CS539-Spring 2018, Final-Exam
2nd May, 2018.
Closed Books. No Calculators
2:00pm - 4:00pm

Prove All Results. Total points: 75+75 (Take-Home).
Sub-questions that must be solved at home are distinctly marked.

1. (a) Find a correlated equilibrium (that is not a mixed NE if possible) in the following hot-potato routing game:

   Table 1: Routing Game

Payoff	HotPotato	Planned
HotPotato	-5,-5	-2,-6
Planned	-6,-2	-1,-1

   (b) Also compute mixed Nash solutions and compare.

   (15+15 pts)

2. (a) Suppose in non-atomic routing in networks, the delay function on edges is of the form ${af_e}^2 + b_e$ where f_e is the flow on edge e. Show an example network where the price of anarchy is exactly

   $$\frac{1}{1 - \frac{2}{3\sqrt{3}}}$$

   (b) **<u>Take-home:</u>** Show that $PoA \leq \frac{1}{1-\frac{2}{3\sqrt{3}}}$.

   (10+20 pts)
   Hint: Use Pigou's Example

3. (a) Recall the local connection game defined by a set of vertices and graph G where edges are to be added to build connections, suppose the cost to user u (i.e. user at node u) is defined to be $C_u = \alpha n_u + \beta \sum_v dist(u,v)$ where n_u is the number of edges incident to u, that u purchases , and $dist(u,v)$ is the shortest distance from u to v. $\alpha, \beta \geq 0$.

Can a path that connects all nodes be a Nash equilibrium solution. If so, then derive a condition on α for a path to occur as a Nash equilibrium, otherwise prove that a path can never be a Nash equilibrium.

(b) **Take-Home:** Suppose the cost function is defined to be $\alpha n_u + \sum_v \sqrt{dist(u,v)}$. Characterize NE solutions as a function of α.

(15+15 pts)

4. (a) In a cloud computing environment, k machines are required to complete the task of finding a solution to a physics problem. Machine owners bid to provide machines for the job where the ith machine owner has valuation v_i for the machine. Show an incentive compatible method to determine the winners.

(b) **Take-Home:** Suppose you need to consider only the cost of connecting the machines. The machines are represent by a vertex set of a graph and edges between vertices represent communication between machines. Each edge is controlled by a player who declares a cost for using the edge. The goal is to determine a connected subgraph to communicate amongst all nodes (hint: Minimum Spanning Tree is one such subgraph). Determine an incentive compatible mechanism for this problem.

(10+20 pts)

5. (a) In the Fisher linear market equilibrium model, let M be a market with n traders and m goods, with each trader i having endowment m_i and $u_{ij} = v_{ij}x_{ij} = r_j s_i x_{ij}$ for good j where $r_j, s_i \in \mathcal{Z}^+$, and $x_{ij} \in \mathcal{R}^+$ i..e. Show that the prices of 2 goods j and j' acquired by a trader is in the ratio $r_j/r_{j'}$, i.e. $p_j/p_{j'} = r_j/r_{j'}$. The available quantity of good j is a_j.

(b) **Take-home:** Use this to determine the price of the goods and then determine an allocation at Market equilibrium.

(10+20 pts)