HW 8

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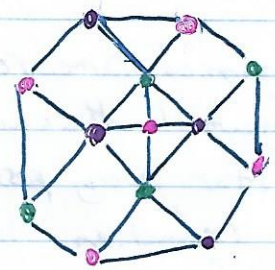
34.4-3

The strategy does not yield a polynomial-time reduction because there are $\Omega(n)$ free variables in which there are $\Omega(2^n)$ rows in the corresponding truth table for every possible assignment. This means that the professor's reduction increases exponentially the problem's size by a factor of 2^n.

34.4-7

Considering a directed graph where vertices correspond to variables in the formula: $\bigwedge_i (x_i \vee y_i)$ in the set $\{\alpha_i\}$. For each x_i, place an edge from $\neg x_i$ to y_i and $\neg y_i$ to x_i. This ensures that all origin and destination edges are true where the formula is satisfiable as there is a path from a vertex to it's negation vertex. in any case by running all subsets by shortest path which in $O(n^2 \lg n)$ time. This is reduced to $\Theta(n^2)$ by checking is a vertex and its negation are contained in a selected connected component or not.

3) ⓐ one instance of a legal 3-coloring of the crossover gadget G showing how opposite corners need to have the same color:



E

ⓑ To verify the properties in #3b, we permute the pattern of colors: For each edge (x,y) in E replace each point with another edge crossing (x,y) with a copy of G, identifying outer ~~edges~~ corners of E' with x & y.

ⓒ Corralary to the above, a graph H is formed from a planar set of (G', E'). If ~~H'~~ is a 3-colorable form of G', then the first property in 3b states that G is a 3-coloring of H. Conversely, this is verified by property 2, which means that this reduction is NP-hard.