Game Theory: Algorithms and Applications
CS 539

Fall 2019
HomeWork 1
Due Thursday 5th September, 11:59pm
Total: 100 points

1. 5 CS labs wish to communicate data with each other. They want to add edges so as to create a connected graph amongst themselves. All possible edges can be used. Each node gains revenue kC, C a large constant, when she is connected to $k \leq 4$ other nodes. However, any node u pays the cost of adding an edge (u, v) with another node v if it wishes to add that edge. Suppose the cost of each edge is the same, c_1. What Nash equilibrium subgraphs can be produced for various values $c_1, 1 \leq c_1 \leq 4C$.
   Can you extend the argument for n Labs.
   Hint: It helps to define the strategy sets and pay-offs for each node.

2. In a sealed-bid e-bay auction, bidders submit bids based on their valuations. Let there be n bidders with valuations $v_1 > v_2 > \ldots > v_n > 0$. The object is won by the bidder with the highest price. In first price auctions the winner is assigned the item at the price that he bids. Find all pure Nash Equilibrium of this game. Does player 1 have a special place?
   Also consider the second-price auction where the highest bidder gets the item at the price of the second-highest bid. Characterize all possible Nash equilibrium for a 3 player game with valuations v_1, v_2 and v_3 respectively.

3. N consumers are to be assigned to P service providers to access the internet. Define the rate of transmission given to a consumer by service provider i to be $R_i = B_i/n_i$ where n_i is the number of consumers accessing service provider i and B_i is the total bandwidth of provider i. Consumers can choose their providers independently optimizing the payoff which is the rate offered, i.e. R_i.

   Determine a pure Nash equilibrium when $B_i = B$ for all base stations. Is there a Nash equilibrium in the general case.

4. Determine Nash Equilibrium for the "Tragedy of Commons/Bandwidth" sharing game involving n players when the utility function for player i is given by

$$U_i = X_i K - \alpha \sum_j X_j^2$$

   where X_i is the units of traffic sent by player i.

5. Determine Nash Equilibrium in a Cournot Game with 2 players where the cost function for producers is given by

$$c_i = x_i^2$$

   where the utility for the i^{th} player is defined to be $U_i = x_i \cdot p(x_1 + x_2) - c_i$, and $p(y) = \max\{0, K - \sqrt{y}\}$, K being a constant (almost as defined in class).