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$\wedge$ = and
$\vee$ = or

Homework 1

32) a. Construct a truth table for $p \to \bar{p}$

p	$\bar{p}$	$p \to \bar{p}$
0	1	1
1	0	0

b. Truth table for $p \leftrightarrow \bar{p}$

p	$\bar{p}$	$p \leftrightarrow \bar{p}$
0	1	0
1	0	0

d. Truth table $(p \wedge q) \to (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \to (p \vee q)$
1	1	1	1	1
1	0	0	1	1
0	1	0	1	1
0	0	0	0	1

74) c. Prove or disprove this statement about the floor & ceiling function: $\lceil \lceil x/2 \rceil /2 \rceil = \lceil x/4 \rceil$ for all real x.

$\lceil x \rceil = n - 1 < x \leq n$

$\frac{n-1}{2} < \frac{x}{2} \leq \frac{n}{2}$

$\therefore \left\lceil \frac{x}{2} \right\rceil = \frac{n}{2}$, $\frac{\lceil x/2 \rceil}{2} = \frac{n/2}{2} = \frac{n}{4}$

$$\boxed{\therefore \left\lceil \frac{\left\lceil \frac{x}{2} \right\rceil}{2} \right\rceil = \frac{n}{4}}$$

3) Given $V_1, V_2, \ldots V_n$ of yes/no votes, describe an algorithm to determine the majority vote.
Find big-theta expression for the speed of the algorithm.

Assume each vote V_n is $= 1$ for a yes vote and $= 0$ for a no vote.

If $\sum_{i=1}^{n} V_n < n/2$, then the majority vote is NO.

If $\sum_{i=1}^{n} V_n \geq n/2$, then the majority vote is YES.

$$\boxed{V_n \rightarrow \Theta(n)}$$

42) If $f(x)$ is $O(g(x))$, does it follow that $2^{f(x)}$ is $O(2^{g(x)})$?

No. Let $f(x) = 2x$, $g(x) = x$

Proof $f(x)$ is $O(g(x))$ since $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{2x}{x}$

$= 2$ (finite),

$\therefore \left| \frac{f(x)}{g(x)} \right| \le 2$ when $x > 0$

~~$|f(x)| \le 2|g(x)|$ when $x>1$~~

and $\boxed{f(x) \text{ is } O(g(x))}$

Proof $2^{f(x)}$ is not $O(2^{g(x)})$

$$\lim_{x\to\infty} \frac{2^{f(x)}}{2^{g(x)}} = \lim_{x\to\infty} \frac{2^{2x}}{2^{x}} = \lim_{x\to\infty} 2^{2x-x} = \lim_{x\to\infty} 2^{x}$$

$= \infty$

$\therefore x > 1$, and $\boxed{2^{f(x)} \text{ is not } O(2^{g(x)})}$

30) Prove that $H_1 + H_2 + \ldots + H_n = (n+1)H_n - n$

Proof $H_1 + H_2 + \ldots + H_n = (n+1)H_n - n$

$H_j = 1 + 1/2 + 1/3 + \ldots + 1/j$

$H_1 = (1+1)H_1 - 1 \longrightarrow$ let this be true for $n=k$

$H_1 + H_2 + \ldots + H_k = (k+1)H_k - k$

$H_1 + H_2 + \ldots H_{k+1} = (H_1 + H_2 + \ldots + H_k) + H_{k+1}$

$= (k+1)H_k - k + H_{k+1} = (k+1)\left[H_{k+1} - \frac{1}{k+1}\right] - k + H_{k+1}$

$= (k+2)H_{k+1} - (k+1)$

$\therefore$ true for $n = k+1$ when true for $n=k$, and

$\boxed{H_1 + H_2 + \ldots + H_n = (n+1)H_n - n}$

6) If Bob has the number 1, and the clock chimes once, he will leave because he knows he has the smallest pos. integer, so Carol must have # 2.

If Bob has the number 2, ~~[illegible]~~ then he knows that Carol must have either 1 or 3. If Carol leaves on the first chime, he knows her number is 1. If she does not, he knows his number (2) must be smaller than hers, so hers must be 3, and he leaves the room.

This logic is true for all $n > 1$, including $n=k$ and $n=k+1$

$\therefore$, a person w/ smaller number (n) will leave the room on the n^{th} chime