1. **Generalized Matching Pennies: Two players choose a number between 1 and N. If the players choose the same number and it is even then player 2 pays $1 to player 1, if odd then player 1 pays $1 to player 2, otherwise no payment is made. Does this game have a pure NE? Find a mixed Nash Equilibrium.**

   PURE: There is a pure NE for this game, unlike the typical matching pennies. Player 1 will always want to choose an even number because choosing an odd number is never advantageous; if player 2 has chosen an odd number, there will be no penalty to either player. However, if player 2 has chosen an even number as well, player 1 receives $1 from player 2. The reverse is true for player 2. Therefore, the pure NE of this game is that player 1 always chooses even, and player 2 always chooses odd, resulting in no payments to either player.

   MIXED: A mixed NE for this game would make a little more sense, as it would allow for money to change hands. For instance, if both players chose even 50% of the time and odd 50% of the time, there is then a 25% chance that they will win $1 from each round, whereas in the pure NE, there is no chance either player will win any money. Of course, there is also a 25% chance they will lose $1 each round as well, so overall, the utility is the same as the pure NE.

2. **Consider a network where there are two players sending one unit of traffic using a single path. P(j, i) represents the path j of player i and each path costs C. If the path chosen by player 1 and the one by player 2 each share an edge, then both the players share the cost of the path, but introduce a congestion cost of B, also shared evenly. The objective of each player is to minimize the objective function defined by: cost of the path + congestion.**
   a. **Find pure and mixed NE when B = C.**

      PURE: When path and congestion costs are equal, there is no incentive to maximize or minimize edge sharing, since the overall cost of a shared edge (BC) is equal to the cost of an unshared edge (2C). Therefore, both players can choose to simply minimize path cost without incentive to introduce more complex strategies.

      MIXED: Another NE exists where both players choose to maximize edge sharing 50% of the time and avoid edge sharing the other 50% of the time.

   b. **What happens when B < C, and B > C.**

      When B < C, there is a strong incentive to maximize edge sharing, since sharing edges reduces the overall cost. Where the cost would have been 2C, the cost is now C + B < 2C. Therefore, all players would choose to share edges as long as it doesn't incur a cost equal or greater than C – B.

      When B > C, there is a strong incentive to minimize edge sharing, since sharing edges inflates the overall cost. Where the cost is 2C for an unshared edge, the cost is C + B > 2C for a shared edge. Therefore, all players would avoid sharing edges as long as it doesn't incur additional cost equal or greater than B – C.

3. **Consider the hot-potato routing/coordination routing game discussed in class. Find a mixed equilibrium of that game.**

MIXED: In the hot potato routing game, in the case where player 1 chooses the planned option, there is a 50% chance of a -5 outcome and a 50% chance of a -2 outcome. In the case where player 1 chooses the hot potato option, there is a 50% chance of a -4 outcome, and a 50% chance of a -1 outcome. Unfortunately, since both the bad and good outcomes for hot potato are respectively better than the bad and good outcomes for planned, the mixed NE is also a pure NE, with 100% hot potato and 0% planned strategies for both players.

4. **Games against Nature: In a network there are k paths, P1, P2, P3 . . . Pk available to send data. On each path there is a compromised edge, on which the player might lose information, the probability of loss of information on each path Pi being li . Sending information along path Pi gives a pay-off of vi. The strategy set of the player sending data is {P1, P2, · · · , Pk}. Mother Nature is the other player with strategies in the set {Lettinginformationflow, Probabilisticallylettinginformationflow}. So path Pi can either have payoff vi or payoff vi(1 − li) to the first player. Find mixed Nash Equilibrium of this game.**

   A mixed NE would choose the path with the highest unobstructed payoff half the time, and the path with the highest obstructed payoff half the time. This way, if "nature" lands on probabilistic flow, we still get the best possible outcome half the time. The reverse is true as well.

5. **Consider the two-player Rock/Paper/Scissors (strategic) game. Remember that the game has the following rule: Rock beats Scissors, Paper beats Rock, and Scissors beats Paper. The winning player receives $10 from the loser. A tie results in zero gain for both players. Find a mixed Nash equilibrium in this game.**

   There is no dominant strategy in a rock/paper/scissors game, so each strategy is given equal weight. A mixed NE is for each player to play rock 1/3 of the time, paper 1/3 of the time, and scissors 1/3 of the time.