CS539-Spring 2019, Mid-Exam
5-7 PM, October 21st, 2019. Open Class Notes

Name : _____ SID : _____

Prove All Results. Do as many problems/sub-problems as possible. Take the rest home at reduced credit (50%). Your take-home part is due 7 PM, Oct. 22nd and overwrites your in-class part. Total Points: 100.

1. (Third-price auction) Consider a third-price sealed-bid auction, which differs from a first- and a second-price auction only in that the winner (the person who submits the highest bid) pays the third highest price. (Assume that there are at least three bidders.)

   (a) Show that the action profile in which each player bids her valuation is not a Nash Equilibrium.

   (b) Find a Nash Equilibrium. (There are ones in which every player submits the same bid.)

2. Consider a cloud computing center with two machines of speed $S_1 = 1$ and $S_2 = s$. The latency job i faces when it is assigned to machine j is $l_j = \frac{\sum_{i' \in j} w_{i'}}{S_j}$. Each job wants to minimize its latency. Given two jobs of weight $w_1 = w_2 = 1$, determine:

   (a) Mixed equilibrium for the following cases:
   (i)$s \leq 1/2$ (ii)$1/2 \leq s \leq 2$ (iii)$s > 2$.

   (b) Determine if there is a correlated equilibrium that is strictly better than the Nash equilibrium when $s = \frac{2}{3}$.

3. Co-ordination Game: Two people decide on policies for the department. Given the options they attempt to make a decision by successively vetoing one option at a time, i.e. person A vetoes one option. Then if options are left, person B vetoes from amongst the ones left and so on.

   Suppose there are 3 options $Hire, Fire, StatusQuo$ with the following ordering for player A over the choice of options, $Hire > Fire > StatusQuo$ while player B has the ordering $StatusQuo > Fire > Hire$. Model this as an extensive game. Find the sub-game perfect equilibrium. Find a Nash equilibrium that is different from the sub-game perfect equilibrium.

4. Two wireless companies are bidding on spectrum. The spectrum could be apportioned to the two companies. Company A has the following profit function $P_A = p(x_A, x_B)x_A - c_A x_A$ where c_A depends on the probability of a company A collaborating with an external agency, while company B has $P_B = p(x_A, x_B)x_B - c_B x_B$ where x_A, x_B is the units of spectrum used by company A and B, respectively.

   (a) Determine the strategic units of spectrum that the companies bid at Nash equilibrium, when $p(x_A, x_B) = 2 \cdot (10 - x_A - x_B), 0 \leq x_A + x_B \leq 10$. For $x_A + x_B > 10$, $p(x_a, x_b) = 0$. Also, $c_A = \alpha, c_B = 2$, where $\alpha = 4$ with probability p and $\alpha = 1$ otherwise.

   (b) What are the Stackelberg equilibrium strategies if company A is the leader and can determine its strategy first?

5. (a) Prove or disprove: If the equilibrium payoff to player 1 in a zero sum game with pure equilibrium is v, then any strategy pair that gives player 1 a payoff of v is an equilibrium.

   (b) Prove or disprove: Suppose the payoff matrix A of player 1 in a strictly competitive game G' has all entries greater than or equal to the corresponding entries in B, in another strictly competitive game G, then G' has no equilibrium that is worse off than in an equilibrium in G

   (c) Find a game which has a mixed equilibrium where the player's equilibrium payoff is greater in value than the max-minimum payoff of the player.