Illinois Institute of Technology
Department of Computer Science

# Third Examination

CS 330 Discrete Mathematics
Spring, 2015

11:25am–12:40pm, Wednesday, April 29, 2015
113 Stuart Building

Print your name and student ID, *neatly* in the space provided below; print your name at the upper right corner of *every* page. Please print legibly.

| | |
|---|---|
| Name: | |
| Student ID: | |

This is an *open book* exam. You are permitted to use the textbook (hardcopy only), hard copies of any class handouts, anything posted on the web page, any of your own assignments, and anything in your own handwriting. Foreign students may use a hardcopy dictionary. *Nothing else is permitted*: No calculators, laptops, cell phones, Ipads, Ipods, communicators, GPSes, etc.!

Do all five problems in this booklet. *All problems are equally weighted, so do not spend too much time on any one question.*

*Show your work!* You will not get partial credit if the grader cannot figure out how you arrived at your answer.

| Question | Points | Score | Grader |
|---|---|---|---|
| 1 | 20 | | |
| 2 | 20 | | |
| 3 | 20 | | |
| 4 | 20 | | |
| 5 | 20 | | |
| Total | 100 | | |

1. **Breadth First Search of Graphs**

   In the lecture of April 1 (see bottom of page 3 of the April 1–6 notes) Professor Reingold stated the following property of the values $d[v]$ assigned by BFS:

   > Suppose that during the execution of BFS on a graph $G = \langle V, E \rangle$, the queue $Q$ contains the vertices $\langle v_1, v_2, \ldots, v_r \rangle$, where $v_1$ is the head of $Q$ and $v_r$ is the tail. Then, $d[v_r] \leq d[v_1] + 1$ and $d[v_i] \leq d[v_{i+1}]$ for $i = 1, 2, \ldots, r - 1$.

   Use induction on the number of queue operations to prove this property.

2. **Depth First Search of Graphs**

   Describe and analyze a DFS algorithm to determine if an undirected graph is 2-colorable, that is, if the vertices can be colored with 2 colors so that no two adjacent vertices have the same color. Your algorithm should either produce the 2-coloring or report the vertices of an odd-length cycle (the only thing that renders a graph not 2-colorable; why?). You will get full credit for an $O(|V| + |E|)$ algorithm and partial credit for slower algorithms.

3. **Graph Structure**

   In the lecture on April 13, Professor Reingold proved that in a planar graph $|E| \leq 3|V| - 6$ (see the middle of page 2 of the notes, or Corollary 1 on page 722 of Rosen). Is the *converse* true? That is, can we conclude that the graph is planar just because $|E| \leq 3|V| - 6$? Prove your answer.

4. **Regular Languages**

State, with proof, whether each of the following languages is regular:

(a) Odd numbers, written in binary, read least significant bit first (that is, from left to right).

(b) Numbers not divisible by 3, written in binary, read most significant bit first (that is, from right to left).

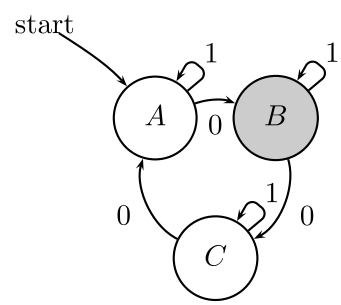
(c) The set of strings of zeros and ones containing at the same number of zeros and ones.

(d) The set $\{0^n 1^m | n \text{ and } m \text{ are congruent modulo } 3\}$.

(e) The set $\{0^n 1^m | n > 2m\}$.

5. **Finite State Machines**

Consider the following finite state machine in which state $A$ is the starting state and state $B$ (shaded) is the only accepting state:




(a) Describe clearly and succinctly *in words* the language recognized.

(b) Construct a regular expression for that language.