Illinois Institute of Technology
Department of Computer Science

First Examination

CS 330 Discrete Mathematics
Fall, 2013

11:25am–12:40pm, Wednesday, September 18, 2013
IT 6C7-1 (sixth floor of IIT Tower, 35th & State)

Print your name and student ID, *neatly* in the space provided below; print your name at the upper right corner of *every* page. Please print legibly.

Name:	
Student ID:	

This is an *open book* exam. You are permitted to use the textbook (hardcopy only), hard copies of any class handouts, anything posted on the web page, any of your own assignments, and anything in your own handwriting. Foreign students may use a hardcopy dictionary. *Nothing else is permitted*: No calculators, laptops, cell phones, Ipads, Ipods, communicators, GPSes, etc.!

Do all five problems in this booklet. *All problems are equally weighted, so do not spend too much time on any one question.*

Show your work! You will not get partial credit if the grader cannot figure out how you arrived at your answer.

Question	Points	Score	Grader
1	20		
2	20		
3	20		
4	20		
5	20		
Total	100		

1. **Mathematical Induction.**

   Suppose we have a statement S_n and we know the following facts:

   (a) S_1 is true.

   (b) If S_n is true, then so is S_{n-1}.

   (c) If S_n is true, then so is S_{2n}

   Prove by induction that S_n is true for all integers $n \geq 1$.

2. **Growth rates.**

   (a) Does $\binom{2n}{n}$ grow slower than, the same as, or faster than n^n? Prove your answer.

   (b) Does $n(H_n)^2$ grow slower than, the same as, or faster than $n \log_2 n$? Prove your answer.

3. **Algorithms.**

   (a) Suppose you compute 2^n using the following recursive function:

```
FUNCTION PowerOfTwo(n)
  BEGIN
    IF n=0 THEN RETURN 1;
    ELSE RETURN PowerOfTwo(n-1) + PowerOfTwo(n-1);
  END
```

   Analyze the number of additions needed to compute 2^n.

   (b) Show how to compute 2^n in $O(n)$ additions.

   (c) Using (b), analyze the number of additions to compute the sum $\sum_{i=0}^{k} 2^i$.

4. **Rules of Sum Product.**

   Use the rules of sum and product to give combinatorial interpretations of the identity

$$\left[\sum_{k=0}^{n} \binom{n}{k}\right]^2 = \sum_{k=0}^{2n} \binom{2n}{k} = 2^{2n} = 4^n.$$

   (*Hint*: Consider the number of ways to select a subset of a set of n couples. Note that you must give combinatorial interpretations for all four expressions.)

5. **Binomial Coefficients.**

   Find the coefficient of x^2 in each of the following polynomials. You need not simplify powers, factorials, or binomial coefficients.

   (a) $(1+2x)^{2013}$

   (b) $(1-x)^{-2013}$

   (c) $(1-x^3+x^9-x^{27}+\cdots)^{2013}$

   (d) $(1-2013x)^{2n+1}$