

# Multivariate regression

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Clear workspace

```
clear all; clc; close all;
```

## Initialize variables

**x1**: 1000 uniformly distributed points between 0 and 10

```
x1 = 10*rand(1000, 1);  
min(x1)
```

```
ans = 0.0211
```

```
max(x1)
```

```
ans = 9.9951
```

**x2**: 1000 uniformly distributed points between 10 and 20

```
x2 = 10+10*rand(1000, 1);  
min(x2)
```

```
ans = 10.0037
```

```
max(x2)
```

```
ans = 19.9906
```

**x3**: 1000 uniformly distributed points between 5 and 10

```
x3 = 5+5*rand(1000, 1);  
min(x3)
```

```
ans = 5.0062
```

```
max(x3)
```

```
ans = 9.9999
```

**eps**: random error --> normal distribution ( /mu=0, /sigma=1) = Standard normal distrib

```
eps = randn(1000, 1);  
std(eps)
```

```
ans = 1.0265
```

**y**: variable to be reconstructed

```
y = 3 + 2*x1 + x2 - x3 + eps;
```

```

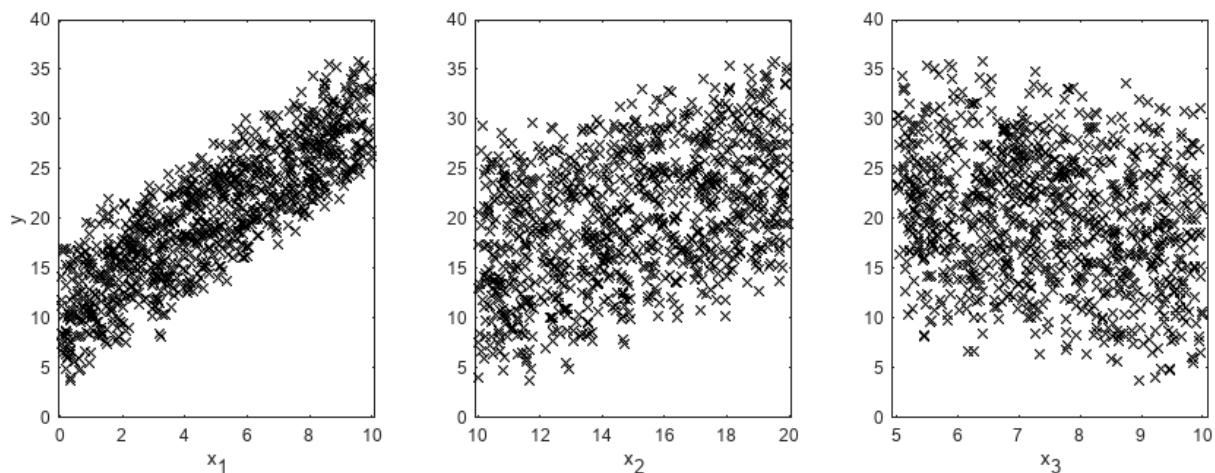
figure(100)
ax1 = subplot(1,3,1);
plot(x1,y,'xk')
xlabel('x_1')
ylabel('y')
xlim([min(x1)-0.1, max(x1)+0.1])

ax2 = subplot(1,3,2);
plot(x2,y,'xk')
xlabel('x_2')
xlim([min(x2)-0.1, max(x2)+0.1])

ax3 = subplot(1,3,3);
plot(x3,y,'xk')
xlabel('x_3')
xlim([min(x3)-0.1, max(x3)+0.1])

linkaxes([ax1, ax2, ax3], 'y');
width=1200;
height=400;
set(gcf,'position',[0,0,width,height])

```



## Lineal adjust regression

```

X = [ones(size(x1,1),1) x1 x2 x3];
[b, bint, r, rint, stats] = regress(y,X);
disp(b)

```

```

2.6349
2.0099
1.0068
-0.9667

```

```
disp(bint)
```

```

2.1404    3.1293
1.9874    2.0325

```

```
0.9851    1.0285
-1.0116   -0.9218
```

```
disp(strcat('r^2: ',num2str(stats(1))))
```

```
r^2:0.97558
```

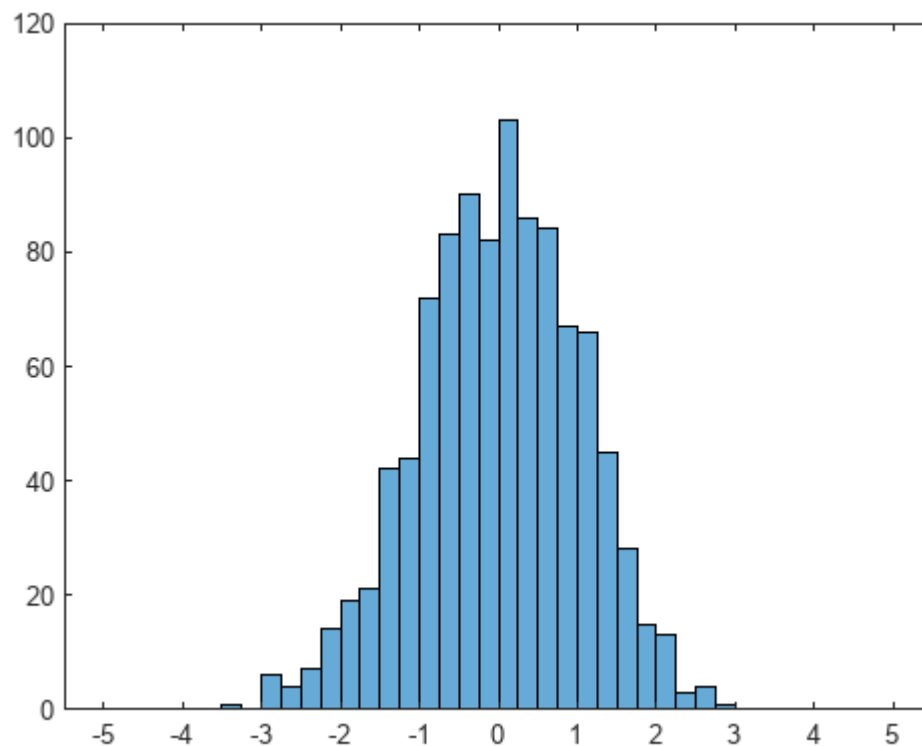
```
disp(strcat('F-stat: ',num2str(stats(2)), ' ==>  p(>F): ',num2str(stats(3))))
```

```
F-stat:13263.8898 ==>  p(>F):0
```

```
disp(strcat('var(err): ',num2str(stats(4))))
```

```
var(err):1.0535
```

```
figure(1)
histogram(r,[-5:0.25:5])
```



## Modify variability of error

```
edges = [-5:0.25:5];
```

### sigma = 0 --> no error

```
y = 3 + 2*x1 + x2 - x3;
```

```
X = [ones(size(x1,1),1) x1 x2 x3];
```

```
[b, bint, r, rint, stats] = regress(y,X);
```

```
disp(b)
```

```
3.0000  
2.0000  
1.0000  
-1.0000
```

```
disp(bint)
```

```
3.0000    3.0000  
2.0000    2.0000  
1.0000    1.0000  
-1.0000   -1.0000
```

```
disp(strcat('r^2: ',num2str(stats(1))))
```

```
r^2:1
```

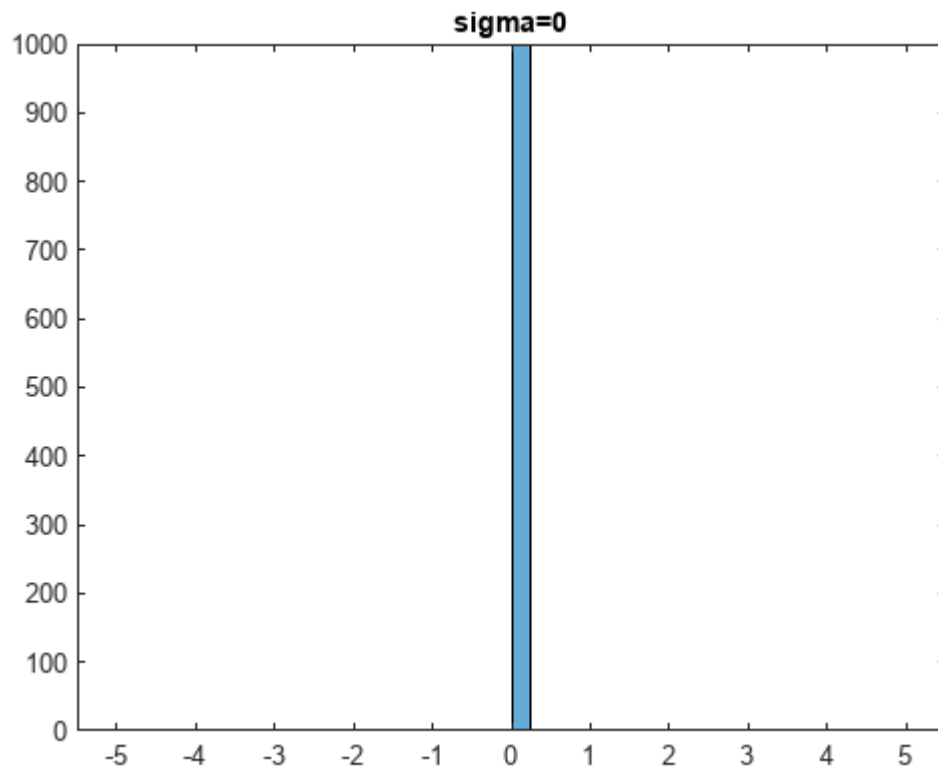
```
disp(strcat('F-stat: ',num2str(stats(2)), ' ==>  p(>F): ',num2str(stats(3))))
```

```
F-stat:2.897369022825098e+32 ==>  p(>F):0
```

```
disp(strcat('var(err): ',num2str(stats(4))))
```

```
var(err):4.7908e-29
```

```
figure(2)  
histogram(r,edges)  
title('sigma=0')  
% saveas(gcf,'03_Multivar/hist_sigma_0.png')
```



## sigma = 0.1

```
y = 3 + 2*x1 + x2 - x3 + 0.1*randn(1000,1);
X = [ones(size(x1,1),1) x1 x2 x3];
[b, bint, r, rint, stats] = regress(y,X);
disp(b)
```

```
3.0216
1.9984
1.0007
-1.0032
```

```
disp(bint)
```

```
2.9739    3.0693
1.9962    2.0006
0.9986    1.0028
-1.0076   -0.9989
```

```
disp(strcat('r^2: ',num2str(stats(1))))
```

```
r^2:0.99977
```

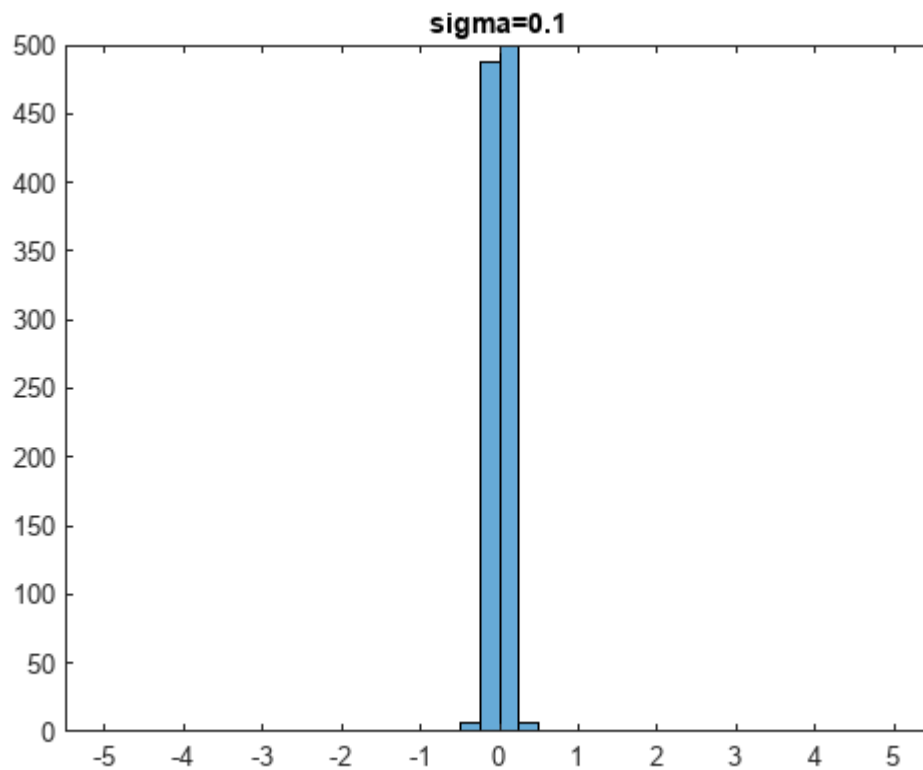
```
disp(strcat('F-stat: ',num2str(stats(2)), ' ==> p(>F): ',num2str(stats(3))))
```

```
F-stat:1414497.7643 ==> p(>F):0
```

```
disp(strcat('var(err): ',num2str(stats(4))))
```

```
var(err):0.0098075
```

```
figure(3)
histogram(r,edges)
title('sigma=0.1')
% saveas(gcf,'03_Multivar/hist_sigma_01.png')
```



### **sigma = 0.5**

```
y = 3 + 2*x1 + x2 - x3 + 0.5*randn(1000,1);
```

```
X = [ones(size(x1,1),1) x1 x2 x3];
[b, bint, r, rint, stats] = regress(y,X);
disp(b)
```

```
2.8621
2.0039
1.0006
-0.9845
```

```
disp(bint)
```

```
2.6156    3.1086
1.9927    2.0152
0.9898    1.0114
-1.0069   -0.9621
```

```
disp(strcat('r^2: ',num2str(stats(1))))
```

```
r^2:0.99379
```

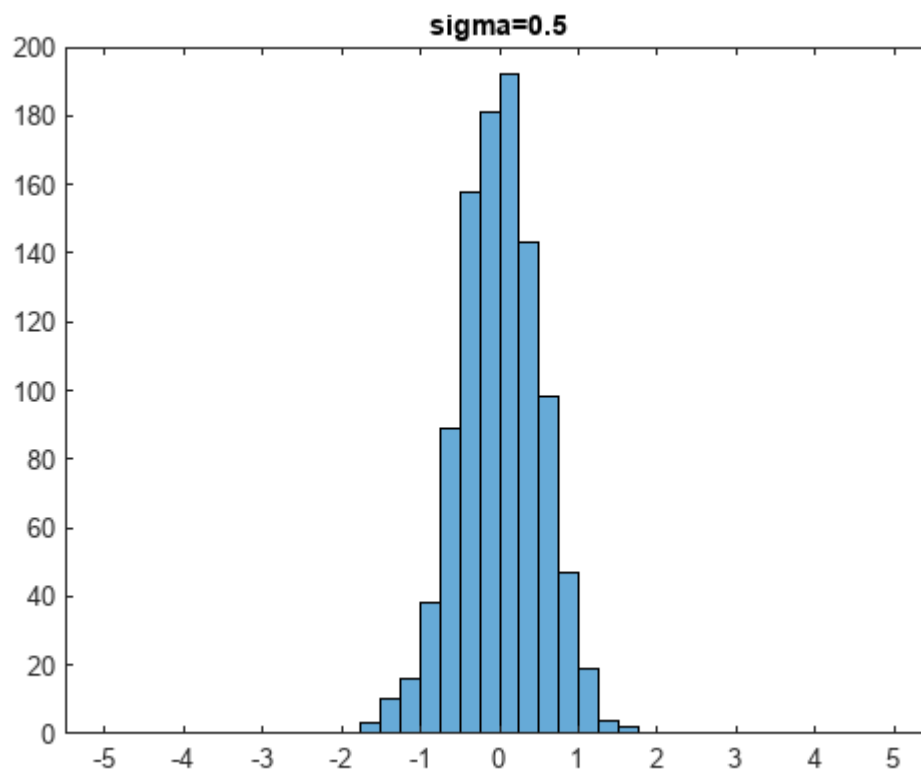
```
disp(strcat('F-stat: ',num2str(stats(2)), ' ==> p(>F): ',num2str(stats(3))))
```

```
F-stat:53109.0524 ==> p(>F):0
```

```
disp(strcat('var(err): ',num2str(stats(4))))
```

```
var(err):0.26177
```

```
figure(4)
histogram(r,edges)
title('sigma=0.5')
% saveas(gcf,'03_Multivar/hist_sigma_05.png')
```



**sigma = 1**

```
y = 3 + 2*x1 + x2 - x3 + 1*randn(1000,1);
```

```
X = [ones(size(x1,1),1) x1 x2 x3];
```

```
[b, bint, r, rint, stats] = regress(y,X);
```

```
disp(b)
```

```
3.1897
```

```
1.9970
```

```
0.9864
```

```
-0.9916
```

```
disp(bint)
```

```
2.7129    3.6666  
1.9752    2.0187  
0.9655    1.0074  
-1.0349   -0.9484
```

```
disp(strcat('r^2: ',num2str(stats(1))))
```

```
r^2:0.97691
```

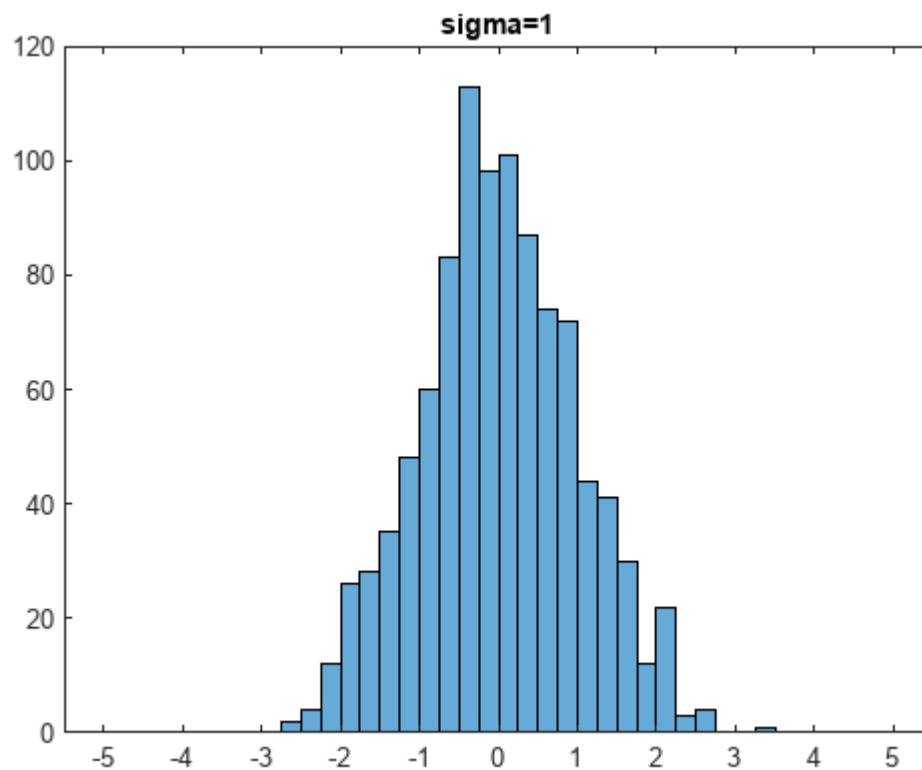
```
disp(strcat('F-stat: ',num2str(stats(2)), ' ==> p(>F): ',num2str(stats(3))))
```

```
F-stat:14047.8203 ==> p(>F):0
```

```
disp(strcat('var(err): ',num2str(stats(4))))
```

```
var(err):0.97976
```

```
figure(5)  
histogram(r,edges)  
title('sigma=1')  
% saveas(gcf,'03_Multivar/hist_sigma_1.png')
```



**sigma = 2**

```
y = 3 + 2*x1 + x2 - x3 + 2*randn(1000,1);
```

```
X = [ones(size(x1,1),1) x1 x2 x3];
```



```
[b, bint, r, rint, stats] = regress(y,X);  
disp(b)
```

```
2.4687  
2.0333  
1.0131  
-0.9650
```

```
disp(bint)
```

```
1.5491    3.3883  
1.9913    2.0752  
0.9728    1.0535  
-1.0485   -0.8816
```

```
disp(strcat('r^2: ',num2str(stats(1))))
```

```
r^2:0.92176
```

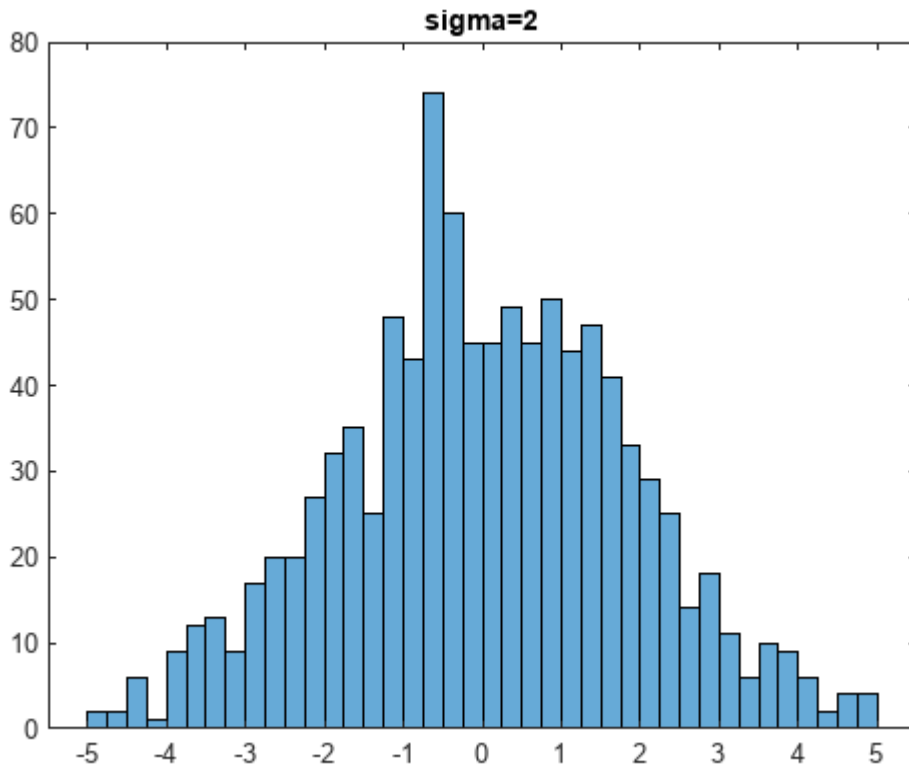
```
disp(strcat('F-stat: ',num2str(stats(2)), ' ==>  p(>F): ',num2str(stats(3))))
```

```
F-stat:3911.1179 ==>  p(>F):0
```

```
disp(strcat('var(err): ',num2str(stats(4))))
```

```
var(err):3.6438
```

```
figure(6)  
histogram(r,edges)  
title('sigma=2')  
% saveas(gcf,'03_Multivar/hist_sigma_2.png')
```



**sigma = 5**

```
edges = [-100:5:100];
```

```
y = 3 + 2*x1 + x2 - x3 + 5*randn(1000,1);
```

```
X = [ones(size(x1,1),1) x1 x2 x3];
[b, bint, r, rint, stats] = regress(y,X);
disp(b)
```

```
4.1274
1.9144
0.9888
-1.0575
```

```
disp(bint)
```

```
1.6878    6.5670
1.8030    2.0258
0.8818    1.0959
-1.2789   -0.8361
```

```
disp(strcat('r^2: ',num2str(stats(1))))
```

```
r^2:0.60489
```

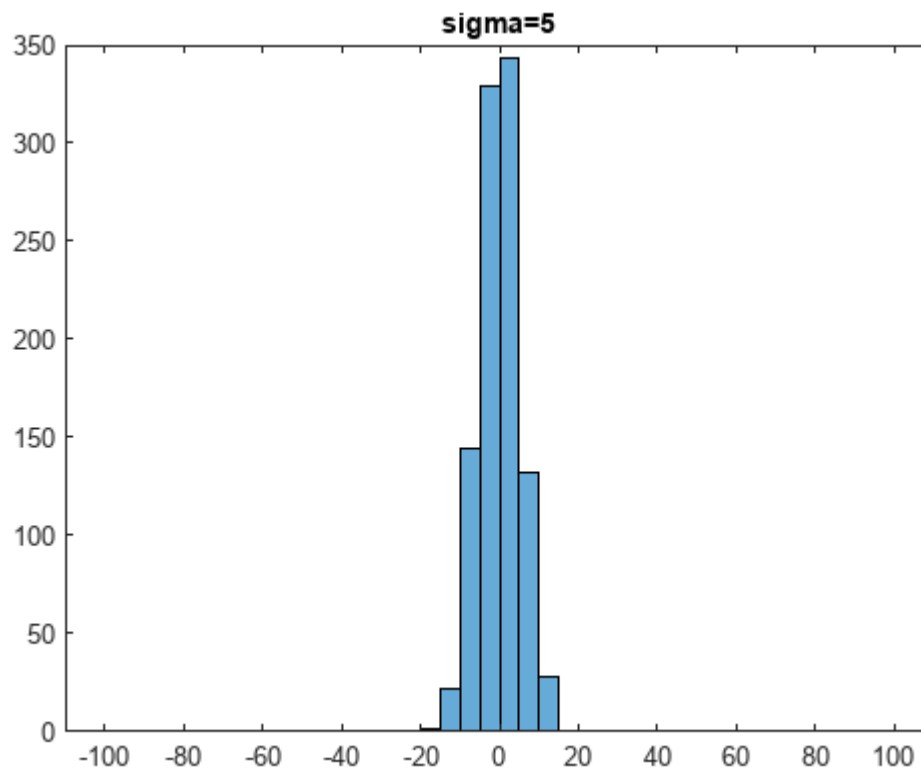
```
disp(strcat('F-stat: ',num2str(stats(2)),', ' ==>  p(>F): ',num2str(stats(3))))
```

F-stat:508.2713 ==> p(>F):2.8732e-200

```
disp(strcat('var(err): ',num2str(stats(4))))
```

var(err):25.6447

```
figure(7)
histogram(r,edges)
title('sigma=5')
% saveas(gcf,'03_Multivar/hist_sigma_5.png')
```



## sigma = 10

```
y = 3 + 2*x1 + x2 - x3 + 10*randn(1000,1);
```

```
X = [ones(size(x1,1),1) x1 x2 x3];
[b, bint, r, rint, stats] = regress(y,X);
disp(b)
```

2.5839  
2.1259  
1.0211  
-1.1077

```
disp(bint)
```

-2.1859    7.3536  
1.9082    2.3436

```
0.8118    1.2304
-1.5406   -0.6749
```

```
disp(strcat('r^2: ',num2str(stats(1))))
```

```
r^2:0.32286
```

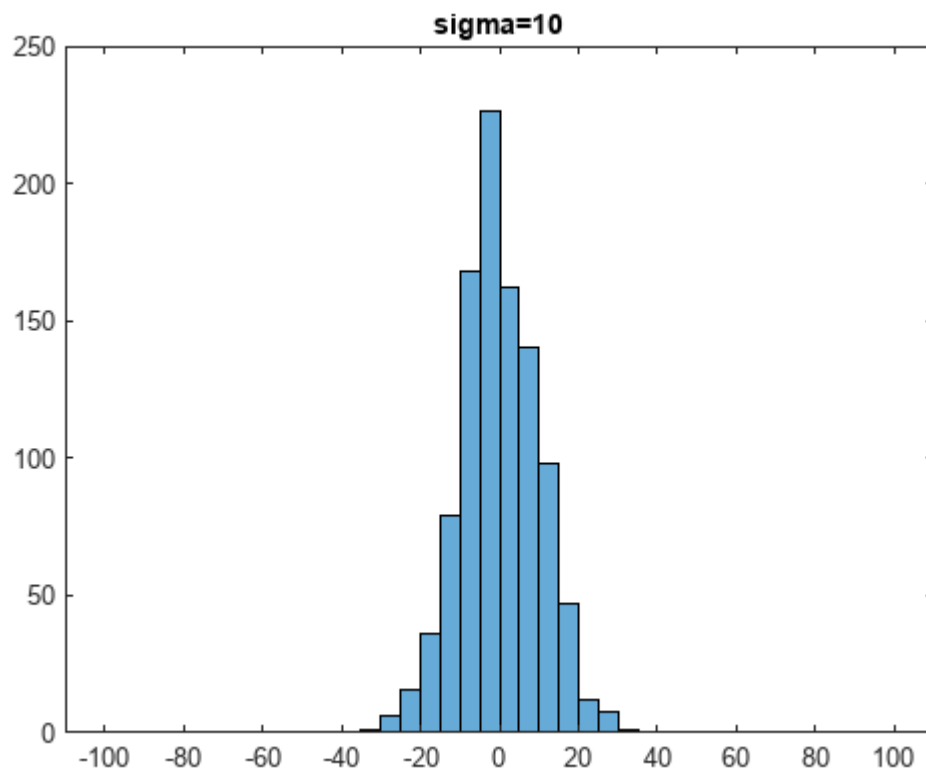
```
disp(strcat('F-stat: ',num2str(stats(2)), ' ==>  p(>F): ',num2str(stats(3))))
```

```
F-stat:158.2938 ==>  p(>F):6.8592e-84
```

```
disp(strcat('var(err): ',num2str(stats(4))))
```

```
var(err):98.028
```

```
figure(8)
histogram(r,edges)
title('sigma=10')
% saveas(gcf,'03_Multivar/hist_sigma_10.png')
```



## sigma = 20

```
y = 3 + 2*x1 + x2 - x3 + 20*randn(1000,1);
```

```
X = [ones(size(x1,1),1) x1 x2 x3];
```

```
[b, bint, r, rint, stats] = regress(y,X);
```

```
disp(b)
```

```
2.1545
1.7714
0.9000
-0.6181
```

```
disp(bint)
```

```
-7.3299  11.6389
 1.3385   2.2044
 0.4839   1.3162
-1.4788   0.2426
```

```
disp(strcat('r^2: ',num2str(stats(1))))
```

```
r^2:0.076485
```

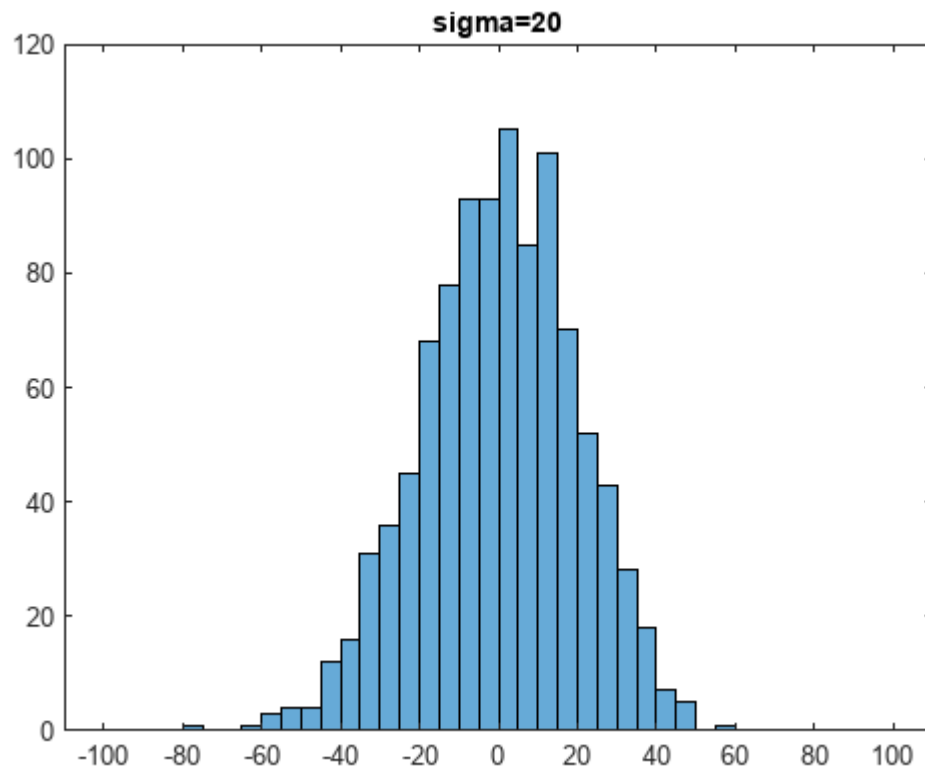
```
disp(strcat('F-stat: ',num2str(stats(2)), ' ==>  p(>F): ',num2str(stats(3))))
```

```
F-stat:27.496 ==>  p(>F):4.3599e-17
```

```
disp(strcat('var(err): ',num2str(stats(4))))
```

```
var(err):387.5961
```

```
figure(9)
histogram(r,edges)
title('sigma=20')
% saveas(gcf,'03_Multivar/hist_sigma_20.png')
```



## **sigma = 50**

```
y = 3 + 2*x1 + x2 - x3 + 50*randn(1000,1);  
  
X = [ones(size(x1,1),1) x1 x2 x3];  
[b, bint, r, rint, stats] = regress(y,X);  
disp(b)
```

```
21.1056  
2.0735  
0.1449  
-1.8854
```

```
disp(bint)
```

```
-2.7092  44.9204  
0.9864   3.1606  
-0.9000   1.1898  
-4.0465   0.2757
```

```
disp(strcat('r^2: ',num2str(stats(1))))
```

```
r^2:0.016808
```

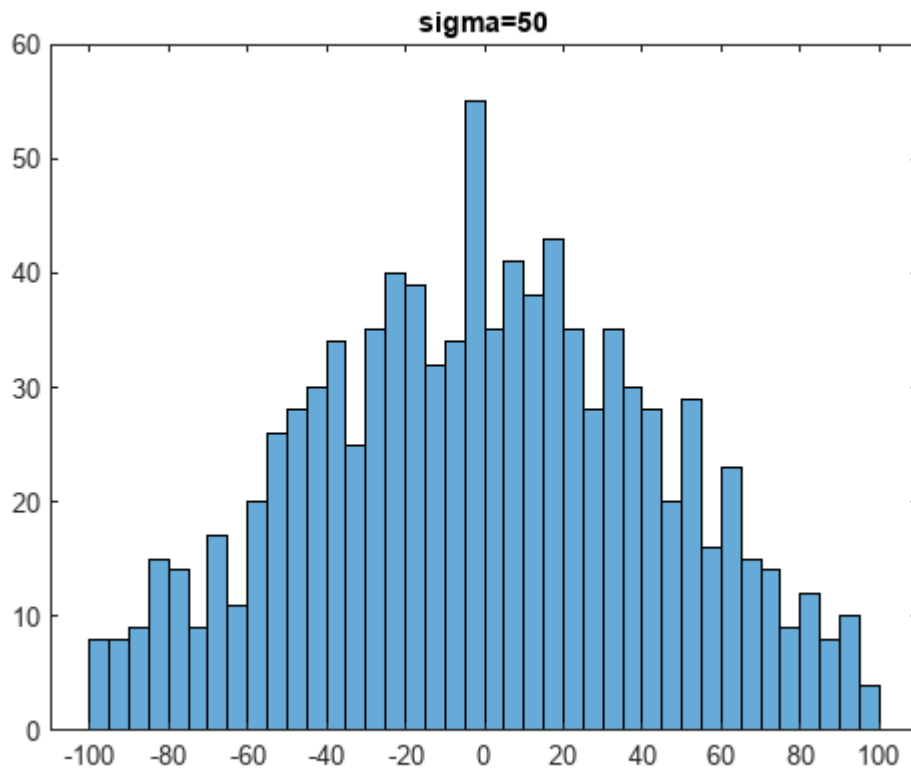
```
disp(strcat('F-stat: ',num2str(stats(2)), ' ==>  p(>F): ',num2str(stats(3))))
```

```
F-stat:5.6755 ==>  p(>F):0.00074414
```

```
disp(strcat('var(err): ',num2str(stats(4))))
```

```
var(err):2443.7441
```

```
figure(10)  
histogram(r,edges)  
title('sigma=50')  
% saveas(gcf,'03_Multivar/hist_sigma_50.png')
```



## Residual analysis for sigma=1

```
y = 3 + 2*x1 + x2 - x3 + randn(1000,1);

X = [ones(size(x1,1),1) x1 x2 x3];
[b, bint, r, rint, stats] = regress(y,X);
disp(b)
```

```
2.9708
1.9966
0.9935
-0.9800
```

```
disp(bint)
```

```
2.4748    3.4668
1.9740    2.0193
0.9717    1.0152
-1.0250   -0.9350
```

```
disp(strcat('r^2: ',num2str(stats(1))))
```

```
r^2:0.9751
```

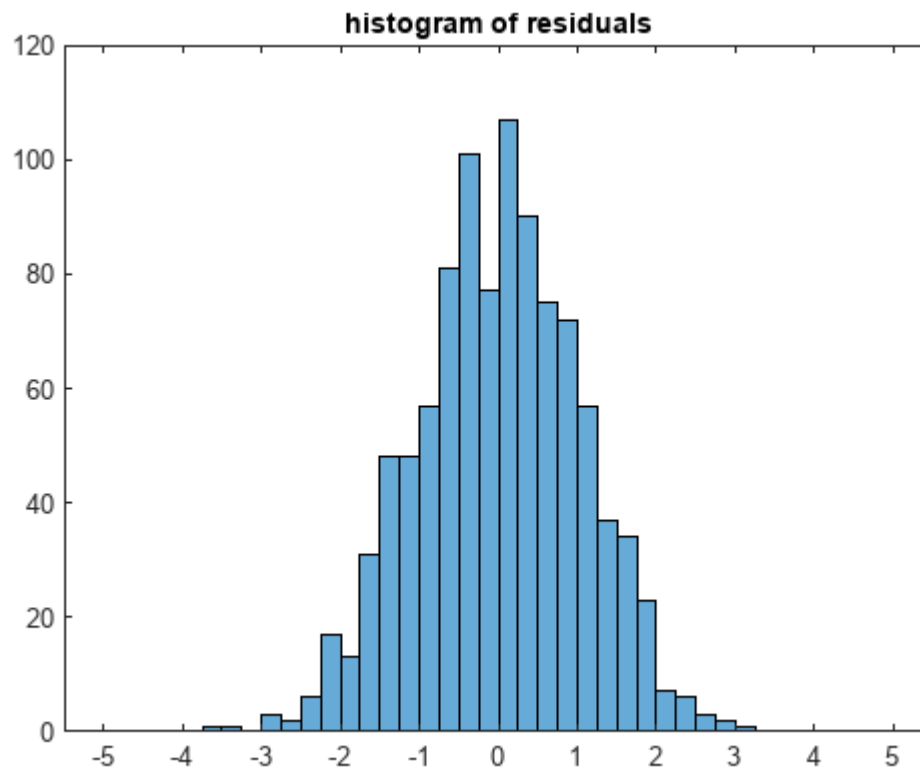
```
disp(strcat('F-stat: ',num2str(stats(2)), ' ==> p(>F): ',num2str(stats(3))))
```

```
F-stat:13001.0876 ==> p(>F):0
```

```
disp(strcat('var(err): ',num2str(stats(4))))
```

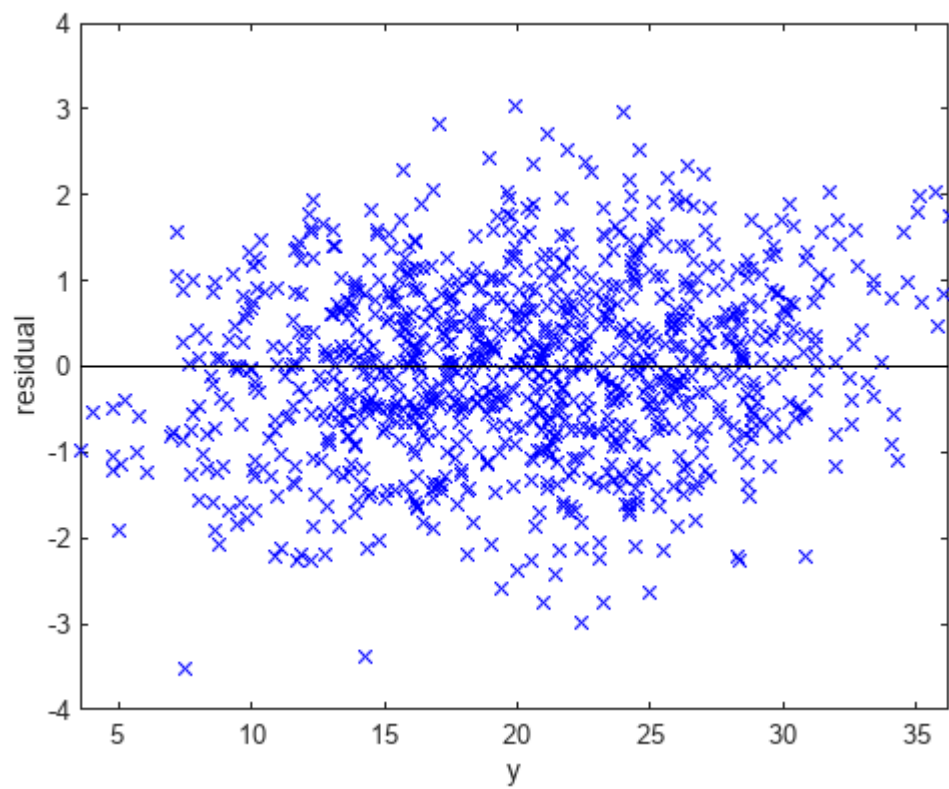
```
var(err):1.06
```

```
figure(11)
histogram(r,[-5:0.25:5])
title('histogram of residuals')
% saveas(gcf,'03_Multivar/histr_multivar.png')
```

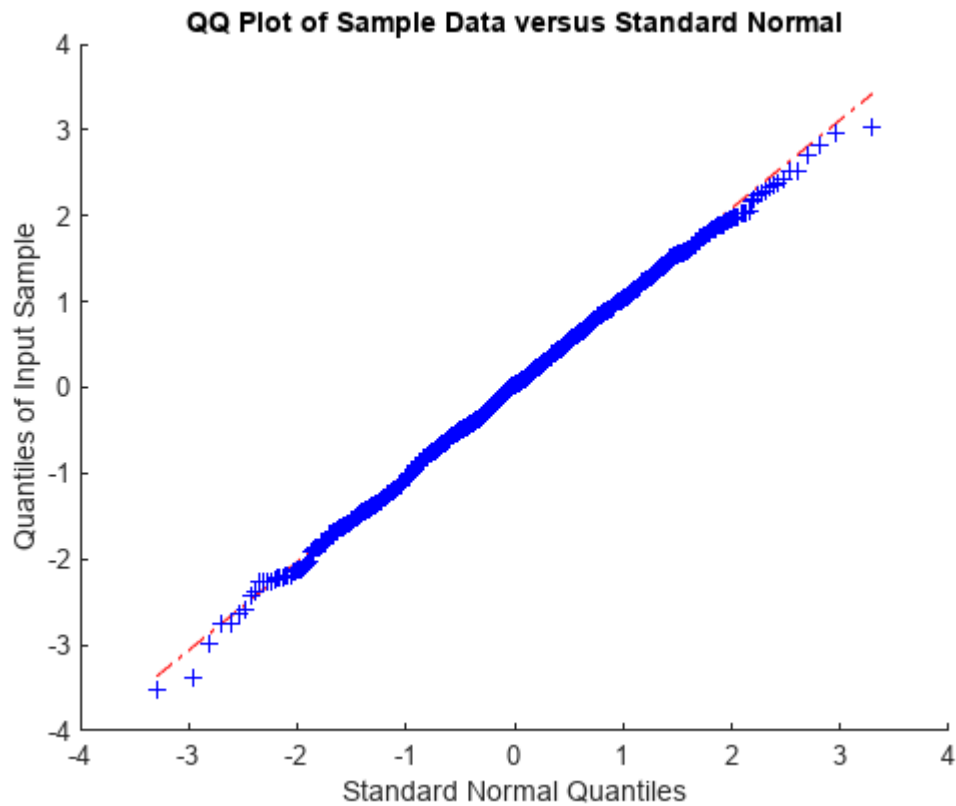


```
figure(12)
plot(y,r,'xb')
hold on
plot([min(y) max(y)], [0 0], 'k')
xlim([min(y) max(y)])
xlabel('y')
ylabel('residual')
hold off
% saveas(gcf,'03_Multivar/residuals_multivar.png')
```





```
figure(13)
qqplot(r)
% saveas(gcf,'03_Multivar/qq_multivar.png')
```

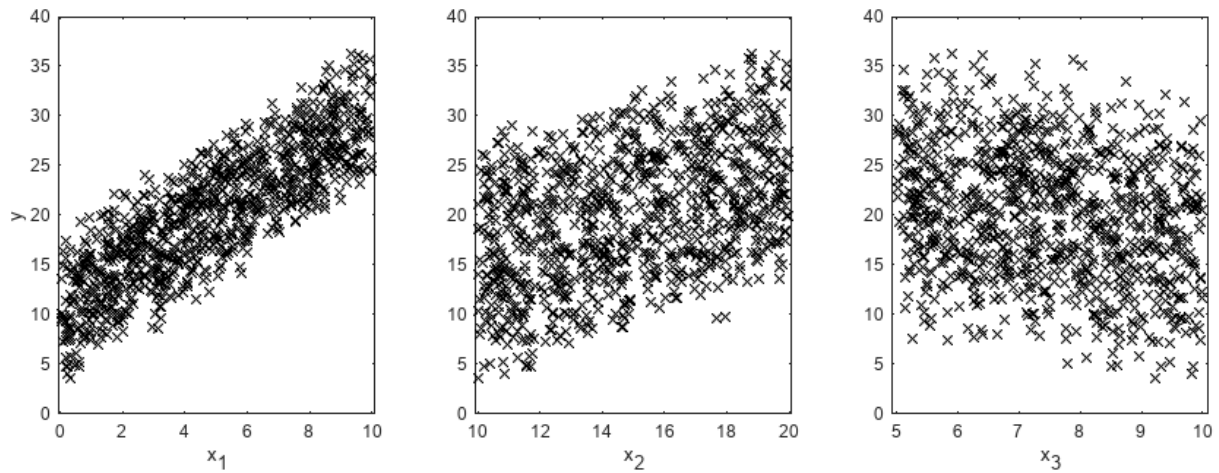


```
figure(101)
ax1 = subplot(1,3,1);
plot(x1,y,'xk')
xlabel('x_1')
ylabel('y')
xlim([min(x1)-0.1, max(x1)+0.1])

ax2 = subplot(1,3,2);
plot(x2,y,'xk')
xlabel('x_2')
xlim([min(x2)-0.1, max(x2)+0.1])

ax3 = subplot(1,3,3);
plot(x3,y,'xk')
xlabel('x_3')
xlim([min(x3)-0.1, max(x3)+0.1])

linkaxes([ax1, ax2, ax3], 'y');
width=1200;
height=400;
set(gcf,'position',[0,0,width,height])
% saveas(gcf,'03_Multivar/multivar_data.png')
```



```

y_adj = X*b;

figure(102)
ax1 = subplot(1,3,1);
hold on
plot(x1,y,'xk')
plot(x1,y_adj,'b.')
plot([min(x1); max(x1)], [b(1)+b(2)*min(x1)+b(3)*mean(x2)+b(4)*mean(x3); ...
                        b(1)+b(2)*max(x1)+b(3)*mean(x2)+b(4)*mean(x3)], 'c', LineWidth=2.5)

hold off
xlabel('x_1')
ylabel('y')
legend('raw data', 'predictions', 'trend', Location='northwest')
xlim([min(x1)-0.1, max(x1)+0.1])

ax2 = subplot(1,3,2);
hold on
plot(x2,y,'xk')
plot(x2,y_adj,'b.')
plot([min(x2); max(x2)], [b(1)+b(3)*min(x2)+b(2)*mean(x1)+b(4)*mean(x3); ...
                        b(1)+b(3)*max(x2)+b(2)*mean(x1)+b(4)*mean(x3)], 'c', LineWidth=2.5)

hold off
xlabel('x_2')
xlim([min(x2)-0.1, max(x2)+0.1])

ax3 = subplot(1,3,3);
hold on
plot(x3,y,'xk')
plot(x3,y_adj,'b.')
plot([min(x3); max(x3)], [b(1)+b(4)*min(x3)+b(2)*mean(x1)+b(3)*mean(x2); ...
                        b(1)+b(4)*max(x3)+b(2)*mean(x1)+b(3)*mean(x2)], 'c', LineWidth=2.5)

hold off
xlabel('x_3')

```

```

xlim([min(x3)-0.1, max(x3)+0.1])

linkaxes([ax1, ax2, ax3], 'y');
width=1200;
height=400;
set(gcf, 'position',[0,0,width,height])
% saveas(gcf,'03_Multivar/multivar_adjust.png')

```

