SIR Model Stability

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1 Model

We can begin with a simplified SIR (Susceptible-Infected-Removed) model with no births or deaths.

$$\frac{dS}{dt} = -\frac{\alpha}{N} SI$$
$$\frac{dI}{dt} = \frac{\alpha}{N} SI - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

Where α is the contact rate, and γ is the recovery rate. The initial conditions would be a constant population, N, and a positive, non-zero S_0 and I_0 and non-negative R_0.

$$S_0 + I_0 + R_0 = N$$
$$S_i + I_i + R_i = N$$
$$S_0 > 0, I_0 > 0, R_0 >= 0$$

In such a model with a closed population, there will be no endemic equilibrium. The disease would die out as the susceptible population is eventually depleted. Thus a more useful model to examine is the SIR model with births and deaths:

References

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