Illinois Institute of Technology
Department of Computer Science

Third Examination

CS 330 Discrete Mathematics
Fall, 2013

11:25am–12:40pm, Monday, November 25, 2013
IT 6C7-1 (sixth floor of IIT Tower, 35th & State)

Print your name and student ID, *neatly* in the space provided below; print your name at the upper right corner of *every* page. Please print legibly.

Name:
Student ID:

This is an *open book* exam. You are permitted to use the textbook (hardcopy only), hard copies of any class handouts, anything posted on the web page, any of your own assignments, and anything in your own handwriting. Foreign students may use a hardcopy dictionary. *Nothing else is permitted*: No calculators, laptops, cell phones, Ipads, Ipods, communicators, GPSes, etc.!

Do all five problems in this booklet. *All problems are equally weighted, so do not spend too much time on any one question.*

Show your work! You will not get partial credit if the grader cannot figure out how you arrived at your answer.

Question	Points	Score	Grader
1	20		
2	20		
3	20		
4	20		
5	20		
Total	100		

1. **Professor Reingold Goes to Boston**

   Professor Reingold is driving to Boston for his grandchild's birthday party. Because he suffers from a weak bladder, he would like to stop at highway exits as often as possible on the way. He realizes, though, that he must travel at least n miles between stops in order to arrive in Boston in time for the party. His map details all highway exits on his route and the distances between them.

   (a) Give a greedy algorithm for finding the exits at which Professor Reingold should stop so as to maximize the number of stops he makes.

   (b) What is the running time of your algorithm if there are m highway exits along the route?

   (c) Argue that your algorithm yields an optimal solution.

2. **Sources**

   A node $s \in V$ of a directed graph $G = (V, E)$ is called a *source* if it has no incoming edges. Given the adjacency structure of a graph, design *and analyze* an $O(|V| + |E|)$ algorithm to determine whether it has a source or not.

3. **BFS/Finite State Machines**

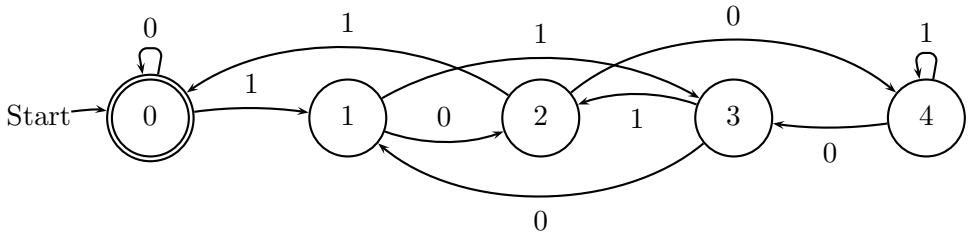
Consider a regular language L over the 26 letters of the English alphabet, and a finite state machine M accepting it. Assume that M has at least 3 states and that its transition diagram has no self-loops or parallel edges.

(a) Describe *and analyze* an algorithm for finding a shortest string in L.

(b) Prove that the transition diagram of M is not planar.

4. **Finite State Machines**

In the lecture of November 18, the following machine was presented to recognize multiples of 5 written in binary and read from left to right (that is, most significant bit first):



(a) Give the formal specification (quintuple) for this FSM.

(b) Draw and give the formal specification of the equivalent FSM which recognizes the same set of numbers (that is, multiples of 5) written in ternary (base 3).

5. **Regular Languages**

Indicate whether the following sets are regular or not, and justify your answer.

(a) Strings with an even number of zeroes and an odd number of ones.

(b) Strings *not* in the Oxford English Dictionary.

(c) $L = \{a^n b^m | n \neq m\}$. (*Hint*: If L is regular, is $(\Sigma^* - L) \cap a^* b^*$ also regular?)