SI Model Stability

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1 Model

$$\frac{dS}{dt} = -\frac{\alpha}{N}SI$$
$$\frac{dI}{dt} = \frac{\alpha}{N}SI$$

2 Stability Analysis

Reduce to one equation. Since $S = N - I$:

$$\frac{dI}{dt} = \frac{\alpha}{N}(N - I)I$$

The equilibria / fixed points are when $\frac{dI}{dt} = 0$.

$$I^* = 0, I^* = N$$

2.1 Disease-Free Equilibrium (DFE)

DFE: $(S, I) = (N, 0)$

Calculate the Jacobian.

$$\begin{vmatrix} 0 & -\alpha \\ 0 & \alpha \end{vmatrix}$$

2.2 Endemic Equilibrium

Endemic equilibrium: $(S, I) = (0, N)$