CS 330 HW 7

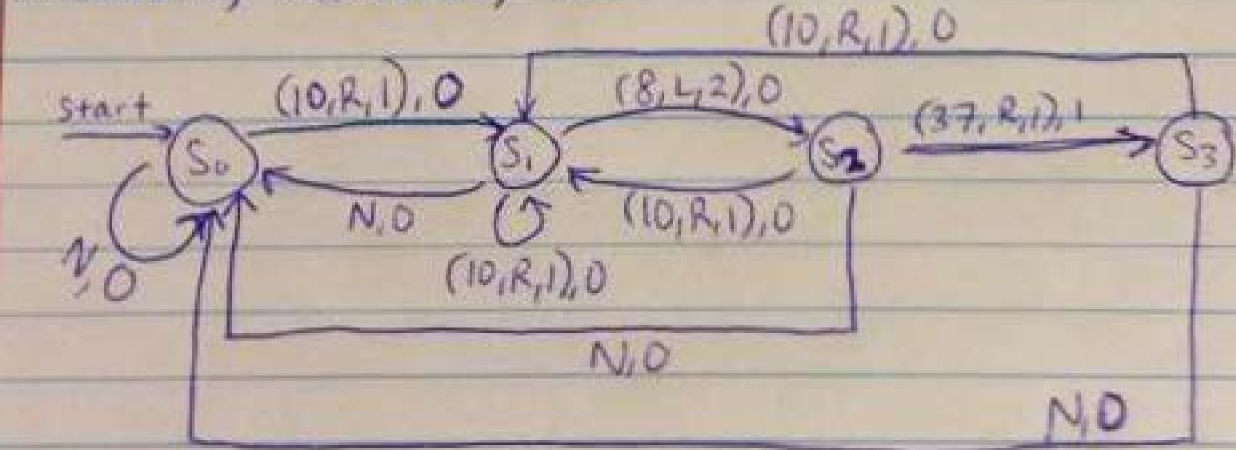
1) Construct an FSM for a combination lock that contains the numbers 1 through 40 and only opens when 10 right, 8 second left, 37 right is entered. Each input is a triple consisting of a number, direction of turn, and number of times the lock is turned in that direction.

The lock opens if input is (10, R, 1)(8, L, 2)(37, R, 1)

There need to be 3 states representing the input of a correct signal.

There is one valid input; all others are N.

Machine outputs 1 when the input is valid and lock is unlocked; otherwise, 0.

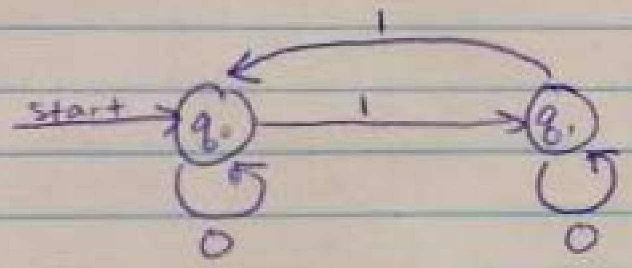


2) Construct a deterministic finite-state machine automation that recognizes the set of all bit strings containing an even # of 1's.

This machine just needs to remember if the # of 1's is even or odd. Thus, it has 2 states.

The start state q_0 is the only final state of the machine. If input bit is 1, then it goes to state q_1. It returns to state q_0 from q_1 if a 1 is seen.

As the # of 0's is irrelevant, the transition for an input of zero (0) on any state is to loop back to the same state.



3) Determine whether 1011 belongs to each regular set:

a) 10^*1^*

Yes, since the set can produce the string $10^1 1^2$ (assuming 0^* can give 0 and 1^* can give 1^2).

b) $0^*(10 \cup 11)^*$

Yes, since the set can produce the string (10)(11), assuming 0^* can give a null string.

c) $1(01)^*1^*$

Yes, since the set can produce the string 1(01)1, assuming $(01)^*$ can give 01 and 1^* can give 1.

d) $1^*01(0 \cup 1)$

Yes, since the 1^* can give 1 and $(0 \cup 1)$ can give 1.

e) $(01)^*(11)^*$

Yes, since the set can produce the string (10)(11)

f) $1(00)^*(11)^*$

No, since the set produces odd-lengthed strings.

g) $(10)^*1011$

Yes, since $(10)^*$ can give a null string.

h) $(1 \cup 00)(01 \cup 0)1^*$

Yes, since $(1 \cup 00)$ can give 1, and $(01 \cup 0)$ can give 01, and 1^* can give 1 to produce 1011.

4) Show that the set $\overbrace{\{1^{n^2} \mid n = 0, 1, 2, \ldots\}}^{S}$ is not regular using the pumping lemma.

The <u>pumping lemma</u> states that if $M = (S, I, f, s_0, F)$ is a deterministic finite state automation and x is a string in $L(M)$, the language recognized by M, with $l(x) \geq |S|$, then there are strings u, v, and w in I^* such that $x = uvw$, $l(uv) \leq |S|$, and $l(v) \geq 1$, and $uv^iw \in L(M)$ for $i = 0, 1, 2, \ldots$

Suppose the set S is regular, recognized by a deterministic finite state ~~machine~~ automation M. Let $x = 1^{n^2}$ for some $n \geq |S|$. <u>By the pumping lemma</u>, we know $x = uvw$ such that uv^iw is in the set for all i. Since there is only one symbol (one is involved), we know $u = 1^x$, $v = 1^y$, and $w = 1^z$, so that the statement that uv^iw is the statement that $(x + z) + iy$ is a perfect square. But this <u>cannot</u> be true, since $(x + z) + iy$ increases by a quantity proportional to i, which is constantly increasing at a constant rate.

<u>Therefore</u>, the number ~~need not be a~~ perfect square for each i. This tells us the set is irregular.