Assignment 1. Space Propulsion

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1 Exercise 1

Consider a spacecraft fitted with a reaction control system (RCS) consisting of thrusters using Aerozine 50 fuel oxidized with nitrogen tetroxide, yielding a combustion temperature $T_c = 3372K$ and a gas with a molecular weight $MW = 0.0226kg/mol$ and adiabatic coefficient (specific heat ratio) $\gamma = 1.24$. The combustion pressure is assumed to be $P_c = 210MPa$, and each thruster produces $F = 100N$ of thrust.

The exit to throat area ratio A_e/A_t has a value in a range from 50 to 100 (to be chosen by the student), the following is requested:

1. Propellant mass flow
2. Specific impulse
3. Throat area

1.1 Solution

From the equation:

$$\frac{A_e}{A_t} = \frac{1}{M_e}\left[\frac{2 + (\gamma - 1)M_e^2}{\gamma + 1}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{1}$$

if we assume a given value for $\frac{A_e}{A_t} = 65$ we can compute the value of the Mach exit number M_e:

$$M_e = 4.870 \tag{2}$$

With this Mach number we can compute the characteristic velocity c^*:

$$c^* = \frac{1}{\sqrt{\gamma}}\left(\frac{\gamma + 1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\sqrt{R_g T_c} \tag{3}$$

and, given that the gas constant is $\frac{R}{MW} = 367.9\ JK^{-1}kg^{-1}$, c^* is:

$$c^* = 1697.39 \quad (4)$$

Now, using that c^* is also defined as:

$$c^* = \frac{P_c A_t}{\dot{m}} \quad (5)$$

we can obtain $\dot{m}$ if we know the area of the throat, which can be calculated, after knowing the vacuum thrust coefficient in the next equation:

$$C_F = \frac{F}{P_c A_t} = 1.8762 \quad (6)$$

Where the value of C_F as a function of γ and M_e has been obtained from the excel table. So, the value of the **throat area** can now be computed as:

$$\boxed{A_t = 2.53810^{-7}\ m^2} \quad (7)$$

And if we remember equation (5), we can now obtain the **propellant mass flow**, $\dot{m}$:

$$\boxed{\dot{m} = \frac{P_c A_t}{c^*} = 0.0314\ Kgs^{-1}} \quad (8)$$

And finally, given that the specific impulse, I_{SP} is given by the equation:

$$\boxed{I_{SP} = \frac{F}{\dot{m} g_0} = 324.6\ s} \quad (9)$$