# Homework Assignment 3

CS 330 Discrete Structures
Spring Semester, 2015

## Due: Friday, February 27, 2015

1. The Mayan Tzolkin calendar combines a cycle of twenty day names with a second cycle of thirteen numbers to produce $20 \times 13 = 260$ unique day designations; for more information, see

   http://en.wikipedia.org/wiki/Tzolkin

   How many people do you need to have the probability be at least 0.5 that some pair of them share a Tzolkin day designation birthday?

2. Page 467, problem 30

3. Page 477, problem 22

4. In the lottery of Bulgaria, a player choses 6 numbers, each number ranges from 1 to 49; order does not matter and a number can only be chosen once.

   (a) How many choices does a player have for his/her picks?

   (b) On September 6, 2009 and then again on the subsequent drawing four days later, the identical set of numbers 4, 15, 23, 24, 35 and 42 were the winning combination—selected (in a different order) by a machine picking balls live on television. The results caused suspicions of manipulation.

      i. What are the odds of this happening?
      ii. Over how many lottery combinations does the probability become 50-50 that *some pair* of the picks will have been the same?
      iii. In the Bulgarian lottery duplication, the set of numbers occurred in successive drawings. Over how many lottery combinations does the probability become 50-50 that some *adjacent pair* of the picks will have been the same?

5. The students in CS 330 decide to send a representative to the dean to complain about Professor Reingold's terrible jokes. But nobody in the class will agree to be the representative, so they decide on the following strategy: Each student will toss his/her coin simultaneously; if all the coins but one show the same face, the person who tossed the odd coin must be the representative. If any other combination of heads and tails occurs, the class will toss coins again.

   (a) Assuming that there are $n$ students in the CS 330 and that all of the coins are fair, what is the expected number of rounds (number of coin tosses each student has to make)? (*Hint*: Explain why the probability of an odd toss in any given round is $p = n/2^{n-1}$. Then explain why the probability of ending on the $k$th round is $(1 - p)^{k-1}p$. Finally, using the method of Example 25 on page 167 of Rosen, get the answer $2^{n-1}/n$.)

   (b) One of the $n$ students decides to cheat and uses a false coin that comes up heads much more often than tails. How does this change the answer in part (a)? Explain.

6. (a) A couple decides to keep having children until they have both sexes. If $p$ is the probability of having a boy, what is the expected number of children they will have? (*Hint*: Use the method of Example 25 on page 167 of Rosen to evaluate the summation, giving the answer $\frac{1-p+p^2}{p(1-p)}$.) Does your answer make sense if $p = 0$ or $p = 1$? Explain.

   (b) A different couple decides to keep having children until they have a child of the same sex as their first child. If $p$ is the probability of having a boy, $1 > p > 0$, what is the expected number of children they will have? What if $p = 0$ or $p = 1$? Explain.