DICE2013R-mc: A Matlab / CasADi Implementation of Vanilla DICE2013R

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Abstract

This brief document provides a description of how to use DICE2013R-mc [2], a Matlab and CasADi-based implementation of the Dynamic Integrated model of Climate and Economy (DICE). DICE2013R-mc provides the same basic functionality as the GAMS code[1] for DICE2013R as available at [4].

1 Software Requirements

This implementation of DICE2013R makes use of the CasADi framework for algorithmic differentiation and numeric optimization [1] in conjunction with Matlab[2]. Version 3.0.0 of CasADi is used and, hence, Matlab 2014a or later is generally required. Appropriate binaries[3] of CasADi v.3.0.0 are available at [1].

Similar to CasADi, DICE2013R-mc is distributed under the GNU Lesser General Public License (LGPL), and hence the code can be used royalty-free even in commercial applications.

2 Model and Optimal Control Problem

The DICE2013R model operates on five year time steps beginning from 2010. To formalize this, let $t_0 = 2010$, $\Delta = 5$, and $i = 1, 2, 3, \ldots$ be the discrete time index. Then

$$t = t_0 + \Delta \times i \tag{1}$$

yields $t = 2010, 2015, 2020, \ldots$ as desired.

The DICE2013R model has six endogenous state variables: two variables to model the global climate in the form of atmospheric and oceanic temperatures (T_{AT} and T_{LO}, respectively, in units of °C), three variables to model the global carbon cycle in the form of carbon concentrations in the atmosphere, upper ocean, and lower ocean (M_{AT}, M_{UP}, and M_{LO}, respectively, in units of GtC),

[1] A manual is available for DICE2013R [5]. However, the description of the model in the manual [5] differs in several respects from the available code [4]. As our aim is replicate the functionality of [4], the description of the model is in reference to the implementation in [4] rather than the description in [5].

[2] For those new to Matlab, MathWorks has several online tutorial resources available at [3].

[3] After downloading an appropriate binary, be sure to add CasADi to your Matlab path as described at [1].

and one state for global capital (K, in units of trillions 2005USD). Decision variables or control inputs are the emissions mitigation rate (μ) and the savings rate (s) where the latter is the ratio of investment to net economic output. Finally, the model is also driven by several exogenous, time-varying terms such as population and total factor productivity. The full dynamics are given by:

$$\begin{bmatrix} T_{\mathrm{AT}}(i+1) \\ T_{\mathrm{LO}}(i+1) \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} T_{\mathrm{AT}}(i) \\ T_{\mathrm{LO}}(i) \end{bmatrix} + \begin{bmatrix} \xi_1 \\ 0 \end{bmatrix} R_F(i) \tag{2}$$

$$\begin{bmatrix} M_{\mathrm{AT}}(i+1) \\ M_{\mathrm{UP}}(i+1) \\ M_{\mathrm{LO}}(i+1) \end{bmatrix} = \begin{bmatrix} \zeta_{11} & \zeta_{12} & 0 \\ \zeta_{21} & \zeta_{22} & \zeta_{23} \\ 0 & \zeta_{32} & \zeta_{33} \end{bmatrix} \begin{bmatrix} M_{\mathrm{AT}}(i) \\ M_{\mathrm{UP}}(i) \\ M_{\mathrm{LO}}(i) \end{bmatrix} + \begin{bmatrix} \xi_2 \\ 0 \\ 0 \end{bmatrix} E(i) \tag{3}$$

$$K(i+1) = (1-\delta)^5 K(i) + 5\left(1 - a\,T_{\mathrm{AT}}(i)^2 - \theta_1(i)\mu(i)^{\theta_2}\right) A(i)K(i)^{\gamma}\left(\tfrac{L(i)}{1000}\right)^{1-\gamma} s(i), \tag{4}$$

where emissions (E in units of $\mathrm{GtCO_2}$) and radiative forcing[4] (R_F) are given by

$$E(i) = \sigma(i)(1-\mu(i))A(i)K(i)^{\gamma}\left(\tfrac{L(i)}{1000}\right)^{1-\gamma} + E_{\mathrm{Land}}(i) \tag{5}$$

$$R_F(i) = \eta \log_2\left(\frac{\zeta_{11}M_{\mathrm{AT}}(i) + \zeta_{12}M_{\mathrm{UP}}(i) + \xi_2 E(i)}{M_{\mathrm{AT},1750}}\right) + F_{\mathrm{EX}}(i). \tag{6}$$

Parameter values can be found in the table at the end of this document.

The exogenous, time-varying signals are given by[5]:

$$\sigma(i+1) = \sigma(i)\exp\left(-0.01 * (0.999)^{5i} * 5\right), \quad \sigma(1) = 0.5491 \tag{7}$$

$$L(i+1) = L(i)\left(\frac{10500}{L(i)}\right)^{0.134}, \quad L(1) = 6838 \tag{8}$$

$$A(i+1) = \frac{A(i)}{1 - 0.079\exp(-0.006 * 5 * (i-1))}, \quad A(1) = 3.8 \tag{9}$$

$$E_{\mathrm{Land}}(i) = 3.3 * 0.8^{(i-1)} \tag{10}$$

$$F_{\mathrm{EX}}(i) = 0.25 + \begin{cases} 0.025(i-1), & i \in [1, 18] \\ 0.45, & i \geq 19. \end{cases} \tag{11}$$

$$\theta_1(i) = \frac{344}{2800} 0.975^{i-1} * \sigma(i). \tag{12}$$

Utility is given by

$$U(C(i), L(i)) = L(i)\left(\frac{\left(\frac{1000C(i)}{L(i)}\right)^{1-\alpha} - 1}{1-\alpha} - 1\right) \tag{13}$$

where the consumption (C) is

$$C(i) = \left(1 - a\,T_{\mathrm{AT}}(i)^2 - \theta_1(i)\mu(i)^{\theta_2}\right) A(i)K(i)^{\gamma}\left(\tfrac{L(i)}{1000}\right)^{1-\gamma}(1-s(i)). \tag{14}$$

[4]The form of the radiative forcing given here is due to the use of an inconsistent discretization of a continuous-time climate model, mixing forward and backward Euler discretizations for the two states, that leads to $T_{\mathrm{AT}}(i+1)$ depending on $M_{\mathrm{AT}}(i+1)$ instead of $M_{\mathrm{AT}}(i)$. Since the aim of this release is to replicate the functionality of [4], we have not corrected this inconsistency.

[5]In [4], θ_1 is called `cost1`.

Optimal pathways are then derived by maximizing the social welfare:

$$\max_{\mathbf{s},\mu} \quad 5 * scale1 * \sum_{i=1}^{60} \frac{U(C(i), L(i))}{(1+\rho)^{5(i-1)}} - scale2 \tag{15}$$

$$\begin{aligned} \text{subject to} \qquad & (2) - (4) \\ & \mu(1) = 0.039 \\ & \mu(i) \geq 0, \qquad i = 2, \ldots, 60 \\ & \mu(i) \leq 1, \qquad i = 2, \ldots, 29 \\ & \mu(i) \leq 1.2, \qquad i = 30, \ldots, 60 \\ & 0 \leq s(i) \leq 1, \qquad i = 1, \ldots, 50 \\ & s(i) = 0.258278, \quad i = 51, \ldots, 60. \end{aligned} \tag{16}$$

The social cost of carbon is given by the ratio of the marginal welfare with respect to emissions and with respect to consumption:

$$\mathrm{SCC}(i) = -1000 \times \frac{\partial W / \partial E(i)}{\partial W / \partial C(i)}. \tag{17}$$

3 Description of Code

DICE2013R-mc consists of three main files:

- `DICE2013R_mc.m` is the top-level file and calls the subsequent two files.
- `set_DICE_parameters.m` is a function that takes the horizon length, N (default $N = 60$), as a parameter and returns all other required parameters[6], including exogenous signals, in the structure `Params`.
- `dice_dynamics.m` is a function that calculates the dynamic states (endogenous signals) of DICE2013R. In addition to the dynamic states, it also calculates the value of the objective function and the quantities required for the social cost of carbon computation as a ratio of marginals; namely the emissions and consumption.

Running DICE2013R-mc in the Matlab command window yields the DICE endogenous states (capital `K`, temperatures `TATM` and `TLO`, and carbon concentrations `MATM`, `MUP`, and `MLO`) and the input values for the mitigation rate (`mu`) and savings rate (`s`). Additionally, the marginals with respect to emissions (`lamE`) and with respect to consumption (`lamC`) are used to calculate the Social Cost of Carbon (`SCC`) and the optimal welfare is given by `J`.

A `clear` command removes many of the variables and other objects used in the solution of the optimal control problem from the workspace. This command can be commented out if these items are required.

As well as the three core component files listed above, two hopefully useful utility files are provided:

- `plot_results.m` generates plots of the exogenous and endogenous signals, as well as the control inputs and social cost of carbon.
- `compute_auxiliary_quantities.m` computes several additional quantities that are available as outputs of the GAMS code [4]. The selected quantities are described below. This file should provide a template for those wishing to define additional quantities of interest.

[6] One minor change in notation has been made in DICE2013R-mc from DICE2013R and this is the indexing into the climate and carbon matrices. DICE2013R uses a non-standard "column-row" numbering for matrices, whereas DICE2013R-mc uses standard "row-column" indexing.

The additional quantities calculated by `compute_auxiliary_quantities.m` are: industrial emissions (IE), net economic output (NEO), per capita consumption (PCC), the damages fraction (DF), atmospheric carbon in parts per million (ACppm), and the marginal cost of abatement (MCA), where

$$\mathrm{IE}(i) = \sigma(i)(1-\mu(i))A(i)K(i)^{\gamma}\left(\frac{L(i)}{1000}\right)^{1-\gamma} \tag{18}$$

$$\mathrm{NEO}(i) = \left(1 - a\,T_{\mathrm{AT}}(i)^2 - \theta_1(i)\mu(i)^{\theta_2}\right)A(i)K(i)^{\gamma}\left(\frac{L(i)}{1000}\right)^{1-\gamma} \tag{19}$$

$$\mathrm{PCC}(i) = \frac{1000 * C(i)}{L(i)} \tag{20}$$

$$\mathrm{DF}(i) = aT_{\mathrm{AT}}(i)^2 \tag{21}$$

$$\mathrm{ACppm}(i) = \frac{M_{\mathrm{AT}}(i)}{2.13} \tag{22}$$

$$\mathrm{MCA}(i) = 344 * (0.975^{i-1}) * \mu(i)^{1.8}. \tag{23}$$

3.1 GAMS Data and Verification Plots

Two additional files are provided with this release for the purpose of demonstrating that DICE2013R-mc replicates the functionality[7] of the publicly available DICE2013R GAMS code [4]. These files are `plot_gams_verification.m` and `GAMS_Results.csv`. The latter contains the output generated by [4] while the former is an extended version of `plot_results.m` that loads and plots the data from DICE2013R against the results of DICE2013R-mc.

The call to `plot_gams_verification.m` is commented out in the release. To view these plots, uncomment the call to `plot_gams_verification.m`.

References

[1] J. Andersson and J. Gillis. CasADi 3.0.0. Available at https://github.com/casadi/casadi/wiki., March 2016.

[2] T. Faulwasser, C. M. Kellett, and S. R. Weller. DICE2013R-mc: Dynamic Integrated model of Climate and Economy 2013R - Matlab and CasADi. Available at https://github.com/cmkellett/DICE2013R-mc., July 2016.

[3] MathWorks. Matlab and simulink tutorials. Online resources at http://www.mathworks.com/support/learn-with-matlab-tutorials.html.

[4] W. D. Nordhaus. Vanilla DICE2103R. Available at http://www.econ.yale.edu/~nordhaus/homepage/Web-DICE-2013-April.htm., November 2013.

[5] W. D. Nordhaus and P. Sztorc. *DICE 2013R: Introduction and User's Manual*, second edition, 31 October 2013.

[7] Note that DICE2013R-mc provides a slightly better (greater) value for the optimal welfare than that provided by the GAMS solution. This may be due to the fact that the default solvers (ipopt for DICE2013R-mc and conopt for DICE2013R) find slightly different local minima.

Parameter	Value	Notes	GAMS Line No.
Climate diffusion parameters			
ϕ_{11}	0.8630	$\mathtt{1 - c1}\left(\frac{\mathtt{fco22x}}{\mathtt{t2xco2}} + \mathtt{c3}\right)$	261
ϕ_{12}	0.0086	$\mathtt{c1} * \mathtt{c3}$	261
ϕ_{21}	0.025	$\mathtt{c4}$	78
ϕ_{22}	0.975	$\mathtt{1} - \mathtt{c4}$	262
Carbon cycle diffusion parameters			
ζ_{11}	0.912*	$\mathtt{b11} = \mathtt{1} - \mathtt{b12}$	139
ζ_{12}	0.03833*	$\mathtt{b21} = \mathtt{b12} * \mathtt{MATEQ}/\mathtt{MUEQ}$ $(= b12 * 588/1350)$	140
ζ_{21}	0.088	$\mathtt{b12}$	55
ζ_{22}	0.9592*	$\mathtt{b22} = \mathtt{1} - \mathtt{b21} - \mathtt{b23}$	141
ζ_{23}	0.0003375*	$\mathtt{b32} = \mathtt{b23} * \mathtt{mueq}/\mathtt{mleq}$ $(= b23 * 1350/10000)$	142
ζ_{32}	0.00250	$\mathtt{b23}$	56
ζ_{33}	0.9996625*	$\mathtt{b33} = \mathtt{1} - \mathtt{b32}$	143
Other parameters			
η	3.8	Forcings of equilibrium CO2 doubling (Wm-2) (`fco22x`)	79
ξ_1	0.098	Multiplier for η (`c1`)	76
ξ_2	5/3.666	Conversion factor for emissions (GtC / GtCO2)	258
$M_{\mathrm{AT},1750}$	588	Pre-industrial carbon in atmosphere (`mateq`)	250
γ	0.3	Capital elasticity in production function (`gama`)	26
θ_2	2.8	Exponent of control cost function (`expcost2`)	89
a	0.00267	Damage quadratic term (`a2`)	85
δ	0.1	Depreciation rate on capital (per year) (`dk`)	30
α	1.45	Elasticity of marginal utility of consumption (`elasmu`)	22
ρ	0.015	Initial rate of social time preference per year (`prstp`)	23
$scale1$	0.016408662	Utility multiplier	107
$scale2$	3855.106895	Utility offset	108