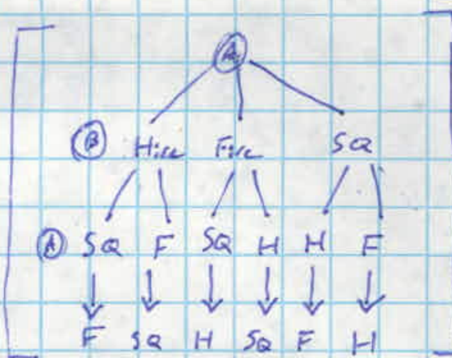
① (b) Three bidders that bid $v_1 = v_2 = v_3$ is a nash equilibrium since no player is incentivized to bid more than their opponents since they can win, and no player wants to bid less than their opponents since they will lose. Therefore three players bidding the same value is NE.

③ game tree:

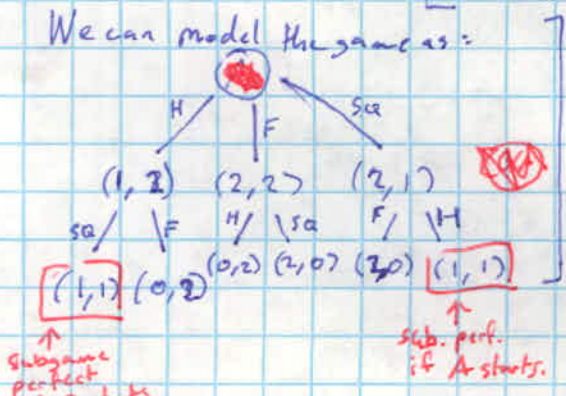


For player A:
let $H = 2$, $F = 1$, $SQ = 0$

For player B:
let $H = 0$, $F = 1$, $SQ = 2$

Player A vetos first:

We can model the game as:



Therefore if P_A vetos first, (1, 1) is the subgame perfect equilibria where (SQ, F) is the strategy.

similarly
If P_B vetos first, (1, 1) is still subperfect, but with strategy (H, SQ).

This is because each player wants to maximize their end gain by vetoing the best option for their competitor. Otherwise, they would maximize their opponent and minimize their gain.

(0, 2) with strategy set (F, H) if P_A starts is NE since if P_A chooses to Fire first, then he has no incentive to SQ first since his relative payoff is the same. However if P_A chooses F, P_B will always choose H since that maximizes their payout.

Similarly (2, 0) with strategy (F, SQ) is NE if P_B starts first.

② assume each job is assigned to Machine$_j$ w/ probability p.

		J_2: M_1	M_2
J_1	M_1	$(2, 0)$	$(1, \frac{1}{5})$
	M_2	$(1, \frac{1}{5})$	$(0, \frac{2}{5})$

Correllated Equilibrium:

For J_1: $\pi(M_1, M_1)\left[2-1\right] + \pi(M_1, M_2)\left[1-0\right] \geq 0$

$\pi(M_2, M_1)[1-2] + \pi(M_2, M_2)[0-1] \geq 0$

$\Rightarrow \boxed{\pi_{11} + \pi_{12} \geq 0 \, , \, -\pi_{21} - \pi_{22} \geq 0}$

For J_2: $\pi_{11}\left[0 - \frac{1}{5}\right] + \pi_{21}\left[\frac{1}{5} - \frac{2}{5}\right] \geq 0$

$\pi_{12}\left[\frac{1}{5} - 0\right] + \pi_{22}\left[\frac{2}{5} - \frac{1}{5}\right] \geq 0$

$\Rightarrow \boxed{\pi_{11}(-\frac{1}{5}) + \pi_{21}(\frac{1}{5}) \geq 0 \, , \, \pi_{12}(\frac{1}{5}) + \pi_{22}(\frac{1}{5}) \geq 0}$

$\sum_{a \in A} \pi_a = 1$

Mixed NE:

$2p^2_{M_1} + 1p^2_2 = 1p^2_{M_1} + 0 \rightarrow p_{M_1} = -p^2_2$

$0p^1_{M_1} + \frac{1}{5}p^1_2 = \frac{1}{5}p^1_{M_1} + \frac{2}{5}p^1_2 \rightarrow \frac{1}{5}p^1_{M_1} = -\frac{1}{5}p^1_2 \rightarrow p^1_2 = -p^1_{M_1}$

when $s = 2/3$, NE is (M_1, M_2) or $(1, 3/2)$.

a) as s increases, the mixed Equilibrium stays at (M_1, M_2) or $(1, \frac{1}{s})$ $\forall s \in \mathbb{R}^+$ therefore there is also no correllated equilibrium better than Mixed NE.

4 A) if $0 \le x_A + x_B \le 10$:

$$\begin{cases} P_A = 20x_A - 2x_A^2 - 2x_A x_B - C_A x_A \\ P_B = 20x_B - 2x_A x_B - 2x_B^2 - C_B x_B \end{cases}$$

$$\frac{\partial P_A}{\partial x_A} = 20 - 4x_A - 2x_B - C_A = 0$$

$$\frac{\partial P_B}{\partial x_B} = 20 - 4x_B - 2x_A - C_B = 0$$

if $x_A + x_B > 10$:

$$\begin{cases} P_A = -C_A x_A \\ P_B = -C_B x_B \end{cases} \Rightarrow \begin{matrix} \partial P_A = -C_A \\ \partial P_B = -C_B \end{matrix} \Rightarrow (-\alpha; -2)$$

$$x_A = \frac{20 - C_A - 2x_B}{4}$$

$$x_B = \frac{20 - C_B - 2x_A}{4}$$

$$2x_A = \frac{20 - C_A}{2} - \frac{20 - C_B - 2x_A}{4}$$

$$8x_A = 40 - 2C_A - 20 + C_B + 2x_A$$

$$\boxed{x_A = \frac{20 - 2C_A + C_B}{6}}$$

$$x_B = \frac{20 - 2C_B + C_A}{6}$$

$$x_A = \begin{cases} 3 & \text{if } 0 \le x_A + x_B \le 10 \ \& \ \alpha = 4 \\ \frac{20}{6} & \text{if } \text{—''—} \ \& \ \alpha = 1 \end{cases}$$

$$x_B = \begin{cases} \frac{20}{6} & \text{if } \text{—''—} \ \& \ \alpha = 4 \\ \frac{17}{6} & \text{if } \text{—''—} \ \& \ \alpha = 1 \end{cases}$$

for $0 \le x_A + x_B \le 10$: if $\alpha = 4$, $(x_A^*, x_B^*) = (3, \frac{20}{6})$
if $\alpha = 1$, —''— $= (\frac{20}{6}, \frac{17}{6})$

for $x_A + x_B > 10$: if $\alpha = 4$, $(x_A^*, x_B^*) = (-4, -2)$
if $\alpha = 1$, —''— $= (-1, -2)$

Hence Expected values if A & B are unsure:

$$\mathbb{E}X_A = 3p + \frac{20}{6}(1-p) = \frac{20}{6} - \frac{2p}{6}$$

$$\mathbb{E}X_B = \frac{20}{6}p + \frac{17}{6}(1-p) = \frac{17}{6} + \frac{p}{2}$$

(4B) if X_A is decided

$$X_B^* = \frac{20-2x_A^*-c_B}{4}$$

$$\to P_A = \frac{1}{2}\left(20x_A - 2x_A^2 + c_B x_A\right) - c_A x_A$$

$$\frac{\partial P_A}{\partial x_A} = -2x_A + \frac{c_B+20-2c_A}{2} = 0 \to X_A = \frac{20+c_B-2c_A}{4}$$

$$\therefore x_B = \frac{c_B}{2} - \frac{20+c_B-2c_A}{4} \to x_B = \frac{3c_B-2c_A+20}{4}$$

$\therefore$ if $0 \le x_A+x_B \le 10$

$$x_B = \frac{26-2\alpha}{4}$$

$$x_B = \begin{cases} 9/2 & \text{if } \alpha = 4 \\ 6 & \text{if } \alpha = 1 \end{cases}$$

$$\mathbb{E} X_B = \frac{9p}{2} + (6(1-p)) = 6 - \frac{3p}{2}$$

else if $x_A+x_B > 10$: $X_B = -2$

$x_B = -2$ regardless of α

$$\mathbb{E} X_B = -2p + (1-p)(-2)$$

$$\mathbb{E} X_B = -2$$

5 (c) Suppose

		P_2 a	b
P_1	a	-5, -1	6, -1
	b	4, -2	3, -2

Pure NE are (6, -1) and (4, -2) [seen in red]

~~or~~

There are infinite mixed strategy NE, where a & b are both parts of a mixed strategy solution where $E\,P_1 = 3.25$. P_2's mixed strategy payout for $(a, b) = (0.2727, 0.7273)$

P_1's maximin strategy payout = 3 which is slightly smaller than the expected mixed payout of 3.25.