Rachael Afenit

Game Theory Midterm

2) $$\boxed{\begin{aligned} P_A &= p(x_A, x_B)x_A - C_A x_A = (2x_A + x_B)x_A - C_A x_A \\ P_B &= p(x_A, x_B)x_B - C_B x_B = (2x_A + x_B)x_B - C_B x_B \end{aligned}}$$ for NE

partially solved in class → i. Find x_A, x_B @ NE

$P_A = 2x_A^2 + x_B x_A - C_A x_A$

$= 2x_A^2 + (x_B - C_A)x_A$

$\frac{dP_A}{dx_A} = 4x_A + x_B - C_A = 0$

$x_A = \frac{C_A - x_B}{4}$ ⟶

$P_B = 2x_A x_B + x_B^2 - C_B x_B$

$= x_B^2 + (2x_A - C_B)x_B$

$\frac{dP_B}{dx_B} = 2x_B + 2x_A - C_B = 0$

$2x_B + 2\left(\frac{C_A - x_B}{4}\right) - C_B = \emptyset$

$2x_B + \frac{1}{2}C_A - \frac{1}{2}x_B - C_B = \emptyset$

$\frac{3}{2}x_B = C_B - \frac{1}{2}C_A$

$3x_B = 2C_B - C_A$

$x_B = \frac{2}{3}C_B - \frac{1}{3}C_A$

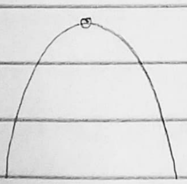
$\frac{3}{12} + \frac{1}{12} = \frac{4}{12}$

$x_A = \frac{1}{4}C_A - \frac{\frac{2}{3}C_B + \frac{1}{3}C_A}{4}$

$x_A = \frac{1}{3}C_A - \frac{1}{6}C_B$

$$\boxed{x_B = -\frac{1}{3}C_A + \frac{2}{3}C_B \qquad x_A = -\frac{1}{6}C_B + \frac{1}{3}C_A}$$

ii. Company A determines strategy first; find Stackelberg eq.



$x_B = \frac{1}{2}C_B - x_A$

$P_A = 2x_A^2 + (\frac{1}{2}C_B - x_A)x_A - C_A x_A$

$= 2x_A^2 + \frac{1}{2}C_B x_A - x_A^2 - C_A x_A$

$= x_A^2 + (\frac{1}{2}C_B - C_A)x_A$

$\frac{dP_A}{dx_A} = 2x_A + \frac{1}{2}C_B - C_A = \emptyset$

$2x_A = C_A - \frac{1}{2}C_B$

$x_B = \frac{1}{2}C_B - \frac{1}{2}C_A + \frac{1}{4}C_B$

$x_B = \frac{3}{4}C_B - \frac{1}{2}C_A$

$$\boxed{x_A = \frac{1}{2}C_A - \frac{1}{4}C_B \qquad x_B = \frac{3}{4}C_B - \frac{1}{2}C_A}$$

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2) iii. Company A knows B's profit function is $2x_A + x_B$ with $\pi = 3/4$, and $x_A + 3x_B$ with $\pi = 1/4$.

$$P_B = (x_A + 3x_B)x_B - C_B x_B$$
$$= x_A x_B + 3x_B^2 - C_B x_B$$
$$= 3x_B^2 + (x_A - C_B)x_B$$
$$\frac{dP_B}{dx_B} = 6x_B + x_A - C_B = 0$$
$$6x_B = C_B - x_A$$
$$x_B = 1/6\, C_B - 1/6\, x_A$$

$$P_A = 2x_A^2 + (1/6\, C_B - 1/6\, x_A)x_A - C_A x_A$$
$$= 2x_A^2 - 1/6\, x_A^2 + 1/6\, C_B x_A - C_A x_A$$
$$= 11/6\, x_A^2 + (1/6\, C_B - C_A)x_A$$
$$\frac{dP_A}{dx_A} = 11/3\, x_A + 1/6\, C_B - C_A = 0$$
$$11/3\, x_A = C_A - 1/6\, C_B$$
$$11x_A = 3C_A - 1/2\, C_B$$
$$22x_A = 6C_A - C_B$$
$$x_{A2} = \frac{6C_A - C_B}{22}$$

75% If B's f(x) is $2x_A + x_B$, A maximizes profit with: ← from ii

$$x_{A1} = 1/2\, C_A - 1/4\, C_B$$

25% If B's f(x) is $x_A + 3x_B$, A maximizes profit with:

$$x_{A2} = 6/22\, C_A - 1/22\, C_B$$

We can then weight these strategies based on π:

$$x_A = 0.75\, x_{A1} + 0.25\, x_{A2}$$
$$= 3/4(1/2\, C_A - 1/4\, C_B) + 1/4(6/22\, C_A - 1/22\, C_B)$$
$$= 3/8\, C_A - 3/16\, C_B + 6/88\, C_A - 1/88\, C_B$$
$$\boxed{x_A = 0.44318\, C_A - 0.1989\, C_B}$$

We can see that this weighted strategy is much closer to x_{A1} than x_{A2}, since this profit function is more likely to occur.

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3) $R_i = \dfrac{1}{\sum_{i \in link} d_i}$ for each user i $\quad \underbrace{d_1 = d_2,\ d_3 = d_4 = 2d_1}_{\text{loads}}$

i. <u>List all NE</u> (this was already listed on my exam)

All of the following can occur in the reverse configuration

e.g. list 1 = {1,2} → list 1 = {3,4} → load $4d_1$
list 2 = {3,4} → list 2 = {1,2} → load $2d_1$

This particular configuration has a difference in loads of 2, which is greater than the smallest load, but none of the players gain anything from switching lists.

list 1 = {1,3} → list 1 = {2,4} → load $3d_1$
list 2 = {2,4} → list 2 = {1,3} → load $3d_1$

$\boxed{\text{list 1} = \{1,4\},\ \text{list 2} = \{2,3\}}$ → list 1 = {2,3} → load $3d_1$; list 2 = {1,4} → load $3d_1$

ii. <u>Find PoA</u>

For worst case, using as an example the circled NE above:

$$R_1 = R_2 = \frac{1}{d_1 + d_1} = \frac{1}{2d_1}$$

$$R_3 = R_4 = \frac{1}{2d_1 + 2d_1} = \frac{1}{4d_1}$$

$$\text{Social utility} = \frac{2}{2d_1} + \frac{2}{4d_1} = \left(\frac{1}{d_1}\right) + \frac{1}{2}\left(\frac{1}{d_1}\right) = 1.5\left(\frac{1}{d_1}\right)$$

For the social optimum, we use the boxed NE above:

$$R_1 = R_2 = R_3 = R_4 = \frac{1}{3d_1}$$

$$\text{social opt} = \frac{4}{3d_1} = \frac{4}{3}\left(\frac{1}{d_1}\right) = 1.33\left(\frac{1}{d_1}\right)$$

Calculate PoA

$$PoA = \frac{1.5\,(1/d_1)}{1.33\,(1/d_1)} = 1.125$$