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CS 330 HW 6

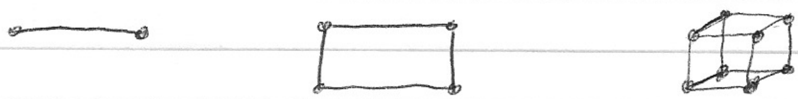
1) Devise a DFS algorithm for constructing Euler circuits in directed graphs.

G = connected graph with even-degree verticies
(defn euler (G)
    c = circuit in G beginning from random vertex v with edges successively added to form a path returning to v
    H = G with edges of c removed
    (while H has edges
        subcircuit = circuit in H beginning at random vertex v that has an endpoint in c
        H = H with edges of subcircuit removed as well as all isolated vertices
        c = c with subcircuit inserted @ appropriate vertex)
    return c)

Euler Circuit

2) Describe the tree produced by BFS and DFS for the n-cube graph Q_n, where n is a positive integer.

1-cube 2-cube 3-cube



So Q_n can be constructed by taking identical (2) copies of Q_{n-1} and joining the corresponding verticies with edges. Thm 1

BFS

1) for $n = 1$,  $= Q_1$

2) BFS on Q_{n-1}

By Thm 1, root of Q_n will have 1 extra child added as the leftmost child.

If vertices of Q_n are represented as a string of n bits, and the root contains all zeroes, then the string for its children can be made by changing one 0 with no ones after it to a 1.

(e.g. root 000, children 001, 010, 100)

DFS

Tree depends on the order in which the verticies are chosen.

All hypercubes contain a Hamiltonian circuit, so the tree obtained by DFS is likely a Hamiltonian Circuit.

3) Show that an edge with smallest weight in a weighted connected graph must be part of any minimum spanning tree.

G = weighted graph
e = least weighted edge in G
T = minimum spanning tree from G without e

prove T cannot exist

Add e to T

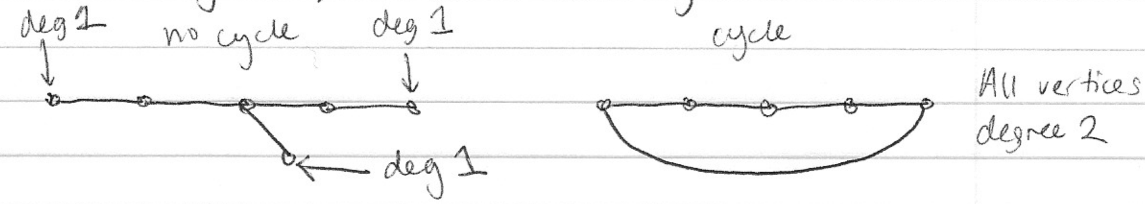
Remove random other edge from T

weight ~~(T) < (original T) weight~~

→ T cannot be a minimum spanning Tree

$\therefore$ The minimum weight edge should always be included in the minimum spanning tree of a graph

4) Prove that if a connected undirected graph has no vertex of degree 1, it contains a cycle.



In order to have a connected graph with no vertices of degree 1, all vertices at ends of main branches must have back edges; these back edges form cycles.