivani of Halayudha (c.950), by Kedaranatha Vasudeva Lakshmana Pansikara, Kavyamala Series No. 91, Bombay 1908, Rep. Chowkhamba 2001.

[7] Jayadeva-Chandas of Jayadeva (800): Ed. with Vivrti of Harsata by H. D. Velankar, Haritosamala, Bombay 1949.

[8] NPTEL lecture notes of Prof. M. D. Srinivasa & Prof. K. Ramasubramanian Binary Numbers in Indian Antiquity, B. Van Nooten, Jour. Ind. Phil. 21, 1993, pp.31-50.

References 2

[1] A HISTORY OF PINGALA'S COMBINATORICS, JAYANT SHAH, Northeastern University, Boston, Mass.

[2] Chandahsastra of Pingala with Comm. Mr. tasanjıvanı of Halayudha Bhatta, Ed. Kedaranatha, 3rd ed. Bombay 1938.

[3] Vrttaratnakara of Kedara with Comms. Narayanı and Setu, Ed. Madhusudana Sastri, Chaukhambha, Varanasi 1994.

[4] R. Sridharan, Sanskrit Prosody, Pingala Sutras and Binary Arithmetic, in G. G. Emch et al Eds., Contributions to the History of Indian Mathematics, Hindustan Book Agency, Delhi 2005, pp. 33-62.

[5] Plofker, Kim (2009). Mathematics in India. Princeton University Press. pp. 55–56. ISBN 978-0-691-12067-6.

[6] Singh, Parmanand (1985). "The So-called Fibonacci Numbers in Ancient and Medieval India" (PDF). Historia Mathematica. Academic Press. 12 (3): 232. doi:10.1016/0315-0860(85)90021-7.

References 3

[1] "Pingala – Timeline of Mathematics". Mathigon. Retrieved 2021-08-21.

[2] Vaman Shivaram Apte (1970). Sanskrit Prosody and Important Literary and Geographical Names in the Ancient History of India. Motilal Banarsidass. pp. 648–649. ISBN 978-81-208-0045-8.

[3] R. Hall, Mathematics of Poetry, has "c. 200 BC"

[4] Mylius (1983:68) considers the Chandas-shāstra as "very late" within the Vedānga corpus.

[5] Susantha Goonatilake (1998). Toward a Global Science. Indiana University Press. p. 126. ISBN 978-0-253-33388-9. Virahanka Fibonacci.

[6] Hall, Rachel Wells (February 2008). "Math for Poets and Drummers". Math Horizons. Taylor & Francis. 15 (3): 10–12. Retrieved 27 May 2022 – via JSTOR.

[7] Edited by K. Ramasubramanian, Takao Hayashi, Clemency Montelle “Bhaskara - prabha : Culture and History of Mathematics 11”, Hindustan Book Agency.

Additional References

[1] P. P. Divakaran "The Mathematics of India Concepts, Methods, Connections, Culture and History of Mathematics 10", Hindustan Book Agency.

[2] C. S. Seshadri "Studies in History of Indian Mathematics, Culture and History of Mathematics 5, Chapter - Combinatorial methods in Indian Music: Protyayas in Sangitaratnakara of Sarngadeva", Hindustan Book Agency.

[3] M. D. Srinivasan NPTEL Combinatorics lecture notes I, II.

[4] M. S. Sriram NPTEL lecture notes on *Lilavati* I, III.

HoMI

Information about HoMI is available here: https://sites.iitgn.ac.in/homi/
For any queries on HoMI, please email us at homi@iitgn.ac.in
P.I and head of HoMI:
Padma Shree Michel Danino

Thank You!