Illinois Institute of Technology
Department of Computer Science

Solutions to Homework Assignment 9

CS 430 Introduction to Algorithms
Spring Semester, 2018

Solution:

1. To prove
$$|OPT| \geq 2 \sum_{i=\lceil \frac{n}{2} \rceil + 1}^{n} l_i,$$
consider n's parity:

   - When n is even, it's the same as the inequality (2) in the lecture. The proof has already been shown in the lecture notes.
   - When n is odd, we denote n as $n = 2k + 1$. What we need to prove is
$$|OPT| \geq 2 \sum_{i=k+2}^{2k+1} l_i$$
Following the same analysis in the lecture, we should have $|OPT| \geq |T_{2k+1}| \geq \sum \min\{l_i, l_j\}$. Each l_i appears in this list at most twice. Still the edges were labeled in decreasing order, we can replace edges in the fist k l_i with members of the last k edges l_i $(l_{k+2}, l_{k+3}, \cdots, l_{2k+1})$, this process yields:
$$|OPT| \geq 2 \sum_{i=k+2}^{2k+1} l_i + l_{k+1} > 2 \sum_{i=k+2}^{2k+1} l_i$$

   So we proved
$$|OPT| \geq 2 \sum_{i=\lceil \frac{n}{2} \rceil + 1}^{n} l_i,$$

2. By the triangle inequality, the last edge is no longer than the sum of the lengths of the other edges; therefore that edge can contribute no more than 1/2 to the ratio $\frac{|NN|}{|OPT|}$. The last edge is thus not the cause of the logarithmic ratio.