

# General Topic: Rational Numbers

## Lesson Overview:

In this lesson, you will learn what **rational numbers** are, how to use them, and how to perform basic operations such as addition, subtraction, multiplication, and division with rational numbers.

## Key Concepts and Subtopics:

### 1. What are Rational Numbers?

- Numbers that can be written in the form  $a/b$  where  $a$  and  $b$  are integers and  $b \neq 0$ .
- This includes whole numbers, fractions, and decimals that terminate or repeat.

### 2. Difference between Rational Numbers, Integers, and Whole Numbers

- All whole numbers and integers are rational numbers, but not all rational numbers are integers or whole numbers.

### 3. Operations with Rational Numbers

- **Addition and Subtraction:** Denominators must be the same before adding or subtracting numerators.
- **Multiplication:** Multiply numerator by numerator and denominator by denominator.
- **Division:** Flip the second rational number (find its reciprocal) and multiply.

**Example 1:**  $\frac{2}{3} + \frac{1}{6}$

- Find the common denominator: 6
- Convert:  $\frac{2}{3} = \frac{4}{6}$
- Add:  $\frac{4}{6} + \frac{1}{6} = \frac{5}{6}$

**Example 2:**  $\frac{3}{4} \times \frac{2}{5}$

- Multiply numerator:  $3 \times 2 = 6$
- Multiply denominator:  $4 \times 5 = 20$
- Result:  $\frac{6}{20} = \frac{3}{10}$  (simplified)

## Summary:

- *Rational numbers are numbers that can be written as fractions with integers in numerator and denominator (denominator  $\neq 0$ ).*
- *Use the correct operations when adding, subtracting, multiplying, or dividing fractions.*
- *Always simplify your answer when possible.*

# General Topic: Integers and Their Operation

## Lesson Overview:

In this lesson, you will learn what integers are and how to perform basic operations such as addition, subtraction, multiplication, and division with integers.

## Key Concepts and Subtopics:

### 1. What are Integers?

- Whole numbers that can be positive, negative, or zero.
- Examples: -3, -2, -1, 0, 1, 2, 3

### 2. Operations with Integers

- Addition
- Subtraction
- Multiplication
- Division

### 3. Rules for Addition and Subtraction of Integers

- When the signs are different, subtract the absolute values and take the sign of the larger number.
- When the signs are the same, add the absolute values and keep the sign.

### 4. Rules for Multiplication and Division of Integers.

- If the signs are the same, the result is positive.
- If the signs are different, the result is negative.

#### **Example 1:** $7 + (-3)$

- Different signs, subtract  $7 - 3 = 4$
- Take the sign of the larger number (7) → positive
- Result: 4

**Example 2:**  $(-5) + (-8)$

- Same signs, add  $5 + 8 = 13$
- Sign is negative
- Result:  $-13$

**Example 3:**  $(-4) \times 6$

- Different signs  $\rightarrow$  *negative*
- Multiply  $4 \times 6 = 24$
- Result:  $-24$

**Example 4:**  $(-12) \div (-3)$

- Same signs  $\rightarrow$  *positive*
- Divide  $12 \div 3 = 4$
- Result:  $4$

**Summary:**

- *Integers are whole numbers that can be positive, negative, or zero.*
- *Follow the rules for adding, subtracting, multiplying, and dividing integers.*
- *Remember the signs to get the correct answer.*

# General Topic: Factors and Multiples

## Lesson Overview:

In this lesson, you will learn about factors and multiples, and how to find them for given numbers. These concepts help in simplifying fractions and solving problems in algebra.

## Key Concepts and Subtopics:

### 1. What are Factors?

- Factors of a number are integers that divide the number exactly without leaving a remainder.
- Example: Factors of 12 are 1, 2, 3, 4, 6, 12.

### 2. What are Multiples?

- Multiples of a number are the products of that number and any integer.
- Example: Multiples of 5 are 5, 10, 15, 20, 25, and so on.

### 3. Prime Numbers

- Prime numbers are numbers greater than 1 that have only two factors: 1 and itself.
- Example: 2, 3, 5, 7, 11.

### 4. Composite Numbers

- Composite numbers are numbers greater than 1 that have more than two factors.
- Example: 4, 6, 8, 9, 12.

### 5. Greatest Common Factor (GCF)

- The largest factor that two or more numbers have in common.

### 6. Least Common Multiple (LCM)

- The smallest multiple that two or more numbers have in common.

**Example 1:** Find the factors of 18.

Factors of 18 are 1, 2, 3, 6, 9, and 18.

**Example 2:** Find the multiples of 4 up to 40.

Multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40.

**Example 3:** Find the GCF of 12 and 18.

Factors of 12: 1, 2, 3, 4, **6**, 12

Factors of 18: 1, 2, 3, **6**, 9, 18

Common factors: 1, 2, 3, **6**

**GCF: 6**

**Example 4:** Find the LCM of 3 and 4.

Multiples of 3: 3, 6, 9, **12**, 15, 18, ...

Multiples of 4: 4, 8, **12**, 16, 20, ...

**LCM: 12**

### **Summary:**

- *Factors are numbers that divide another number exactly.*
- *Multiples are products of a number and any integer.*
- *GCF is the greatest factor shared by two or more numbers.*
- *LCM is the smallest multiple shared by two or more numbers.*

# General Topic: Algebraic Expressions

## Lesson Overview:

In this lesson, you will learn about algebraic expressions, their parts, and how to simplify them.

## Key Concepts and Subtopics:

### 1. What are Algebraic Expressions?

- Combinations of numbers, variables, and operations (like addition and multiplication).
- Example:  $3x + 5$ ,  $2a - 7$ ,  $4y^2 + 3y - 1$ .

### 2. Terms

- The parts of an expression separated by plus (+) or minus (-) signs.
- Example: In  $3x + 5$ , the terms are  $3x$  and  $5$ .

### 3. Coefficients

- The numbers multiplied by variables in a term.
- Example: In  $3x$ ,  $3$  is the coefficient.

### 4. Constants

- Numbers without variables.
- Example: In  $3x + 5$ ,  $5$  is the constant.

### 5. Like Terms

- Terms that have the same variable raised to the same power.
- Example:  $2x$  and  $5x$  are like terms;  $3y$  and  $4y^2$  are not.

### 6. Simplifying Expressions

- Combine like terms by adding or subtracting their coefficients.

**Example 1:** Simplify  $4x + 3x$

- Both terms are like terms (x).
- Add coefficients:  $4 + 3 = 7$ .
- Result:  $7x$

**Example 2:** Simplify  $5a^2 + 2a - 3a^2 + 4$

- Combine like terms:  $5a^2 - 3a^2 = 2a^2$
- Other terms:  $2a$  and  $4$  stay as is.
- Result:  $2a^2 + 2a + 4$

**Summary:**

- *Algebraic expressions combine numbers, variables, and operations.*
- *Like terms have the same variables and powers.*
- *Simplify expressions by combining like terms.*

# General Topic: Linear Equations in One Variable

## Lesson Overview:

In this lesson, you will learn how to solve linear equations that have one variable.

## Key Concepts and Subtopics:

### 1. What is a Linear Equation?

- An equation that makes a straight line when graphed.
- It has variables with the highest power of 1.
- Example:  $2x + 3 = 7$

### 2. Goal of Solving Linear Equations

- Find the value of the variable that makes the equation true.

### 3. Properties Used in Solving

- Addition Property: You can add the same number to both sides.
- Subtraction Property: You can subtract the same number from both sides.
- Multiplication Property: You can multiply both sides by the same number (except zero).
- Division Property: You can divide both sides by the same number (except zero).

### 4. Steps in Solving Linear Equations

- Simplify both sides (remove parentheses, combine like terms).
- Use properties to isolate the variable on one side.
- Solve for the variable.
- Check the solution by substituting the value back into the original equation.

**Example 1:** Solve  $2x + 3 = 7$

- Subtract 3 from both sides:  $2x = 4$
- Divide both sides by 2:  $x = 2$
- Check:  $2(2) + 3 = 7$  ✓

**Example 2:** Solve  $5y - 4 = 16$

- Add 4 to both sides:  $5y = 20$
- Divide both sides by 5:  $y = 4$
- Check:  $5(4) - 4 = 16$  ✓



**Summary:**

- *Linear equations have variables with power 1.*
- *Use properties of equality to isolate the variable.*
- *Always check your solution.*

# General Topic: Ratio, Proportion, and Percentage

## Lesson Overview:

In this lesson, you will learn about ratios, proportions, and percentages. These concepts help you compare quantities and solve real-life problems involving parts, wholes, and rates.

## Key Concepts and Subtopics:

### 1. Ratio

- A ratio compares two quantities showing how many times one value contains or is contained within the other.
- Written as  $a:b$  or  $a/b$ .
- Example: If there are 6 boys and 3 girls in a class, the ratio of boys to girls is 6:3 or simplified to 2:1.

### 2. Proportion

- An equation that states two ratios are equal.
- Example:  $3/4 = 6/8$
- Proportions can be solved using cross multiplication.

### 3. Percentage

- A ratio that compares a number to 100.
- Expressed with the symbol %.
- Example: 45% means 45 parts out of 100.
- Can be converted to fractions or decimals.

#### **Ratio Example:**

If there are 10 apples and 5 oranges, what is the ratio of apples to oranges?

- Ratio =  $10:5 = 2:1$  (simplified)

#### **Proportion Example:** Solve for x: $4/5 = x/15$

- Cross multiply:  $4 \times 15 = 5 \times x$
- $60 = 5x$
- Divide both sides by 5:  $x = 12$
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**Percentage Examples:**1. Find 25% of 200

- Convert percentage to decimal:  $25\% = 0.25$
- Multiply:  $0.25 \times 200 = 50$

2. Convert 0.4 to percentage

- $0.4 \times 100 = 40\%$

3. Convert  $\frac{3}{5}$  to percentage

- Divide:  $3 \div 5 = 0.6$
- Multiply by 100 = 60%

**Summary:**

- *Ratio compares two quantities.*
- *Proportion states two ratios are equal and can be solved with cross multiplication.*
- *Percentage shows parts per hundred and can be converted to decimals or fractions.*

# General Topic: Geometry – Basic Concepts and Properties

## Lesson Overview:

In this lesson, you will learn the basic concepts of geometry such as points, lines, angles, and shapes. You will also understand the basic properties of these figures.

## Key Concepts and Subtopics:

### 1. Points

- *The smallest unit in geometry.*
- *Has no size or dimension, just a location.*
- *Represented by a dot (·).*

### 2. Lines

- *Extends infinitely in both directions with no end points.*
- *Types of lines:*
  - *Line segment (has two endpoints)*
  - *Ray (has one endpoint and extends infinitely in one direction)*
  - *Straight line (extends infinitely in both directions)*

### 3. Angles

- *Formed when two lines meet at a point.*
- *Types of angles:*
  - *Acute angle (less than 90 degrees)*
  - *Right angle (exactly 90 degrees)*
  - *Obtuse angle (greater than 90 degrees but less than 180 degrees)*

### 4. Basic Geometric Shapes

- *Triangle (3 sides)*
- *Quadrilateral (4 sides)*
- *Circle (all points are equal distance from the center)*

## 5. Properties of Shapes

- *Triangle: has 3 sides, the sum of interior angles is 180 degrees.*
- *Quadrilateral: has 4 sides, the sum of interior angles is 360 degrees.*
- *Circle: all points on the circle are the same distance from the center (radius).*

**Example 1:** *Identify types of angles.*

- *Angle A measures  $45^\circ \rightarrow$  acute angle*
- *Angle B measures  $90^\circ \rightarrow$  right angle*
- *Angle C measures  $120^\circ \rightarrow$  obtuse angle*

**Example 2:** *Sum of interior angles of a triangle.*

- *Angles are  $50^\circ$ ,  $60^\circ$ , and  $70^\circ$*
- *$50 + 60 + 70 = 180^\circ \rightarrow$  correct*

**Example 3:** *Sum of interior angles of a quadrilateral.*

- *Angles are  $90^\circ$ ,  $80^\circ$ ,  $100^\circ$ , and  $90^\circ$*
- *$90 + 80 + 100 + 90 = 360^\circ \rightarrow$  correct*

### **Summary:**

- *Points are locations with no size.*
- *Lines can extend infinitely or have endpoints.*
- *Angles are formed by two lines meeting at a point.*
- *Triangles and quadrilaterals have specific sums for their interior angles.*
- *Circles have a radius, which is the distance from the center to any point on the circle.*

# Statistics and Probability (Basic)

## Lesson Overview:

In this lesson, you will learn the basics of statistics and probability. You will understand how to collect, organize, and interpret data, and how to calculate simple probabilities.

## Key Concepts and Subtopics:

### 1. Statistics

- The science of collecting, organizing, analyzing, and interpreting data.
- Types of data:
  - Qualitative (descriptive)
  - Quantitative (numerical)

### 2. Data Representation

- Tables
- Bar graphs
- Pie charts
- Line graphs

### 3. Mean (Average)

- Sum of all data values divided by the number of values.
- Formula:  $\text{Mean} = (\text{Sum of values}) \div (\text{Number of values})$

### 4. Median

- The middle value when data is arranged in order.
- If even number of data points, median is the average of the two middle values.

### 5. Mode

- The value that appears most frequently in a data set.

### 6. Probability

- The chance or likelihood that an event will happen.
- Formula:  $\text{Probability} = (\text{Number of favorable outcomes}) \div (\text{Total number of possible outcomes})$
- Probability ranges from 0 (impossible) to 1 (certain).

**Example 1:** Find the mean of 4, 6, 8, 10

- $\text{Sum} = 4 + 6 + 8 + 10 = 28$
- $\text{Number of values} = 4$
- $\text{Mean} = 28 \div 4 = 7$

**Example 2:** Find the median of 3, 7, 9, 11, 15

- Arrange data (already arranged)
- Middle value = 9 (3rd number)
- Median = 9

**Example 3:** Find the mode of 2, 4, 4, 6, 8

- Mode = 4 (appears twice)

**Example 4:** Calculate probability of rolling a 3 on a 6-sided die

- Favorable outcomes = 1 (only one side has 3)
- Total outcomes = 6
- Probability =  $1 \div 6 \approx 0.167$

### **Summary:**

- *Statistics helps us understand data through measures like mean, median, and mode.*
- *Probability tells us how likely an event is to happen.*
- *Both are important for making decisions based on data and chance.*