DICE2013R-mc: A Matlab / CasADi Implementation of Vanilla DICE2013R

Christopher M. Kellett[1], Timm Faulwasser[2], and Steven R. Weller[1]

[1] *School of Electrical Engineering and Computing, University of Newcastle, Callaghan, New South Wales 2308, Australia, email:* {`Chris.Kellett, Steven.Weller`}`@newcastle.edu.au`

[2] *Institute for Applied Computer Science, Karlsruhe Institute of Technology, 76344 Eggenstein-Leopoldshafen, Germany, e-mail:* `timm.faulwasser@kit.edu`

July 22, 2017

Abstract

This brief document provides a description of how to use DICE2013R-mc [2], a Matlab and CasADi-based implementation of the Dynamic Integrated model of Climate and Economy (DICE). DICE2013R-mc provides the same basic functionality as the GAMS code[1] for DICE2013R as available at [4]. DICE2016R essentially represents an update of model parameters from DICE2013R and either version can be run in DICE2013R-mc by setting an appropriate parameter in the main file.

1 Software Requirements

This implementation of DICE2013R (DICE2016R) makes use of the CasADi framework for algorithmic differentiation and numeric optimization [1] in conjunction with Matlab[2]. Version 3.2.1 of CasADi is used and, hence, Matlab 2014a or later is generally required. Appropriate binaries[3] of CasADi v.3.2.1 are available at [1].

Similar to CasADi, DICE2013R-mc is distributed under the GNU Lesser General Public License (LGPL), and hence the code can be used royalty-free even in commercial applications.

2 Model and Optimal Control Problem

The DICE2013R model operates on five year time steps beginning from 2010 (DICE2016R starts from 2015). To formalize this, let $t_0 = 2010$, $\Delta = 5$, and $i = 1, 2, 3, \ldots$ be the discrete time index. Then

$$t = t_0 + \Delta \times i \tag{1}$$

yields $t = 2010, 2015, 2020, \ldots$ as desired.

The DICE2013R model has six endogenous state variables: two variables to model the global climate in the form of atmospheric and oceanic temperatures (T_{AT} and T_{LO}, respectively, in units

[1] A manual is available for DICE2013R [5]. However, the description of the model in the manual [5] differs in several respects from the available code [4]. As our aim is replicate the functionality of [4], the description of the model is in reference to the implementation in [4] rather than the description in [5].

[2] For those new to Matlab, MathWorks has several online tutorial resources available at [3].

[3] After downloading an appropriate binary, be sure to add CasADi to your Matlab path as described at [1].

of °C), three variables to model the global carbon cycle in the form of carbon concentrations in the atmosphere, upper ocean, and lower ocean (M_{AT}, M_{UP}, and M_{LO}, respectively, in units of GtC), and one state for global capital (K, in units of trillions 2005USD[4]). Decision variables or control inputs are the emissions mitigation rate (μ) and the savings rate (s) where the latter is the ratio of investment to net economic output. Finally, the model is also driven by several exogenous, time-varying terms such as population and total factor productivity. The full dynamics are given by:

$$\begin{bmatrix} T_{\mathrm{AT}}(i+1) \\ T_{\mathrm{LO}}(i+1) \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} T_{\mathrm{AT}}(i) \\ T_{\mathrm{LO}}(i) \end{bmatrix} + \begin{bmatrix} \xi_1 \\ 0 \end{bmatrix} R_F(i) \tag{2}$$

$$\begin{bmatrix} M_{\mathrm{AT}}(i+1) \\ M_{\mathrm{UP}}(i+1) \\ M_{\mathrm{LO}}(i+1) \end{bmatrix} = \begin{bmatrix} \zeta_{11} & \zeta_{12} & 0 \\ \zeta_{21} & \zeta_{22} & \zeta_{23} \\ 0 & \zeta_{32} & \zeta_{33} \end{bmatrix} \begin{bmatrix} M_{\mathrm{AT}}(i) \\ M_{\mathrm{UP}}(i) \\ M_{\mathrm{LO}}(i) \end{bmatrix} + \begin{bmatrix} \xi_2 \\ 0 \\ 0 \end{bmatrix} E(i) \tag{3}$$

$$\begin{aligned} K(i+1) &= (1-\delta)^{\Delta} K(i) \\ &\quad + \Delta \left(1 - a_2\, T_{\mathrm{AT}}(i)^{a_3} - \theta_1(i)\mu(i)^{\theta_2}\right) A(i)K(i)^{\gamma} \left(\tfrac{L(i)}{1000}\right)^{1-\gamma} s(i), \end{aligned} \tag{4}$$

where emissions (E in units of $\mathrm{GtCO_2}$) and radiative forcing[5] (R_F) are given by

$$E(i) = \sigma(i)(1-\mu(i))A(i)K(i)^{\gamma} \left(\tfrac{L(i)}{1000}\right)^{1-\gamma} + E_{\mathrm{Land}}(i) \tag{5}$$

$$R_F(i) = \eta \log_2 \left(\frac{\zeta_{11} M_{\mathrm{AT}}(i) + \zeta_{12} M_{\mathrm{UP}}(i) + \xi_2 E(i)}{M_{\mathrm{AT},1750}} \right) + F_{\mathrm{EX}}(i). \tag{6}$$

Parameter values can be found in the table at the end of this document.

The exogenous, time-varying signals are given by[6]:

$$\sigma(i+1) = \sigma(i) \exp\left(-g_{\sigma} \cdot (1-\delta_{\sigma})^{\Delta i} \cdot \Delta\right), \quad \sigma(1) = \sigma_0 \tag{7a}$$

$$L(i+1) = L(i) \left(\frac{L_a}{L(i)}\right)^{\ell_g}, \quad L(1) = L_0 \tag{7b}$$

$$A(i+1) = \frac{A(i)}{1 - g_a \exp(-\delta_A \cdot \Delta \cdot (i-1))}, \quad A(1) = A_0 \tag{7c}$$

$$E_{\mathrm{Land}}(i) = E_{L0} \cdot (1-\delta_{EL})^{(i-1)} \tag{7d}$$

$$F_{\mathrm{EX}}(i) = f_0 + \min\left\{ \frac{f_1 - f_0}{t_f}(i-1),\ f_1 - f_0 \right\} \tag{7e}$$

$$\theta_1(i) = \frac{p_b}{1000 \cdot \theta_2}(1-\delta_{pb})^{i-1} \cdot \sigma(i). \tag{7f}$$

Utility is given by

$$U(C(i), L(i)) = L(i) \left(\frac{\left(\frac{1000C(i)}{L(i)}\right)^{1-\alpha} - 1}{1-\alpha} - 1 \right) \tag{8}$$

where the consumption (C) is

$$C(i) = \left(1 - a_2\, T_{\mathrm{AT}}(i)^{a_3} - \theta_1(i)\mu(i)^{\theta_2}\right) A(i)K(i)^{\gamma} \left(\tfrac{L(i)}{1000}\right)^{1-\gamma} (1 - s(i)). \tag{9}$$

[4]DICE2016R measures capital in units of trillions 2010USD.

[5]The form of the radiative forcing given here is due to the use of an inconsistent discretization of a continuous-time climate model, mixing forward and backward Euler discretizations for the two states, that leads to $T_{\mathrm{AT}}(i+1)$ depending on $M_{\mathrm{AT}}(i+1)$ instead of $M_{\mathrm{AT}}(i)$. Since the aim of this release is to replicate the functionality of [4], we have not corrected this inconsistency.

[6]In [4], θ_1 is called `cost1`.

Optimal pathways are then derived by maximizing the social welfare:

$$\max_{\mathbf{s},\mu} \quad \Delta * scale1 * \sum_{i=1}^{N} \frac{U(C(i), L(i))}{(1+\rho)^{\Delta(i-1)}} - scale2 \tag{10}$$

$$\begin{aligned} \text{subject to} \quad & (2) - (4) \\ & \mu(1) = \mu_0 \\ & \mu(i) \geq 0, \quad i = 2, \ldots, N \\ & \mu(i) \leq 1, \quad i = 2, \ldots, 29 \\ & \mu(i) \leq 1.2, \quad i = 30, \ldots, N \\ & 0 \leq s(i) \leq 1, \quad i = 1, \ldots, N-10 \\ & s(i) = s^*, \quad i = N-9, \ldots, N. \end{aligned} \tag{11}$$

The social cost of carbon is given by the ratio of the marginal welfare with respect to emissions and with respect to consumption:

$$\mathrm{SCC}(i) = -1000 \times \frac{\partial W / \partial E(i)}{\partial W / \partial C(i)}. \tag{12}$$

3 Description of Code

DICE2013R-mc consists of three main files:

- `DICE2013R_mc.m` is the top-level file and calls the subsequent two files.
- `set_DICE_parameters.m` is a function that takes the desired version (2013, 2016, or 1 for a custom version of parameters), as a parameter and returns all other required parameters[7], including exogenous signals, in the structure `Params`.
- `dice_dynamics.m` is a function that calculates the dynamic states (endogenous signals) of DICE2013R. In addition to the dynamic states, it also calculates the value of the objective function and the quantities required for the social cost of carbon computation as a ratio of marginals; namely the emissions and consumption.

Running DICE2013R-mc in the Matlab command window yields the DICE endogenous states (capital `K`, temperatures `TATM` and `TLO`, and carbon concentrations `MATM`, `MUP`, and `MLO`) and the input values for the mitigation rate (`mu`) and savings rate (`s`). Additionally, the marginals with respect to emissions (`lamE`) and with respect to consumption (`lamC`) are used to calculate the Social Cost of Carbon (`SCC`) and the optimal welfare is given by `J`.

A `clear` command removes many of the variables and other objects used in the solution of the optimal control problem from the workspace. This command can be commented out if these items are required.

As well as the three core component files listed above, two hopefully useful utility files are provided:

- `plot_results.m` generates plots of the exogenous and endogenous signals, as well as the control inputs and social cost of carbon.
- `compute_auxiliary_quantities.m` computes several additional quantities that are available as outputs of the GAMS code [4]. The selected quantities are described below. This file should provide a template for those wishing to define additional quantities of interest.

[7] One minor change in notation has been made in DICE2013R-mc from DICE2013R and this is the indexing into the climate and carbon matrices. DICE2013R uses a non-standard "column-row" numbering for matrices, whereas DICE2013R-mc uses standard "row-column" indexing.

The additional quantities calculated by `compute_auxiliary_quantities.m` are: industrial emissions (IE), net economic output (NEO), per capita consumption (PCC), the damages fraction (DF), atmospheric carbon in parts per million (ACppm), and the marginal cost of abatement (MCA), where

$$\mathrm{IE}(i) = \sigma(i)(1-\mu(i))A(i)K(i)^{\gamma}\left(\frac{L(i)}{1000}\right)^{1-\gamma} \tag{13}$$

$$\mathrm{NEO}(i) = \left(1 - a_2\, T_{\mathrm{AT}}(i)^{a_3} - \theta_1(i)\mu(i)^{\theta_2}\right) A(i)K(i)^{\gamma}\left(\frac{L(i)}{1000}\right)^{1-\gamma} \tag{14}$$

$$\mathrm{PCC}(i) = \frac{1000 * C(i)}{L(i)} \tag{15}$$

$$\mathrm{DF}(i) = a_2 T_{\mathrm{AT}}(i)^{a_3} \tag{16}$$

$$\mathrm{ACppm}(i) = \frac{M_{\mathrm{AT}}(i)}{2.13} \tag{17}$$

$$\mathrm{MCA}(i) = p_b * (1-\delta_{pb})^{i-1} * \mu(i)^{\theta_2 - 1}. \tag{18}$$

3.1 GAMS Data and Verification Plots

Two additional files are provided with this release for the purpose of demonstrating that DICE2013R-mc replicates the functionality[8] of the publicly available DICE2013R GAMS code [4]. These files are `plot_gams_verification.m` and `GAMS_Results.csv`. The latter contains the output generated by [4] while the former is an extended version of `plot_results.m` that loads and plots the data from DICE2013R against the results of DICE2013R-mc.

The call to `plot_gams_verification.m` is commented out in the release. To view these plots, uncomment the call to `plot_gams_verification.m`. Note that this only works with `Param_set = 2013`; i.e., verification plots are not provided for the 2016R version or, obviously, custom parameter settings.

4 Release notes

1.0 Initial release, requires *casadi-matlabR2014b-v3.0.0* or *casadi-matlabR2014b-v3.1.1*.

1.1 Fixes compatability issue with *casadi-matlabR2014b-v3.2.1*.

2.0 Adds parameters for DICE2016R and switch statement to use 2013R, 2016R, or custom parameter values.

References

[1] J. Andersson and J. Gillis. CasADi 3.0.0. Available at https://github.com/casadi/casadi/wiki., March 2016.

[2] T. Faulwasser, C. M. Kellett, and S. R. Weller. DICE2013R-mc: Dynamic Integrated model of Climate and Economy 2013R - Matlab and CasADi. Available at https://github.com/cmkellett/DICE2013R-mc., July 2016.

[3] MathWorks. Matlab and simulink tutorials. Online resources at http://www.mathworks.com/support/learn-with-matlab-tutorials.html.

[8] Note that DICE2013R-mc provides a slightly better (greater) value for the optimal welfare than that provided by the GAMS solution. This may be due to the fact that the default solvers (ipopt for DICE2013R-mc and conopt for DICE2013R) find slightly different local minima.

[4] W. D. Nordhaus. Vanilla DICE2103R. Available at http://www.econ.yale.edu/~nordhaus/homepage/Web-DICE-2013-April.htm., November 2013.

[5] W. D. Nordhaus and P. Sztorc. *DICE 2013R: Introduction and User's Manual*, second edition, 31 October 2013.

Parameter	Value DICE2013R	Value DICE2016R	Notes	GAMS Line No. (2013R)
Δ	5		Time step in years (`tstep`)	16
t_0	2010	2015	Initial year	
N	60	100	Horizon length	11
μ_0	0.039	0.03	Initial mitigation rate (`miu0`)	43
Climate diffusion parameters				
ϕ_{11}	0.8630	0.8718	$1 - \mathtt{c1}\left(\frac{\mathtt{fco22x}}{\mathtt{t2xco2}} + \mathtt{c3}\right)$	261
ϕ_{12}	0.0086	0.0088	$\mathtt{c1} \cdot \mathtt{c3}$	261
ϕ_{21}	0.025	0.025	`c4`	78
ϕ_{22}	0.975	0.975	$1 - \mathtt{c4}$	262
Carbon cycle diffusion parameters				
ζ_{11}	0.912*	0.88	$\mathtt{b11} = 1 - \mathtt{b12}$	139
ζ_{12}	0.03833*	0.196	$\mathtt{b21} = \mathtt{b12} \cdot \mathtt{MATEQ/MUEQ}$ $(= b12 \cdot 588/1350)$	140
ζ_{21}	0.088	0.12	`b12`	55
ζ_{22}	0.9592*	0.797	$\mathtt{b22} = 1 - \mathtt{b21} - \mathtt{b23}$	141
ζ_{23}	0.0003375*	0.001465	$\mathtt{b32} = \mathtt{b23} \cdot \mathtt{mueq/mleq}$ $(= b23 \cdot 1350/10000)$	142
ζ_{32}	0.00250	0.007	`b23`	56
ζ_{33}	0.9996625*	0.99853488	$\mathtt{b33} = 1 - \mathtt{b32}$	143
Other parameters				
η	3.8		Forcings of equilibrium CO2 doubling (Wm-2) (`fco22x`)	79
ξ_1	0.098		Multiplier for η (`c1`)	76
ξ_2	5/3.666		Conversion factor for emissions (GtC / GtCO2)	258
$M_{\mathrm{AT},1750}$	588		Pre-industrial carbon in atmosphere (`mateq`)	250
γ	0.3		Capital elasticity in production function (`gama`)	26
θ_2	2.8	2.6	Exponent of control cost function (`expcost2`)	89
a_2	0.00267	0.00236	Damage multiplier term (`a2`)	85
a_3	2		Damage exponent (`a3`)	86
δ	0.1		Depreciation rate on capital (per year) (`dk`)	30
α	1.45		Elasticity of marginal utility of consumption (`elasmu`)	22
ρ	0.015		Initial rate of social time preference per year (`prstp`)	23
$scale1$	0.016408662	0.030245527	Utility multiplier	107
$scale2$	3855.106895	10993.704	Utility offset	108
Exogenous signal parameters				
L_0	6838	7403	Global population in 2010 (`pop0`)	27
L_a	10500	11500	Asymptotic global population (`popasym`)	29
ℓ_g	0.134		Population growth rate (`popadj`)	28
E_{L0}	3.3	2.6	Initial land use emissions (`eland0`)	40
δ_{EL}	0.2	0.115	Land use emissions decrease rate (`deland`)	41
A_0	3.80	5.115	Initial total factor productivity (`a0`)	33
g_a	0.079	0.076	Initial TFP growth (`ga0`)	34
δ_A	0.006	0.005	Decline rate of TFP (`dela`)	35
p_b	344	550	Cost of backstop 2005$ per tCO2 2010 (`pback`)	90
δ_{pb}	0.025		Decline rate of backstop cost (`gback`)	91
σ_0	0.5491	0.3503	Initial emissions intensity, calculated as $\frac{\mathtt{global\ emissions}}{\mathtt{global\ output} \times (1 - \mathtt{mitigation\ rate})} = \frac{33.61}{63.69(1-0.039)}$	
g_σ	0.01	0.0152	Emissions intensity base rate (`gsigma1`)	38
δ_σ	-0.001		Emissions intensity decline rate (`dsig`)	39
f_0	0.25	0.5	2010 forcings of non-CO2 GHG (Wm^2) (`fex0`)	68
f_1	0.70	1.0	2100 forcings of non-CO2 GHG (Wm^2) (`fex1`)	69
t_f	18	17	Number of periods that exogenous forcings increase	

- Initial conditions

	$T_{\text{AT}}(0)$	$T_{\text{LO}}(0)$	$M_{\text{AT}}(0)$	$M_{\text{UP}}(0)$	$M_{\text{LO}}(0)$	$K(0)$
2013R	0.8	0.0068	830.4	1527	10010	135
2016R	0.85	0.0068	851	460	1740	223

- Carbon cycle parameters to compute diffusion coefficients

	b12	b23	mateq	mueq	mleq
2013R	0.088	0.0025	588	1350	10000
2016R	0.12	0.007	588	360	1720

- Climate parameters to compute diffusion coefficients

	c1	c3	c4	fco22x	t2xco2
2013R	0.098	0.088	0.025	3.8	2.9
2016R	0.1005	0.088	0.025	3.6813	3.1

- Parameters for calculating $\sigma_0 = \frac{e0}{q0(1-\mu_0)}$:

	e0	q0	miu0
2013R	33.61	63.69	0.039
2016R	35.85	105.5	0.03

- Optimal savings rate calculated as

$$s^* = \frac{\delta + 0.004}{\delta + 0.004\alpha + \rho}\gamma$$