PASS
PEER ASSISTED STUDY SESSIONS

MOCK EXAM
FOR PRACTICE ONLY

COURSE: ECOR 2606 **FACILITATOR:** Neil Douglas

It is **most beneficial** to you to write this mock midterm **UNDER EXAM CONDITIONS**. This means:

- Complete the midterm in **3** hour(s).
- Work on your own.
- Keep your notes and textbook closed.
- Attempt every question.

After the time limit, go back over your work with a different colour or on a separate piece of paper and try to do the questions of which you are unsure. Record your ideas in the margins to remind yourself of what you were thinking when you take it up at PASS.

The purpose of this mock exam is to give you practice answering questions in a timed setting and to help you to gauge which aspects of the course content you know well and which are in need of further development and review. Use this mock exam as a ***learning tool*** in preparing for the actual exam.

Please note:

- Come to the PASS workshop with your mock exam complete. During the workshop you can work with other students to review your work.
- Often, there is not enough time to review the entire exam in the PASS workshop. Decide which questions you most want to review – the Facilitator may ask students to vote on which questions they want to discuss in detail.
- Facilitators do not bring copies of the mock exam to the session. Please print and complete the exam before you attend.
- **Facilitators do not produce or distribute an answer key for mock exams.** Facilitators help students to work together to compare and assess the answers they have. If you are not able to attend the PASS workshop, you can work alone or with others in the class.

Good Luck writing the Mock Exam!

Dates and locations of mock exam take-up: Monday Feb. 26, 6pm to 9pm (MC5050)

Bisection Search

Use a Bisection Search to find the root of $x^2 + 4x = 8$ that lies in between x=-9 and x=-3.

$f(-9) =$ ______________

$f(-3) =$ ______________

Step	X_L	X_U	X_R	$f(X_R)$	E_{MAX}
1	-9.000	-3.000			
2					
3					

Q1 What is the purpose of calculating $f(-9)$ and $f(-3)$ at the beginning?

(a) $f(-9)$ and $f(-3)$ must have the same sign (both positive or both negative) in order for a root to lie in between them.

(b) $f(-9)$ and $f(-3)$ must have different signs in order for a root to lie in between them.

(c) We have to make sure that $f(-3)$ is larger than $f(-9)$.

(d) We have to make sure that $f(-9)$ is larger than $f(-3)$.

Q2 What are the values for Step 2 in the table above?

(a) -6.000; -3.000; -4.500; -5.750; 1.500

(b) -9.000; -6.000; -7.500; 18.250; 1.500

(c) -6.000; -3.000; -1.500; -11.75; -4.500

Q3 What is our best guess at the root after three iterations?

(a) -6.000

(b) -5.250

(c) -1.438

(d) -4.500

Q4 How many total steps will it take to reduce E_{MAX} to less than 0.01?

(a) 7

(b) 8

(c) 9

(d) 10

Q5 What is the maximum error on the root after seven iterations?

(a) 0.47

(b) 0.047

(c) 0.0047

(d) 4.7

False-Position Search

Use a False-Position Search to find the root of $f(x) = \sin(x) + \cos(3x)$ that lies between x=2.1 and x=3.1. x is measured in radians.

$f(2.1)$ = ______________

$f(3.1)$ = ________________

Step	X_L	X_H	X_R	f(X_R)
1	2.100	3.100		
2				

Q6 What are the values for Step 2 in the table above?

(a) 2.100; 2.762; 2.745; 0.014

(b) 2.762; 3.100; 2.745; 0.014

(c) 2.100; 2.762; 2.431; 1.184

Q7 We set up a False-Position Search on some function with the initial bracketing points, X_L and X_H . If $f(X_L) * f(X_H)$ is a positive value, which of the following is true?

(a) A root is guaranteed to lie between X_L and X_H

(b) A root is not guaranteed to lie between X_L and X_H

(c) It is impossible for any roots to lie between X_L and X_H

(d) We should go ahead and perform a false position search using these bracketing points

Newton-Raphson Search

Use a Newton-Raphson Search to find a root of $f(x) = x^3 + 2x^2 - 5$ given a chosen initial value of x=0.5.

$$f'(x) = \underline{\hspace{6cm}}$$

Step	X_i	$f(X_i)$	$f'(X_i)$	E_a
0	0.500			
1				
2				

Q8 What is our best guess at the root and the approximate error on this root after two iterations?

(a) 2.091; 1.591

(b) 1.491; 0.600

(c) 1.491; -0.600

Q9 What is the relative error on the root in Step 2?

(a) 0.600

(b) -0.600

(c) -40.241%

(d) 40.241%

Q10 What does it mean if $f'(X_i)$ = 0 on any iteration of a N-R Search on some function? (Assume X_i is not a root of the function)

(a) N-R Search has failed, and it is impossible to find a root of this function using a Newton-Raphson Search

(b) The root of the function does not exist

(c) You may still be able to find the root if you continue with the same N-R Search

(d) N-R Search has failed, but it may be possible to use an N-R Search to find a root of this function if you choose a different initial x-value

Secant Search

Use the Secant Search to find a root of $x^2 - 6 = 2x$ given the initial x-values in the table below.

Step	X_{k-1}	X_k	X_{k+1}	f(X_{k+1})	E_a
1	-2.000	1.000			
2					
3					

Q11 What are the values for X_{k-1} and X_k in Step 2?

(a) -2.000; 1.000

(b) 1.000; -1.333

(c) N/A; -1.333

(d) -2.000; -1.333

Q12 What is our best guess at the root after three iterations?

(a) -1.625

(b) -0.109

(c) -2.000

(d) -1.333

Q13 Secant Search would fail if f(X_{k-1}) = f(X_k). True or false?

(a) True

(b) False

Golden Section Search

Use a Golden Section Search to find the minimum of $f(x) = x^2 + 10x - 5$ using the initial bracketing points in the table below.

Step	xL	x2	x1	xU	f(xL)	f(x2)	f(x1)	f(xU)	Emax
1	-9.00			-2.00					
2									

Q14 The values in Step 1 of the Golden Section Search are:

(a) -9.00; -6.33; -4.67; -2.00; -14.00; -28.23; -29.89; -21.00; 3.50

(b) -9.00; -6.33; -4.67; -2.00; -14.00; -28.23; -29.89; -21.00; -0.83

(c) -9.00; -4.67; -6.33; -2.00; -14.00; -28.23; -29.89; -21.00; 3.50

(d) -9.00; -4.67; -6.33; -2.00; -14.00; -28.23; -29.89; -21.00; 0.83

Q15 The values in the non-shaded boxes in Step 2 are:

(a) -6.33; -4.67; -2.00; -2.00; 2.16

(b) -9.00; -7.35; -6.33; -4.67; 2.16

(c) -6.33; -4.67; -3.65; -2.00; 2.16

(d) -6.33; -4.67; -3.65; -2.00; 1.33

Q16 You are performing a Golden Section Search. Initially, x_U=3.020 and x_2=1.080. What are x_L and E_{MAX}?

(a) -1.201; 2.111

(b) 1.201; 0.910

(c) 0.119; 1.451

(d) -0.119; 1.570

Q17 How would you use Golden Section Search to find the maximum of a function?

(a) A G-S Search can only be used to find the minimum of a function

(b) Negate the function, and use the same logic you use when finding a minimum

(c) Using the following logic: If f(x1) < f(x2), move xU to x1. Otherwise, move xL to x2

(d) Use either b or c

MATLAB Question 1

(a) Write a MATLAB function that will calculate the roots of a polynomial represented by row vector p and output the absolute value of the root with the greatest absolute value. It should return an error if length(p) is not greater than 1.

The first line of the function should be:

function [rmax] =Maxroot(p)

(b) Write a script file (or void function file) that will calculate the greatest absolute root of f(x)= x^3+k x^2- 1/k x +2 for k=1, 3, 5, ...15 and print the results in a nicely formatted table. You may want to make use of Maxroot.

MATLAB Question 2

(a) Write a MATLAB function that will return the difference between the maximum and minimum y values of a function f over an interval [xl, xu]. It should also plot f. It should return an error if xu is not 2 greater than xl. The first line of the function should be:

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function [diff]=diffy(f,xl, xu)
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(b) Write a script file (or void function file) that will calculate the difference between the maximum and minimum values of f(x)= x^3 - 2x^2 – (½)x + 2 over the domains x=[k-1 k+1] for k=0:10, plot each function and print the results in a nicely formatted table. You may want to make use of diffy.

MATLAB Reference:

error ('message') aborts m-file execution with specified error message
fminbnd (f, l, h) locates minimum of function *f* between *l* and *h*
fplot (f, range**)** plot function *f* over interval *range*
range is a 2 element vector. ex: fplot (f, [0, 10])
fprintf ('format', v1, v2, v3...) outputs values using format defined by '*format'*
format value fields: %f (fixed point), %e, %E (scientific), %g (general), %d (integer)
width and precision control: %12.3f format controls: \n (newline) \t (tab)
fzero (f, loc) returns a root of function *f*
if *loc* is a 2 element vector, looks for a root between *loc*(1) and *loc*(2)
otherwise looks for a root in the vicinity of *loc*
input ('prompt') outputs *prompt*, returns a value obtained from the user
length (x) returns the length of row or column vector *x*
linspace(begin, end, n) returns a vector containing *n* equally spaced values running from *begin* to *end*. if *n* is omitted 100 values are generated
max(*s*1, *s*2) returns the maximum of *s*1 and *s*2 (*s*1, and *s*2 scalars)
max (*v*) returns the maximum value in vector *v*
use [*maxVal*, *maxPos*] = **max** (*v*) to get both max value and its position
min(...) analogous to **max**
ones(r, c) creates a matrix *r* by *c* matrix of zeroes
ones(size) creates a matrix of zeros with the dimensions contained in 2 element vector *size*
plot (x, y) plot *y* values against *x* values. *x* are *y* are vectors of the same length
polyval (p, x) evaluates the polynomial defined by vector *p* for value *x*.
if *x* is a matrix the result will be a matrix of the same size.
roots (p) returns a column vector containing all of the roots of the polynomial defined by vector *p*
polyint(p) creates a row vector that represents the integral of the polynomial represented by p
polyder(p) creates a row vector that represents the derivative of the polynomial represented by p
size (x) returns a 2 element row vector containing the dimensions of *x* (# rows, # cols)
zeros(r,c) creates a matrix *r* by *c* matrix of zeroes
zeros(size) creates a matrix of zeros with the dimensions contained in 2 element vector *size*

Formulas:

$$x_R = x_U - \frac{f(x_U)(x_U - x_L)}{f(x_U) - f(x_L)} \quad x_{i+1} = x_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)} \quad x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$E_{MAX} = \frac{\Delta x^0}{2^N} \quad N = \log_2 \frac{\Delta x^0}{E_{MAX}} = \frac{\log(\Delta x^0 / E_{MAX})}{\log 2}$$

$$X_2 = X_U - d, \quad X_1 = X_L + d, \quad d = (\phi - 1)(X_U - X_L)$$

$$\phi = \frac{1+\sqrt{5}}{2} = 1.6180339887 \quad E_{MAX} = \frac{\Delta x^0 (\phi - 1)^{N-1}}{2} \text{ (assuming midpoint)}$$