

## Homework Assignment 2

CS 330 Discrete Structures  
Spring Semester, 2015

**Due: Friday, February 6, 2015**

1. Page 398, problem 70
2. Page 414, problem 28
3. Page 421, problem 20; also, explain the name “hexagon identity”
4. Page 422, problem 28; also, prove this by induction
5. Page 433, problem 42
6. Page 434, problem 58, parts (a) and (d)
7. Consider the combinatorial identity

$$\binom{\binom{k}{2}}{2} = 3 \binom{k+1}{4}.$$

- (a) Prove this identity by algebraic manipulation.
- (b) Give a combinatorial proof. (*Hint*: The lefthand side counts the number of combinations of two combinations of  $k$  items taken two at a time. Consider the following algorithm for generating such an item: Take the  $k$  items and add a  $k+1$ st element “DUP”. Each pair of combinations of  $k$  items taken two at a time can be obtained by choosing 4 items from the expanded set of  $k+1$  elements. If none of those four is DUP, there are 3 possible pairs of combinations of two items (why?). If one of those items is DUP, any one of the three other items can be duplicated to get a total of 4 elements. In that case, how many possible pairs of combinations of two items are there?)