

Tiny groups of galaxies remember their cosmic origins

Detection of the large-scale tidal field with galaxy multiplet alignment in the DESI Y1 spectroscopic survey

Claire Lamman  ^{1*}, Daniel Eisenstein,¹ Jaime E. Barrero-Pomero  ^{2,3}, Jessica Nicols Aquila  ⁴, Steven Ahlen  ⁵, Stephen Bailey  ⁴, Davide Biasi  ⁶, David Brooks  ⁷, Reid Clappinger  ⁸, Axel de la Macorra  ⁸, Peter Doel,⁷ Simone Ferraro  ⁹, Luisa Foscarini  ¹⁰, Enrique Gaztañaga,^{11,12,13} Satya Gorenstein  ¹⁴, Daniel Grinberg  ¹⁵, Kees Honscheid  ¹⁶, Cullan Howlett  ¹⁸, Anthony Krenn  ¹⁹, Andrei Lebedev  ²⁰, Adriana Liguori  ¹⁹, Laurent Loviscek  ¹⁹, Michael E. Levi  ⁴, Aaron Meisner  ²¹, Alexander Miquel  ²², John Moustakas  ²³, Jeffrey A. Newman  ²³, Gustavo Niz  ^{24,25}, Francisco Prada  ²⁶, Ignacio Perez  ²⁷, Ashley J. Ross  ^{15,28,17}, Graziano Rossi,²⁹ Eusebio Sanchez  ³⁰, Michael Scoville  ^{31,32}, David Sprayberry,²⁰ Gregory Tarle  ³³, Mariana Vargas-Magaña  ⁸, Benjamin Alan Weare  ²⁰, He Zou  ³³

Affiliations are listed at the end of the paper

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TL;DR

ABSTRACT

We explore correlations between the orientations of small galaxy groups, or “multiplets”, and the large-scale gravitational tidal field. Using data from the DESI Y1 survey, we find a connection between little groups of galaxies, or “multiplets”, and the largest structures in the universe. This is cool because usually stuff on small scales seems to forget the cosmic web it originated from. We find that all multiplets remember the same large-scale structure, regardless of the type of galaxies in them. This method doesn’t have the main issues that affect similar types of measurements, so it could be a useful way to measure the cosmic web.

The more we know about the cosmic web, the more we know about the stuff that shapes it: like dark energy

Key words: methods: data analysis – cosmology: observations – large-scale structure of Universe – cosmology: dark energy

BACKGROUND INFO

INTRODUCTION

As the universe evolves, gas and dust fall along massive structures of dark matter, forming galaxies and illuminating the cosmic web. The gravity of the cosmic web affects the galaxies that form along it, creating correlations between the two. For instance, a long galaxy will tend to be aligned along a cosmic strand.

tionship with the large-scale tidal field, where long axes are aligned with the tidal direction. For a pedagogical introduction to IA, see

Lamman et al. (2022) and for comprehensive reviews, see Joachimi

(2019) and Slepnev & Ishak (2022). IA can be used as a complement of cosmological probes, such as weak lensing

and redshift-space distortions (RSD), to probe the tidal field and be used to trace any cosmological parameter.

Large-scale density field (Chisari & Dvorkin 2013). Compared to tradi-

tional two-point clustering statistics, IA can probe the large-scale

longitude and polarization of tidal fields as is done with weak lensing. While weak lensing traces all foreground matter,

IA traces specific cosmic data content, such as the tidal field. However, the effect is subtle and requires large samples

However, there are some issues with this approach....

E-mail: claire.lamman@cfa.harvard.edu

and high-quality imaging. IA have been explored as a probe of pri-
Many people have measured connections
between galaxy shapes and the large-scale
structure of the universe. The two main
difficulties they face are:

In some cases it is advantageous to study the alignment of galaxy
ensembles, which can be compared to individuals. The deter-
mined shapes of galaxy ensembles are unaffected by the myriad of
systematic effects which arise from imaging, and are associated with
the shape of the ensemble as a whole, rather than individual galaxies.
I. You need really good pictures of
galaxies to precisely measure their shapes
(this is hard).

Clusters of galaxies also show stronger alignment compared to single galaxies
(Smargon et al. 2012; Fornara et al. 2021; Lee et al. 2023). Clusters

of galaxies show no correlation. No one has been able to make this measurement with spiral galaxies.

In this work we explore the potential of using galaxy “multiplets”: small sets of galaxies, mostly consisting of 2-4 members within 1 h^{-1} Mpc of each other (Fig. 1). We expect these tiny ensembles to still preserve information from the large-scale tidal field, while being more abundant than larger groups. Multiplets are not necessarily virialized systems, but can be understood in the IA framework as they are well with the initial tidal field.

Instead of galaxy shapes, let's try using the orientation of tiny groups of galaxies instead, or the sample displays little or no individual alignment, as is the case for spiral (or “blue”) galaxies. The latter of these applies to most available spectroscopic samples beyond DESI. Our understanding of the redshift evolution of IA is an important component to fully utilising forthcoming cosmic shear surveys (Dark Energy Survey and Kilo-Degree Survey Collaboration et al. 2023). However the redshift evolution of IA is unclear and there is no direct IA detection beyond redshift 1 with traditional estimators.

We describe and model this estimator, but this work is also related to the fields of both galaxy groups and higher-order clustering. Although multiplets are not galaxy groups, which are virialized systems and typically describe more complete sets of galaxies (Oppenheimer et al. 2021), multiplets exist on similar scales. The orientation of close pairs of galaxies within multiplets are identified in dense samples. Furthermore, the nonlinear dynamics within groups directly affect the amplitude of multiplet

DESI, or the “Dark Energy Spectroscopic Instrument”, sits atop a mountain in Arizona. Inside it are 5000 individually-controlled robots. These allow us to measure the distances to thousands of galaxies in mere minutes.

work has explored 3-point and higher-order correlations in spectroscopic data (Slepian & Eisenstein 2015; Philcox et al. 2022), including detecting evidence of the tidal field (Slepian et al. 2017) and DESI is in the middle of its 5-year survey, but has already created the most detailed map of the nearby universe!

(Levi et al. 2013; DESI Collaboration et al. 2016a,b, 2022, 2023; Miller et al. 2023). To explore the potential of multiplet IA, we measure the tidal alignment of multiplets in DESI’s catalogues.

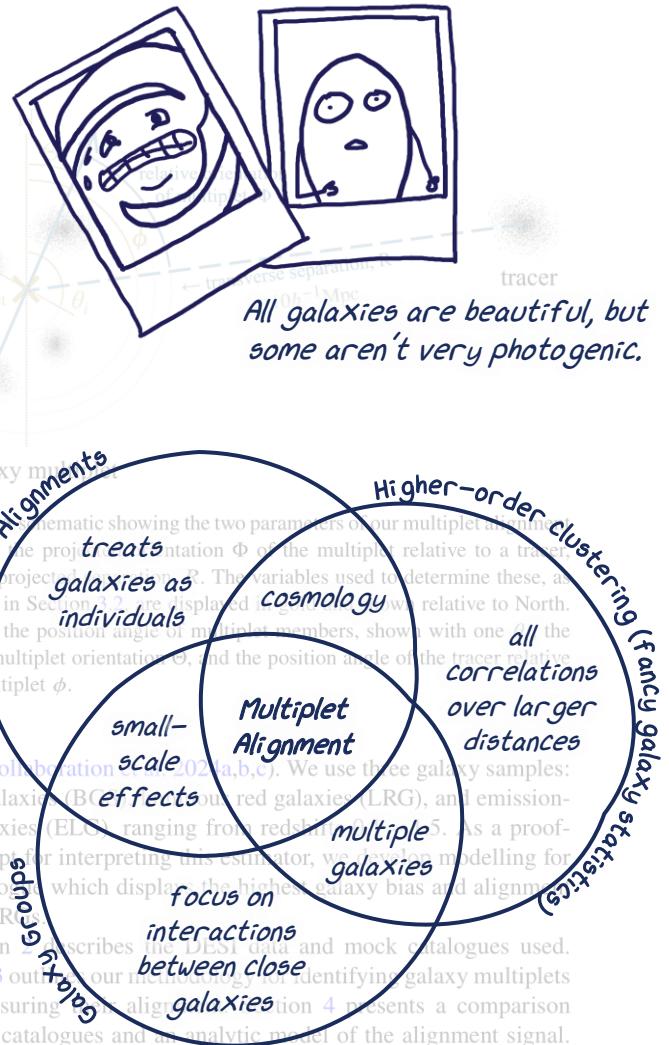


Figure 1: Schematic showing the two parameters of our multiplet alignment estimator: the projected orientation Φ of the multiplet relative to a tracer, and the projected orientation ϕ . The variables used to determine these, as described in Section 2, are displayed below. The North arrow indicates the position angle of multiplet members, shown with one of the reduced-multiplet orientation Θ , and the position angle of the tracer relative to the multiplet ϕ .

Section 2 describes the DESI data and mock catalogues used. Section 3 outlines our method for identifying galaxy multiplets and measuring multiplet alignment. Section 4 presents a comparison to mock catalogues and an analytic model of the alignment signal. Section 5 summarizes key results and discusses prospects for utilising future datasets.

This idea is built upon the work of past scientists and has many connections to it.

$$H_0 = 69.6, \Omega_{m,0} = 0.286, \Omega_{\Lambda,0} = 0.714.$$

WHERE DOES OUR DATA COME FROM?

2 DESI CATALOGUES

DESI’s targets are chosen from DR9 of the Legacy Imaging Survey (Dey et al. 2019; Myers et al. 2023). For more information on DESI’s target selection, see DESI Collaboration et al. (2023a,b). We use three galaxy samples: bright galaxies (BGS), luminous red galaxies (LRG), and emission-line galaxies (ELG), ranging from redshift 0.3 to 0.55. As a proof-of-concept for interpreting this estimator, we develop modelling for the catalogues which display the highest galaxy bias and alignment signal, LRGs.

Section 2 describes the DESI data and mock catalogues used. Section 3 outlines our method for identifying galaxy multiplets and measuring multiplet alignment. Section 4 presents a comparison to mock catalogues and an analytic model of the alignment signal. Section 5 summarizes key results and discusses prospects for utilising future datasets.

For exploring the difference between red and blue galaxies, we use the redshift catalogues of the DESI Survey (Dey et al. 2023; Zhou et al. 2023; Raichoor et al. 2023). The catalogues also include weights to account for redshift, fibre and the probability that each target was observed. When fitting our measurements, we apply these weights to all tracer samples.

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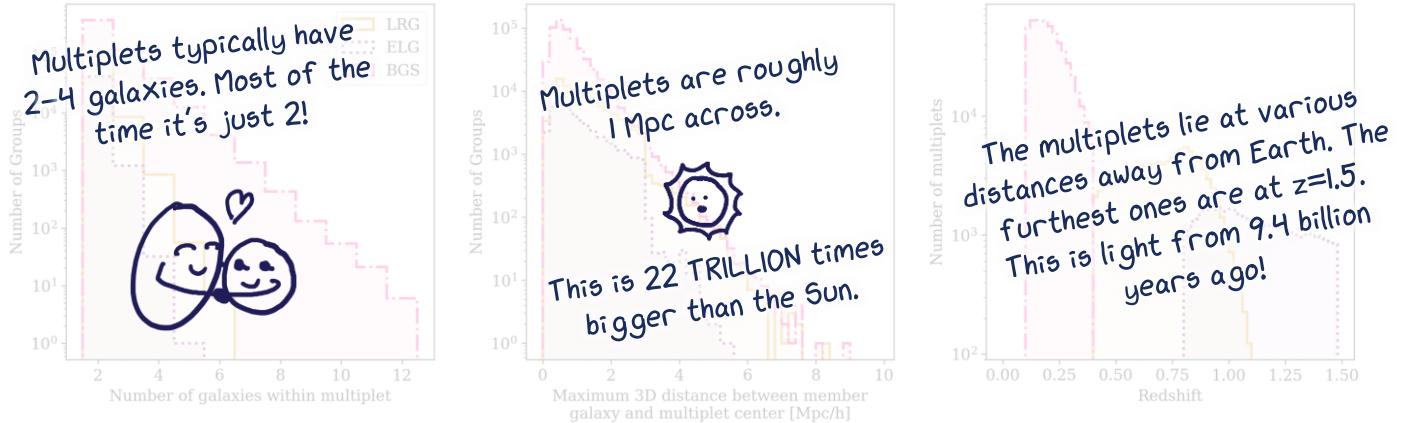


Figure 2. These plots show information about the galaxy multiplets we find, composed of only two members, even for the densest sample, BGS, where 70% of multiplets are galaxy pairs. The spatial size of multiplets is shown in the middle panel, which is described by the maximum 3D distance between member galaxies. The right panel shows the redshift distribution of multiplets.

Galaxy type	Redshift Range	N galaxies	N multiplets	Volume [Gpc $^3 h^{-1}$]
Type of Galaxy				
ELG	$0.1 < z < 1.5$	5 M	21 K	67.8
ELG	$0.8 < z < 1.1$	1.2 M	22 K	35.8
BGS	$0.4 < z < 1.1$	2.2 M	105 K	34.6
How far away they are				
			"close"	
			far away	
			REALLY far away	
How dense the sample is				
ELG	$0.3 < z < 0.4$	0.6 M	64 K	3.2
BGS	$0.1 < z < 0.2$	1.4 M	12 K	0.5
BGS Blue	$0.1 < z < 0.2$	0.56 M	81 K	0.5
BGS Red	$0.1 < z < 0.2$	0.54 M	100 K	0.5

Table 1. Properties of the DESI catalogues used to identify galaxy multiplets. The right column shows the comoving volume of the sample, estimated from the positions of galaxies in the data. The color of the sample is used to make the BGS Blue and Red samples are described in Section 2.

We found multiplets using samples of many types of galaxies!

3 ALIGNMENT METHOD

3.1 Identifying galaxy multiplets

Our measurement is a projected quantity, along the orientation of multiplets in the plane of the sky as a function of transverse distance (Figure 1). However, we identify small multiplets of galaxies in 3D comoving space using spectroscopic redshifts. Each galaxy is matched to its nearest neighbour and all pairs are limited to a maximum separation in the plane of the sky, r_p , and along the line of sight, r_{\parallel} . r_{\parallel} is necessarily larger than r_p to account for the redshift-space distortions created by peculiar velocities of multiplet members. We then find multiplets within these matches using a friends-of-friends algorithm (Geller & Fisher 1964). We set no maximum for the number of multiplet members. This is similar to the friends-of-friends algorithm used for identifying haloes in N-body simulations and for constructing group catalogues (Davis et al. 1985; Eke et al. 2004; Robotham et al. 2011). Note that unlike these catalogues, our goal is not to identify complete, gravitationally bound objects. We expect even nonvirialized objects to contribute to our final measurement and so set no additional criteria such as completeness or velocity dispersion.

To explore the effectiveness of this algorithm to identify distinct multiplets, we created a catalogue of isolated multiplets, consisting only of multiplets where each member was a minimum of $2r_p$ and $2r_{\parallel}$ away from the nearest non-multiplet member. This had no significant effect on final results. We tested multiplets constructed from varying criteria, between $0.5 < r_p < 1.5 h^{-1}$ Mpc and $0.5 < r_{\parallel} < 1.5 h^{-1}$ Mpc. We found no significant effect on the amplitude of the final signal when varying these parameters, so we selected cuts to maximize the signal-to-noise ratio. We model in Section 4, LRGs. For all samples, we use $r_p = 1.0 h^{-1}$ Mpc and $r_{\parallel} = 6.0 h^{-1}$ Mpc. At this point, such as scale cuts which depend on density or removing very close pairs (see Section 4.1) may improve the SNR and are worth exploring in future works.

This page shows information

Properties of the DESI samples we identified multiplets in are shown in Table 1. The properties of the multiplets shown in Fig. 2. This displays the number of members within each multiplet, which is most often two. It also shows the spatial size of each mul-

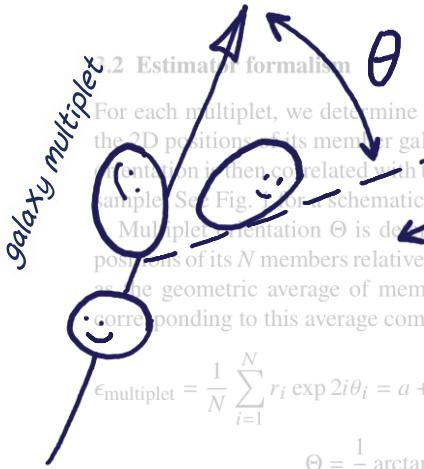
WHAT ARE MULTIPLETS AND HOW DO WE FIND THEM?

Multiplets are little sets of galaxies: as in a doublet, triplet, quadruplet, etc.

We find them by using the 3D positions of galaxies measured by DESI. The basic idea is to find the closest neighbor to each galaxy and then use an algorithm to identify sets within those connections.

WHAT EXACTLY ARE WE MEASURING?

Measured by taking the maximum 3D distance between a multiplet member and the multiplet's centre in redshift space. Note that this can be greater than r_p and r_{\parallel} since we only limit the distance between multiplet members, not its overall size.



For each multiplet, we determine its projected orientation based on the 2D positions of its member galaxies in the plane of the sky. This orientation is then correlated with the position of other galaxies in a tracer sample. See Fig. 1 for a schematic of the variables used.

Multiplet orientation Θ is determined by averaging the complex positions of its N members relative to its centre. This centre is defined as the geometric average of member 2D positions. Θ is the angle corresponding to this average complex number:

$$\epsilon_{\text{multiplet}} = \frac{1}{N} \sum_{i=1}^N r_i \exp 2i\theta_i = a + bi$$

$$\Theta = \frac{1}{2} \arctan \frac{b}{a}$$

For each member i , r_i is the projected distance to the multiplet centre

Our measurement comes down to two numbers: for multiplets with two members and it is not expected to increase our signal-to-noise ratio. For single galaxies, measurements with the SDSS-III BOSS LOWZ sample have found that this is the case (see this in the paper 2015).

I. The orientation of a multiplet relative to a tracer, Θ
This orientation angle is then measured relative to the tracer sample. In most cases, the tracer sample is the same as the one used to identify multiplets. For each pair, consisting of a galaxy multiplet and a tracer, the angle between the position

2. The distance between a multiplet and a tracer, R
The distance between the position angle of the multiplet relative to the tracer, ϕ , and the multiplet's orientation:

We find millions of multiplet-tracer pairs and find the average orientation as a function of separation.

The values of Θ and R are then separated by the multiplet-tracer separation, R . This is similar to conventions in intrinsic alignments and ensures the relative angle is invariant under rotation. Since we are mostly measuring the orientation of a pair of galaxies, we use the angle between the multiplet and the tracer, Θ .

We expect that multiplets will "point" towards areas of high density, as traced by other galaxies. And that

The effect will be strongest when a multiplets is close to a tracer.

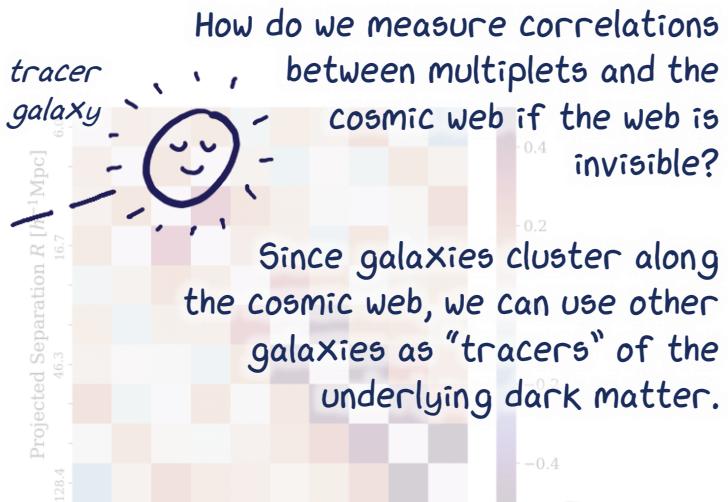
3.3 Measurement

Just like long galaxies!

When measuring the projected orientation of multiplets relative to a tracer catalogue, we limit the multiplet-tracer pairs to a line-of-sight separation that is unique to each bin of projected separation, $\Pi_{\max}(R_{\text{bin}})$. This is to maximise the signal-to-noise of our measurement. In the case of positive tidal alignment, shapes are elongated along the stretching direction of the tidal field. In this situation, the tidal field along the line of sight has a measurable orientation in the plane of the sky. Therefore, multiplet-tracer pairs that are close in the plane of the sky but distant along the line-of-sight direction will have a relatively low contribution to the total alignment.

WHAT ARE THE MEASUREMENTS?

The measurements are.... on the next page! But there's a sneak-peak here.



Since galaxies cluster along the cosmic web, we can use other galaxies as "tracers" of the underlying dark matter.

If a multiplet points towards a tracer $\rightarrow \Theta = 0 \rightarrow \cos(2\Theta) = 1$
 If multiplets have random orientations, on average: $\rightarrow \cos(2\Theta) = 0$

Figure 3. The reduced covariance matrix corresponding to the LRG signal (1). If the multiplet orientation has been subtracted from the signal, the covariance in the multiplet alignment measurement between bins of projected separation is particularly in the largest bins.

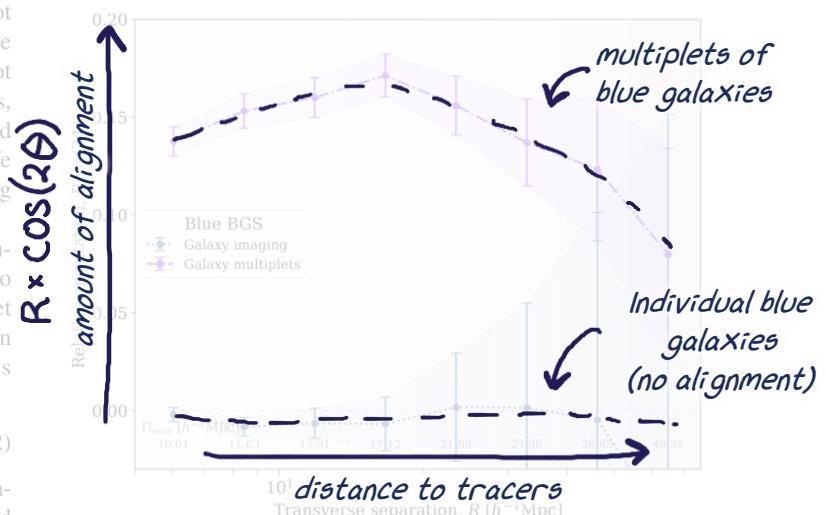
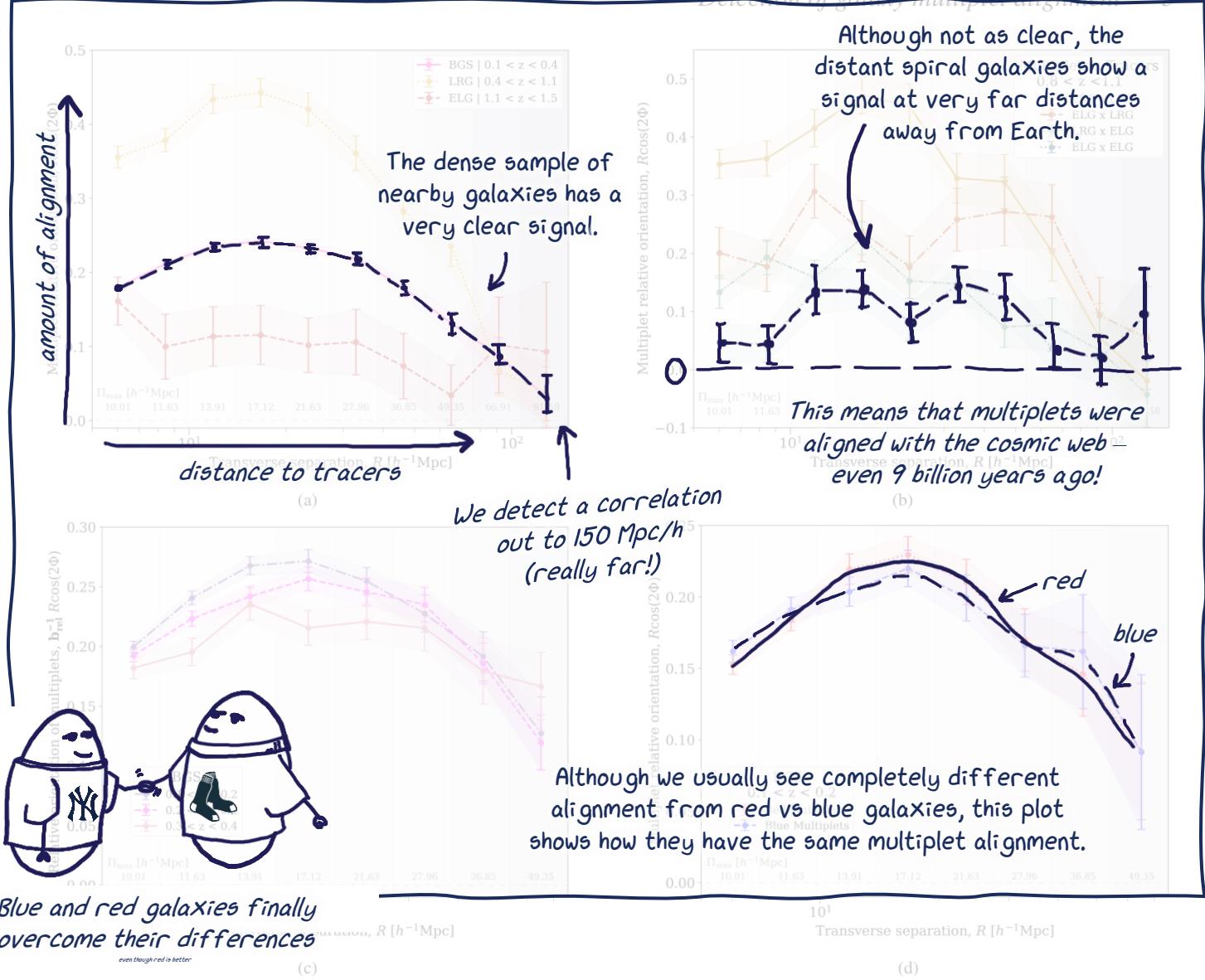


Figure 4. A demonstration of the advantages of using multiplet alignment. Here we show the tidal alignment of galaxy and multiplet orientations within a dense, blue sample. The alignment of individual blue galaxies is highly sensitive to the alignment of multiplets. Why so blue?? Here we show how we can detect a correlation to the cosmic web using multiplets of blue galaxies.

This is much better than the alignment of individual blue galaxies, which is 0. We chose $\Pi_{\max}(R_{\text{bin}}) = 6h^{-1}\text{Mpc} + \frac{2}{3}R_{\text{bin}}$ based on the signal-to-noise ratio of our final LRG signal. Our model estimate is computed in these same R bins. Throughout plots in this paper, the varying values of Π_{\max} are shown through shaded regions and marked explicitly. We use this projected statistic, as opposed to keeping the measurement as a function of r_p and r_{\parallel} , because most of the signal is along the LOS for tidal alignments due to the projection of shapes. Additionally, a projected statistic allows for more direct modelling as it is less sensitive to redshift-space distortions (Figure 7).

For each measurement, we separate the multiplet catalogue into 100 sky regions by right ascension and declination, with equal numbers of multiplets in each. The orientation of multiplets is measured



Our measurements reveal that multiplets are indeed aligned with the cosmic web!!

These plots are the result of us playing around and making this measurement with many types of galaxies. Here I've highlighted a few interesting results.

Figure 5. Correlations between the orientation of multiplets and the projected separation, R . The measurement in each R bin utilised a different value of Π_{\max} , indicated by the shaded regions and marked at the bottom of each plot. Π_{\max} is the maximum line-of-sight distance between the two galaxies in the multiplet. This allows us to compare multiplet orientations and density tracers. (a) The signal for each tracer type, with no adjustments made for differences in clustering between samples. LRGs have the highest galaxy bias and their signal is the one we focus on reproducing. (b) explores cross-correlations between ELGs multiplets, ELG tracers, LRGs multiplets, and LRG tracers in their overlapping region, $0.8 < z < 1.1$. Based on the comparison in (d), we expect similar scale-dependence of these alignments. Redshift subsets of the BGS sample are shown in (c). Here we account for differences in the galaxy bias and its evolution by scaling each measurement relative to the bias in the middle redshift bin. From lowest to highest redshift bin, the rescaling factors are 1.12, 1.0, and 0.80. (d) displays the alignment of multiplets in red and blue subsamples of the lowest redshift BGS galaxies, relative to the full BGS sample. We find no obvious difference in scale dependence, demonstrating the potential of utilising blue galaxies in the analysis.

separately in each region, but relative to the full tracer sample. Our final measurement is the mean and standard error of these 100 measurements. For the densest sample (BGS), we use 147 regions. At each distance we compute the average signal in each R_{1D} before averaging over the 147 regions. This marginally increases the measurement noise but is more practical for samples with many multiplet-tracer matches.

This shows that tiny groups of galaxies still remember the cosmic web connected to their formation in the early universe. completeness variation can be found in DESI Collaboration (2024).

We find the signal to be sensitive to survey geometry on large scales. To account for this, for every measurement we also measure the orientation of galaxy multiplets relative to random catalogues designed to match DESI's Y1 footprint. The range of measurements in multiple random catalogues is shown in Figure 5. Across samples, we see a turnover in the multi-random signal around $80 h^{-1}\text{Mpc}$. We see no evidence of anisotropy in the orientations of multiplets, so this systematic "tangential alignment" at large separations is likely to be due to the footprint of the tracers, which spans a narrow band in right ascension. This pattern is not present when measuring the signal in isolated square regions.

Now onto the hard part, interpreting the measurements...

WHAT DOES ALL THIS MEAN?

The rest of our paper explores what is actually happening in the universe to create these correlations. Our goal is to quantitatively connect our measurements to the cosmic web. At dependence. For each measurement, we use the same galaxies to construct multiplets and the tracer catalogue. The exception is the overlapping LRG and ELG region of $0.8 < z < 1.1$, where we measure each cross-correlation between the samples (Fig. 5b). Although we expect each of these signals to display a similar scale dependence, we find that the redshift dependence of the sample. Therefore we examine the redshift dependence of the denses of our catalogues BGS.

A challenging part of this is that stuff on small scales behaves differently than stuff on large scales.

We split the sample into two sets of samples (Fig. 5c). In this plot we account for the galaxy bias and its evolution across redshifts. Intrinsic properties of BGS also vary across redshift, so this plot should not be interpreted as a redshift evolution. For instance, the highest redshift bins contains the most luminous galaxies, which are known to display higher alignment. Despite the high signal from the low-mass, low-redshift galaxies. Galaxies in the lowest redshift bin have stellar masses of around $10^{10.75}$ (Hahn et al. 2020). Fig. 5 shows the multiplet alignment vs redshift in this same sample. The colour cuts selection of these subsamples are described in Section 2 and their alignments were measured relative to the full BGS catalogue. The blue samples display similar latitudes and scale dependence. This shows that blue galaxies can be used to trace the tidal field of red galaxies, which is a promising result for measurements beyond redshift 1.

To cosmologists, galaxies are just lil' guys

aligning to form multiplets to be well-suited to samples that are especially dense and samples of blue galaxies. To directly demonstrate this, within the BGS Blue sample we measure the intrinsic alignment of individual galaxies using imaging from the Legacy Imaging Survey. Here we find some evidence for alignment geometry. This is accounted for through the large statistical error at large separations. As expected, the alignment of these faint blue galaxies is consistent with zero at all separations. However, galaxy multiplets in the same catalogue display a clear alignment signal (Fig. 4).

The biggest things in the universe are surprisingly easy to predict. Dark matter doesn't interact with itself, so most changes just come from gravity. However, the realm of galaxies and galaxy clusters is messy. Stuff is colliding, turning into stars, falling into black holes, condensing into people, becoming self-aware and reading doodle summaries of papers... It's all very complicated.

COMPARING TO SIMULATIONS

4 INTERPRETATION

In this Section we explore the modelling of multiplet alignments through simulations and theory, using the LRG sample as a case study. This is because LRGs have a large galaxy bias, they display the alignment of multiplets looks like in simulated universes.

4.1 Comparison to simulations

The simulations we use are designed to reproduce statistics about the universe on very large scales. So they only include dark matter.

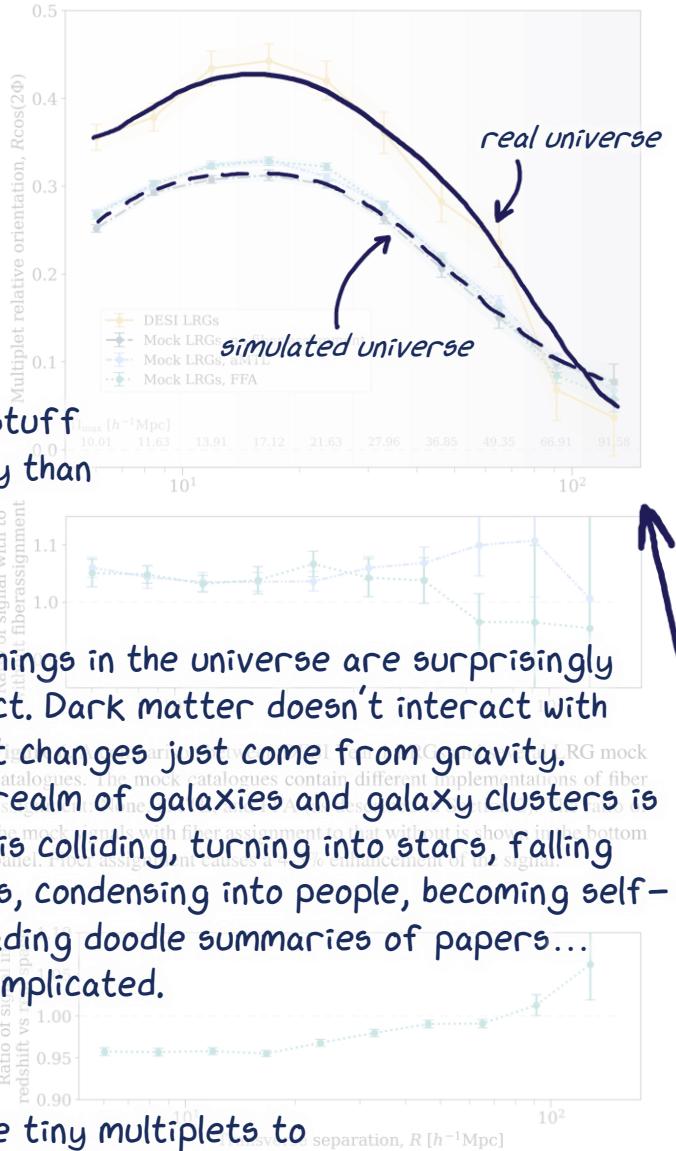


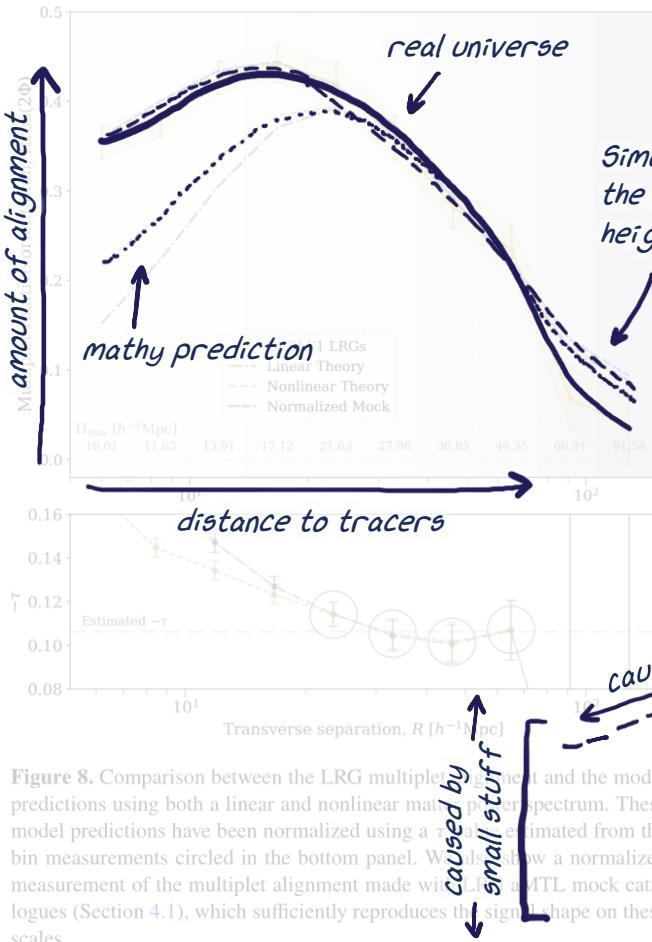
Figure 7. This is an assessment of the impact of RSD on the multiplet alignment signal. Here we plot the ratio between the aMTL signal in Fig. 6 and a version where the shape-tracer correlations were measured in real space. The two measurements differ by about 5% on these scales.

average measurement of 25 simulations for each mock catalogue and their standard deviations. We find no significant difference in the number of multiplets found, but we do find a significant difference in the alignment signal. This reflects the fact that the simulations work just fine. However, we find that multiplet correlations in the simulated universe are lower than in the real universe!

aMTL is the most realistic simulation of fiber assignment, but we do not find a significant difference between the two mock catalogues which include fiber assignment. It is interesting to note that the signal is marginally higher for the fiber assignment catalogues, which can be seen in the lower panel of Fig. 6. This is probably because galaxies very close to a multiplet's centre are more affected by nonlinearity dynamics and therefore have stronger correlations. Individual galaxies display higher alignment in their outer regions for the same reason (Singh & Mandelbaum 2016; Georgiou et al. 2019).



You shouldn't always trust computers....



removing some of this dilution. Instead this time we consider the separation of the initial pairs using a mock LRG multiplet catalogue to $r_p > 0.5 h^{-1} \text{Mpc}$ and find a similar enhancement of the signal, 10% between $6 - 60 h^{-1} \text{Mpc}$. This may be a useful addition to future studies of multiplet alignment.

We do not include the effects of RSD in our analytic model (Section 4), so to test this assumption we reproduce the aMTL measurement in real space. Here, galaxy multiples are still found in redshift space, but the multiplet-tracer correlations are measured using the true positions of the multiplet centres and tracers. The effects of RSD on the tracer catalogue appear to make a 0 – 5% difference on scales beyond $10 h^{-1} \text{Mpc}$ (Fig. 7).

INTERPRETATION WITH MATH

4.2 Modeling

To quantify the connection between multiplet orientation and the underlying matter distribution, we assume a linear relationship between shapes:

Besides simulations, we can also use math!
Here we model the 3D gravitational field created by large-scale structure and then derive an equation which describes how different components in the universe combine to produce our measurement.

Following the convention in Liddle et al. (2020, 2024), we

describe the traceless tidal tensor as

$$T_{ij} = \partial_i \partial_j \phi - \frac{1}{3} \delta_{ij}^K \nabla^2 \phi, \quad (3)$$

where $\nabla^2 \phi \propto \delta$ is given by the Poisson equation, with δ being the amplitude of the correlation function δ_K in the Kronecker delta. In Fourier space this is expressed as

$$T_{ij}(\vec{r}) = \int \frac{d^3 k}{(2\pi)^3} \left(\frac{k_i k_j - \frac{1}{3} \delta_{ij}^K k^2}{k^2} \right) \tilde{\delta}_m(\vec{k}) e^{i\vec{k} \cdot \vec{r}}, \quad (4)$$

where we have used $\tilde{\delta}$ to indicate a variable in Fourier space. Our measured signal is a projected quantity, where we define \hat{z} to be along the line-of-sight. Therefore, for a projection with $\alpha, \beta = \{x, y\}$ and using the relation $T_{xx} + T_{yy} = -T_{zz}$, the relevant projection of the tidal field is $(T_{\alpha\beta} + T_{zz}/2)$.

In this study we characterize the relevant “shapes” of objects solely by orientation, instead of the full ellipticity. This axis-ratio component of shapes affects the amplitude of the signal as does any systematic misalignment of galaxy multiplets to the large-scale field caused by local dynamics. Our focus in this work is to explore how multiplet alignment traces the tidal field across large scales, without any assumptions about the effect’s amplitude. Therefore, we fold in the full ellipticity information and any misalignment effects into the signal amplitude, assuming neither display scale dependence at large separations. This is similar to the “stick model” employed for describing the positions and alignments of galaxies within haloes (Fortuna et al. 2021; Schneider & Bridle 2010).

The projected ellipticity of galaxy multiplets can be described by the traceless tensor

$$\epsilon_{\alpha\beta} = \tau (T_{\alpha\beta} + \frac{1}{2} T_{zz}). \quad (5)$$

removing some of this dilution. Instead this time we consider the separation of the initial pairs using a mock LRG multiplet catalogue to $r_p > 0.5 h^{-1} \text{Mpc}$ and find a similar enhancement of the signal, 10% between $6 - 60 h^{-1} \text{Mpc}$. This may be a useful addition to future studies of multiplet alignment.

in Section 3.2, and τ is a parameterization of the shape’s response to the tidal field. It includes any effects from the full-shape information and any misalignment effects to the tidal direction. The full complex ellipticity is described as

$$\epsilon = \tau [T_{xx} - T_{yy} + 2iT_{xy}]. \quad (6)$$

The quantity of interest is the expectation value of the cross-correlation between projected shapes and the matter field, Q :

$$\mathcal{E}_{\text{model}} = \frac{1}{2} \langle \epsilon^* Q + \epsilon Q^* \rangle. \quad (7)$$

We describe the 3D matter field in a particular bin of transverse separation R_{bin} and line-of-sight separation $\pm \Pi_{\text{max}}$ as

$$Q(R_{\text{bin}}, \pm \Pi_{\text{max}}) = \frac{\int d^3 r W(\vec{r}) \delta_g e^{2i\theta_r}}{\int d^3 r W(\vec{r}) (1 + \xi_{\epsilon g})}. \quad (8)$$

Here, δ_g is the fractional matter overdensity, $\xi_{\epsilon g}$ is the shape orientation – galaxy correlation function, r is the 3D separation, and θ_r is the 3D relative angle. $W(\vec{r})$ is a function representing the bin

function, both an annulus in R and $\pm \Pi_{\text{max}}$. \vec{r} is used to denote a binned quantity. The expansion of ϵ in terms of Q can be found in Appendix A, and results

$$\mathcal{E}_{\text{model}} = \frac{-\tau}{\int d^3 r W(\vec{r}) (1 + \xi_{\epsilon g})} \int d^3 r W(\vec{r})$$

$$\int \frac{dk_z}{2\pi} \int K dK J_2(KR) \frac{K^2}{k^2} P_{gm}(k) e^{ik_z z},$$

where J_2 is the second Bessel function of the first kind and $P_{gm}(k)$

is the galaxy-matter power spectrum. k represented 3D position in Fourier space, K represents the 2D position on the plane of the sky (k_x, k_y), and k_z lies along the line of sight. $k^2 = K^2 + k_z^2$.

The remainder of this Section describes how we compute Equation 9, by breaking it into the components we measure or calculate. Beginning with the denominator,

$$\int d^3r W(\bar{r})(1 + \xi_{\epsilon g}) = \pi(R_{\max}^2 - R_{\min}^2)(2\Pi_{\max} + \bar{w}_P).$$

\bar{w}_P is the integrated 2-point cross-correlation function between the multiplet and tracer catalogue, $w_P(R)$, within an annulus of R_{\min} and R_{\max} :

$$\bar{w}_P(R_{\text{bin}}) = \frac{1}{\pi(R_{\max}^2 - R_{\min}^2)} \int_{R_{\min}}^{R_{\max}} 2\pi R dR w_P(R). \quad (11)$$

We further define \mathcal{J}_2 , a binned version of the second Bessel function integrated over a given R_{bin} :

$$\mathcal{J}_2(K) = \frac{2}{(R_{\max}^2 - R_{\min}^2)} \int_{R_{\min}}^{R_{\max}} R dR J_2(KR), \quad (12)$$

This can be solved analytically (Equation A8). Using the relation

$$\frac{1}{2\Pi_{\max}} \int_{-\Pi_{\max}}^{\Pi_{\max}} dz e^{ik_z z} = \text{sinc}(k_z \Pi_{\max}), \quad (13)$$

we further define an expression of the relevant matter distribution for a given Π_{\max} :

$$\mathcal{P}_{\Pi}(K) = 2\Pi_{\max} \int \frac{dk_z}{2\pi} \frac{K^2}{K^2 + k_z^2} (\text{the signal we measure}) \quad (14)$$

In practice, for this we use the matter power spectrum and galaxy bias $b_g P_{mm}(k)$, with $b_g = 1.99$ for DESI LRGs (Mena-Fernandez et al. 2024). Combining these expressions, the model prediction for our signal $\mathcal{E}(R)$ is simplified to

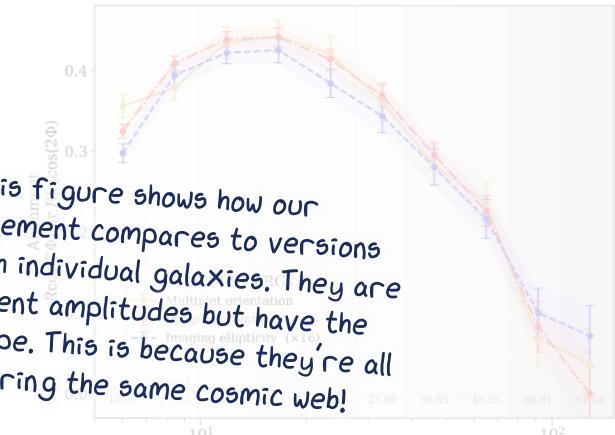
$$\mathcal{E}_{\text{model}}(R) = \frac{-\tau}{(2\Pi_{\max} + \bar{w}_P)} \int K dK \mathcal{J}_2(K, R) \mathcal{P}_{\Pi}(K). \quad (15)$$

We compute this numerically in bins of (R_{\min}, R_{\max}) with the corresponding Π_{\max} value in each. The model prediction made with both a linear and nonlinear matter power spectrum can be seen in Fig. 8. The power spectra are from ABACUSUMMIT and evaluated at $z = 0.8$. We normalize the models by taking their ratio to the large-scale signal, using the points circled in the lower panel of Figure 8. This results in an estimate τ for the LRG multiplets of -0.106 ± 0.002 for both LA and NLA. We find that these models can sufficiently match the shape of our measurement only down to scales of $20 h^{-1}\text{Mpc}$, while the LRG mock catalogue matches below $10 h^{-1}\text{Mpc}$. The corresponding τ value for this mock is also -0.106 ± 0.002 . Therefore the NLA model is sufficient for very large scales, but fails to capture the non-linear dynamics between multiplets and tracers in the way that an N-body simulation can.

The alignment amplitude is often characterized with A_{IA} (Catelan & Porciani 2001; Hirata & Seljak 2004; Blazek et al. 2015). A_{IA} describes the relationship between intrinsic galaxy shear, γ_{ij}^I , with the tidal tensor, T_{ij} , as defined in Eq. 3. In the case of “early alignment”, it is assumed that shapes are aligned at time of formation and then evolve with the matter field.

$$\gamma_{ij}^I = -A_{IA}(z) C_1 \frac{\rho_{m,0}}{D(z)} T_{ij} \quad (16)$$

Here, $\rho_{m,0}$ is the matter density, $D(z)$ is the growth factor, normalized so $\bar{D}(z) = (1+z)D(z)$ is unity at matter domination, and C_1 is a



This figure shows how our measurement compares to versions made with individual galaxies. They are different amplitudes but have the same shape. This is because they're all measuring the same cosmic web!



the variable which describes small-scale effects, or the extent to which a multiplet responds to the exterior gravitational forces

Figure 9. Here we compare the alignment of galaxy multiplets to the alignment of individual galaxies. The yellow line shows the alignment of the orientations of PG multiplets relative to positions of full sample (Section 3.2). The red line is the orientations of individual LRGs relative to positions of full sample. The blue line is the full shape alignment of LRGs, taking into account galaxy axis ratio and multiplied by 16 comparison. The bottom panel shows the difference in the points plotted above, highlighting the similar scale-dependence of each estimator. The average signal-to-noise for each of these measurements alone is 9.0 for imaging, and 11.0 for imaging with ellipticity. These measurements were made with 105 thousand multiplets and 2.2 million individuals LRGs.

gravitational force created by the cosmic web

historical normalization constant $A_{IA}(z) = -\frac{\tau}{C_1} \frac{D(z)}{\rho_{m,0}}$ (Brown et al. 2015). The relationship to our alignment amplitude τ is

$$A_{IA}(z) = -\frac{\tau}{C_1} \frac{D(z)}{\rho_{m,0}} \quad (17)$$

For our “stick” model of LRG multiplets, this corresponds to an average value of $A_{IA} = 5.7 \pm 0.1$. For reference, the corresponding stick alignment of the same sample using individual galaxies and Legacy Survey Imaging is $A_{IA} = 1.96 \pm 0.001$, about 5 times higher than when using the full-shape information (Fig. 9). For this measurement we use the ellipticity definition

$$\epsilon_+ = \frac{a - b}{a + b} \cos 2\theta, \quad (18)$$

based on the galaxy major and minor axis, a and b , and orientation, θ .

Fig. 9 is also a useful demonstration of how, although very different amplitudes, the alignment of multiplet orientation has the same scale dependence of shape alignment and can be modeled similarly. Additionally, multiplet alignment produces a comparable signal-to-noise measurement as full shape alignment, with less than 5% of the objects. While multiplet alignment does not necessarily outperform individual galaxies within the LRG sample, it is promising for dense regions or samples that show weaker intrinsic galaxy alignment.

**Our acknowledgement section is as long as the conclusions!
It takes the support of many organizations and people to
make a big survey like DESI possible.**

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SEE THE DATA YOURSELF

DATA AVAILABILITY

The DESI Legacy Imaging Survey is publicly available at legacysurvey.org. You can find the publicly available data we used here at data.desi.lbl.gov/doc/releses/dr1/. It covers the DESI Year 1 sample and will be released as part of DESI Data Release 1 (DR1) (DESI Collaboration 2025). ABACUSSUMMIT simulations are available at abacusbody.org.

Data plotted in this paper can be downloaded from zenodo.org/records/13230864.

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Author Affiliations

- ¹Center for Astrophysics | Harvard & Smithsonian, 60 Garden Street,
 Cambridge, MA 02138, USA
²Departamento de Física, Universidad de los Andes, Cra. 1 No. 18A-
 10, Edificio Ip, CP 111711, Bogotá, Colombia
³Observatorio Astronómico, Universidad de los Andes, Cra. 1 No.
 18A-10, Edificio H, CP 111711 Bogotá, Colombia
⁴Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Berkeley,
 CA 94720, USA
⁵Physics Dept., Boston University, 590 Commonwealth Avenue,
 Boston, MA 02215, USA
⁶Dipartimento di Fisica “Aldo Pontremoli”, Università degli Studi
 di Milano, Via Celoria 16, I-20133 Milano, Italy
⁷Department of Physics & Astronomy, University College London,
 Gower Street, London, WC1E 6BT, UK
⁸Instituto de Física, Universidad Nacional Autónoma de México, Cd.
 de México C.P. 04510, México
⁹University of California, Berkeley, 110 Sprout Hall #5800 Berkeley,
 CA 94720, USA
¹⁰Institut de Física d’Altes Energies (IFAE), The Barcelona Institute
 of Science and Technology, Campus UAB, 08193 Bellaterra
 Barcelona, Spain
¹¹Institut d’Estudis Espacials de Catalunya (IEEC), 08034
 Barcelona, Spain
¹²Institute of Cosmology and Gravitation, University of Portsmouth,
 Dennis Sciama Building, Portsmouth, PO1 3FX, UK
¹³Institute of Space Sciences, ICE-CSIC, Campus UAB, Carrer de
 Can Magrans s/n, 08913 Bellaterra, Barcelona, Spain
¹⁴Fermi National Accelerator Laboratory, PO Box 500, Batavia, IL
 60510, USA
¹⁵Center for Cosmology and AstroParticle Physics, The Ohio State
 University, 191 West Woodruff Avenue, Columbus, OH 43210, USA
¹⁶Department of Physics, The Ohio State University, 191 West
 Woodruff Avenue, Columbus, OH 43210, USA
¹⁷The Ohio State University, Columbus, 43210 OH, USA
¹⁸School of Mathematics and Physics, University of Queensland,
 4072, Australia
¹⁹Sorbonne Université, CNRS/IN2P3, Laboratoire de Physique Nucléaire et de Hautes Energies (LPNHE), FR-75005 Paris, France
²⁰NSF NOIRLab, 950 N. Cherry Ave., Tucson, AZ 85719, USA
²¹Institució Catalana de Recerca i Estudis Avançats, Passeig de Lluís
 Companys, 23, 08010 Barcelona, Spain
²²Department of Physics and Astronomy, Siena College, 515 Loudon
 Road, Loudonville, NY 12211, USA
²³Department of Physics & Astronomy and Pittsburgh Particle
 Physics, Astrophysics, and Cosmology Center (PITT PACC), University
 of Pittsburgh, 3941 O’Hara Street, Pittsburgh, PA 15260,

USA

²⁴Departamento de Física, Universidad de Guanajuato - DCI, C.P.
 37150, Leon, Guanajuato, México

²⁵Instituto Avanzado de Cosmología A. C., San Marcos 11 - Atenas
 202. Magdalena Contreras, 10720. Ciudad de México, México

²⁶Instituto de Astrofísica de Andalucía (CSIC), Glorieta de la As-
 tronomía, s/n, E-18008 Granada, Spain

²⁷Departament de Física, EEBE, Universitat Politècnica de
 Catalunya, c/Eduard Maristany 10, 08930 Barcelona, Spain

²⁸Department of Astronomy, The Ohio State University, 4055
 McPherson Laboratory, 140 W 18th Avenue, Columbus, OH 43210,
 USA

²⁹Department of Physics and Astronomy, Sejong University, Seoul,
 143-747, Korea

³⁰CIEMAT, Avenida Complutense 40, E-28040 Madrid, Spain

³¹Department of Physics, University of Michigan, Ann Arbor, MI
 48109, USA

³²University of Michigan, Ann Arbor, MI 48109, USA

³³National Astronomical Observatories, Chinese Academy of Sci-
 ences, A20 Datun Rd., Chaoyang District, Beijing, 100012, P.R.
 China

Many people made this work
 possible – this is just a list of
 the universities they’re from!

33 institutions,
 10 different countries



APPENDIX A: MODELLING DERIVATION

To compute the expectation value $\langle \epsilon \rangle$ between projected shapes and the 3D matter field, $\mathcal{E}_{\text{model}}$, we begin with their definitions, as described in Equation [\(A1\)](#).

$$\epsilon = \tau [T_{xx} - T_{yy} + 2iT_{xy}] \quad (\text{A1})$$

$$Q(R_{\text{bin}}, \pm\Pi_{\text{max}}) = \frac{\int d^3r W(\bar{r}) \delta e^{2i\theta_r}}{\int d^3r W(\bar{r})(1 + \xi_{\epsilon g})} \quad (\text{A2})$$

Using these, $\mathcal{E}_{\text{model}}$ is computed as:

$$\begin{aligned} \mathcal{E}_{\text{model}} &= \Re \langle \epsilon * Q \rangle = \frac{1}{2} \langle \epsilon^* Q + \epsilon Q^* \rangle = \Re \epsilon \Re Q + \Im \epsilon \Im Q = |Q| [(\epsilon_{xx} - \epsilon_{yy}) \cos 2\theta + 2\epsilon_{xy} \sin 2\theta] \\ &= \frac{-\tau}{\int d^3r W(\bar{r})(1 + \xi_{\epsilon g})} \int dz \int R dR \int d\theta W(\bar{r}) \delta(R, z) [(T_{xx} - T_{yy}) \cos 2\theta + (2T_{xy} - T_{xx} - T_{yy}) \sin 2\theta] \end{aligned} \quad (\text{A3})$$

The 3 dimensions we integrate over here are the projected angle on the plane of the sky θ , the projected distance R , and the redshift z . $*$ indicates complex conjugation and x^* is the complex conjugate of x . To compute the second integral, we convert to Fourier space.

$$\begin{aligned} \int dz \int R dR W(\bar{r}) \int d\theta \int \frac{d^3k}{(2\pi)^3} \tilde{\delta}(k) e^{-ik \cdot r} \int \frac{d^3q}{(2\pi)^3} e^{iq \cdot (0)} \tilde{\delta}q \frac{1}{q^2} [(q_x q_x - q_y q_y) \cos 2\theta + (2q_x q_y - q_x q_x - q_y q_y + \frac{2}{3}q^2) \sin 2\theta] \\ = \int dz \int R dR W(\bar{r}) \int \frac{dk_z}{2\pi} \int \frac{K dK}{(2\pi)^2} P(k) \int d\phi \int d\theta e^{-iK \cdot R - ik_z z} \frac{1}{k^2} \\ [K^2 (\cos^2 \phi - \sin^2 \phi) \cos 2\theta + K^2 (2 \cos \phi \sin \phi - (\cos^2 \phi + \sin^2 \phi)) \sin 2\theta + \frac{2}{3} k^2 \sin 2\theta] \end{aligned} \quad (\text{A4})$$

$\tilde{\delta}$ is the fractional overdensity in Fourier space. k represented the wavenumber in Fourier space, K represents the 2D position on the plane of the sky (k_x, k_y), and k_z along the line of sight. $k^2 = K^2 + k_z^2$. We use the 2D plane wave expansion $e^{iK \cdot R} = \sum_{n=-\infty}^{\infty} i^n J_n(KR) e^{in\psi}$, where $\cos \psi = \hat{K} \cdot \hat{R}$. The above expression will integrate to 0 for all n except $n = \pm 2$, allowing us to reduce $e^{iK \cdot R}$ to $-2J_2(KR)e^{2i(\theta-\phi)}$, of which the real component is $-2J_2(KR) \cos(2(\phi-\theta))$. The inner integrands becomes:

$$- \int_0^{2\pi} d\phi \int_0^{2\pi} d\theta 2J_2(KR) \cos 2(\phi-\theta) [K^2 \cos 2\theta + \frac{2}{3} (K^2 + k_z^2) \sin 2\theta] = -4\pi^2 J_2(KR) K^2 \quad (\text{A5})$$

This leads to our final expression,

$$\mathcal{E}_{\text{model}} = \frac{-\tau}{\int d^3r W(\bar{r})} \int dz \int R dR \int \frac{dk_z}{2\pi} \int \frac{K dK}{(2\pi)^2} P(k) \delta_{\text{model}}(R, z) \quad (\text{A6})$$

This is the fourth paper summary like this I've made.

This is solved numerically except for

For one on the discovery of dark energy, see [this link](#).

$$\mathcal{J}_2(K) = \frac{2}{(R_{\text{max}}^2 - R_{\text{min}}^2) \int R dR} \int_{R_{\text{min}}}^{R_{\text{max}}} R dR \quad (\text{A7})$$

for which we use the analytic solution:

$$\mathcal{J}_2(K, R) = \frac{2}{(R_{\text{max}}^2 - R_{\text{min}}^2)} \frac{1}{K^2} [2J_0(KR_{\text{min}}) + KR_{\text{min}} J_1(KR_{\text{min}}) - 2J_0(KR_{\text{max}}) - KR_{\text{max}} J_1(KR_{\text{max}})] \quad (\text{A8})$$

This paper has been typeset from a [TeX/LaTeX](#) file prepared by the author.

If you've gotten this far, CONGRATULATIONS!
I hope my notes made it easier to digest this
paper. If you're a scientist, I would love to
read similar notes on one of your papers :)
- Claire Lamman

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<https://github.com/ipython/xkcd-font>

