FACIL: Neil Douglas

WEEK: 11

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COURSE: ECOR2606

OFFICE HOURS: Fridays 3:00 – 4:00

Opener

Today we will be looking at the following core concepts:

- Interpolation using splines
- Integration
    - Using MATLAB
    - Trapezoidal integration
    - Simpson's rules

Activity 1 - Interpolation using splines

Find the values of a, b, c, d, e, f, g, h (yes, 8 unknowns!) such that $s_1(x)$ and $s_2(x)$ form a pair of cubic splines that pass through (1,2), (2,3), and (3,5). Note that (2,3) is the knot and these splines have the "natural" end condition.

The general equation for a cubic spline is:

$$s_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$$

For this problem, we have two cubic splines:

$$s_1(x) = a + b(x-1) + c(x-1)^2 + d(x-1)^3 \qquad x \in [1,2]$$

$$s_1(x) = e + f(x-2) + g(x-2)^2 + h(x-2)^3 \qquad x \in [2,3]$$

Credit for creating this problem: YouTube: The Math Guy

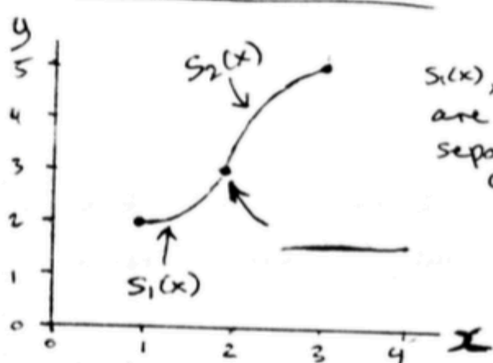
Video title: "Interpolation – Cubic Splines – example", posted Oct 30, 2017

$$S_1(x) = a + b(x-1) + c(x-1)^2 + d(x-1)^3$$

$$S_2(x) = e + f(x-2) + g(x-2)^2 + h(x-2)^3$$

8 unknowns $\longrightarrow$ Need 8 equations to solve for these unknowns

Let's Graph This:



What does the "natural" end condition tell us?

What is a Knot?

Our 8 equations:

1.

2.

3.

4.

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$S_1'(x) = b + 2c(x-1) + 3d(x-1)^2$ (First Derivative)

$S_1''(x) = 2c + 6d(x-1)$ (Second Derivative)

$S_2'(x) = f + 2g(x-2) + 3h(x-2)^2$ (First Derivative)

$S_2''(x) = 2g + 6h(x-2)$ (Second Derivative)

5.

6.

To get the last two equations, think about the boundary conditions given by our "natural" end condition.

7.

8.

We have 5 equations that still need to be solved. Sub in $c = 0$ into these and you get the following series of linear equations:

$$b + d = 1$$
$$f + g + h = 2$$
$$b + 3d = f \Rightarrow b + 3d - f = 0$$
$$3d = g \Rightarrow 3d - g = 0$$
$$2g + 6h = 0$$

Let's make our augmented matrix:

$$\begin{array}{c} \begin{array}{ccccc} \underline{b} & \underline{d} & \underline{f} & \underline{g} & \underline{h} \end{array} \\ \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 2 \\ 1 & 3 & -1 & 0 & 0 & 0 \\ 0 & 3 & 0 & -1 & 0 & 0 \\ \underbrace{0 \quad 0 \quad 0 \quad 2 \quad 6}_{A} & & & & & \underbrace{0}_{b} \end{array}\right] \end{array}$$

Solve for $x = \begin{bmatrix} b \\ d \\ f \\ g \\ h \end{bmatrix}$

$(Ax = b)$

Solve for b, d, f, g, h using the methods we learned for solving series of linear equ'ns.

Or, for a much quicker solution, solve using MATLAB:

```
A=[1 1 0 0 0;0 0 1 1 1;1 3 -1 0 0;0 3 0 -1 0;0 0 0 2 6]
b=[1;2;0;0;0]
X = ______
```

$$x = \begin{bmatrix} b \\ d \\ f \\ g \\ h \end{bmatrix} = \begin{bmatrix} 3/4 \\ 1/4 \\ 3/2 \\ 3/4 \\ 1/4 \end{bmatrix}$$

Solution $a = 2,\ b = \frac{3}{4},\ c = 0,\ d = \frac{1}{4},\ e = 3,\ f = \frac{3}{2},\ g = 3/4,\ h = 1/4$

$$S_1(x) = 2 + \frac{3}{4}(x-1) + \frac{1}{4}(x-1)^3 \qquad x \in [1,2]$$

$$S_2(x) = 3 + \frac{3}{2}(x-2) + \frac{3}{4}(x-2)^2 + \frac{1}{4}(x-2)^3 \qquad x \in [2,3]$$

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Activity 2 – Different ways to integrate

For this activity, we will be looking to solve the following definite integral in four different ways.

$$\int_{2}^{4}(7x^2 + 3x + 2)dx$$

This is a simple integral to solve without MATLAB or any special numerical method. The answer is 152.66. However, let's solve this integral in MATLAB and using a couple numerical methods to see how they work and make sure we are getting an answer this is exactly, or very close to, 152.66.

(a) Solve this integral in MATLAB using the *integral* function.

(b) Solve this integral using trapezoidal integration. You are given 5 evenly spaced points from 2 to 4 (i.e., you will have four intervals, so n=4).

$$h = \frac{b-a}{n} = \frac{4-2}{4} = \frac{1}{2}$$

x	2.0	2.5	3.0	3.5	4.0
y	36.00	53.25	74.00	98.25	126.00

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Did this manual method give us a perfectly accurate answer? _______________

Why?

(c) Solve this integral in MATLAB using the *trapz* function.

(d) Solve this integral using either Simpson's 1/3 rule or Simpson's 3/8 rule.

x	2.0	2.5	3.0	3.5	4.0
y	36.00	53.25	74.00	98.25	126.00

Did this manual method give us a perfectly accurate answer? _________________

Why?

Now, we had a really simple integral where we wouldn't need any of these special methods to solve it, but these methods are really useful for solving complex integrals, such as the following integral from the lecture slides that you have been looking at in class.

$$s = \int_4^7 v(t)dt = \int_4^7 \sqrt{\frac{gm}{c_d}} \tanh\left(\left(\sqrt{\frac{gc_d}{m}}\right)t\right)dt$$

Imagine trying to solve this without any of these special methods!!

Closer – Let's make a study plan!

You've learned a lot in this course, so it's best to start studying as soon as you can! Try answering the following questions to help guide your studying.

1. How many days do you think you should spend studying for this class?
2. What day will you start studying for this class?
3. What times of the day do you study the best?
4. Where can you study that you will allow you to focus the best?
5. What is the most efficient way to study for this class?
6. What resources do you have? (Ex., PASS mock-final, Prof. Goheen's review, old tests, etc…)
7. What times can you leave for last minute review in the day or two before the exam?
8. What rewards will you give yourself for productive study days throughout this exam period?
9. What are some ways that you can relieve some stress during this exam period? (Remember to always prioritize good mental health!)