Game Theory: Algorithms and Applications
CS 539

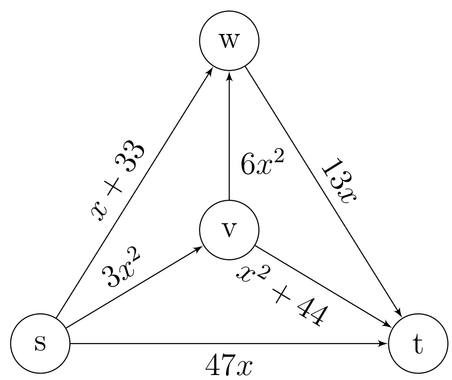
Spring 2018
HomeWork 4
Due 11:59pm, April 11th
Total: 100 points

1. (a) Compute the Pigou bound for linear functions of the form $c_e(x) = a_e x + b_e, a, b \geq 0$ which form a class $\mathcal{C}$. The Pigou bound is

$$\alpha(\mathcal{C}) = \sup_{c \in \mathcal{C}} \sup_{x,r \geq 0} \frac{rc(r)}{xc(x) + (r-x)c(r)}$$

   (b) Show that the Pigou bound for nondecreasing , nonnegative, concave functions is at most 4/3.

2. Use the following figure to show that Nash equilibria need not exist in Atomic Congestion Games. Use two players between s and t, with requirement 1 and 2 units.



3. Let (G, K, c) be an atomic instance with affine cost functions where G defines the network, K the set of source-sink pairs and c the cost function. Show that (G, K, c) admits at least one equilibrium flow. Use the following potential function

$$\Phi(f) = \sum_{e \in E} (c_e(f_e) f_e + \sum_{i \in P} c_e(r_i) r_i)$$

   where P is the set of players that choose a path that includes e. Note the difference w.r.t. the theorem proved in class. That was specific to all flow requirements being the same value R.

4. (a) In the local connection game where a node incurs cost $\alpha d_u + \sum_v dist(u, v)$, where d_u is the degree of node u and $dist(u, v)$ the shortest distance from u to v, show that if $\alpha > n^2$ then all Nash Equilibria are trees. Show that the *Price of Anarchy* is bounded by a constant.

   (b) Show that the Price of stability is bounded by 4/3.

5. Global Connection game:

(a) Show that the price of anarchy in the global connection game can never exceed k where k is the number of players.

(b) In the connection game, where players (having same source and sink) are to choose a flow path each from a set of flow paths, which are disjoint, and split the benefits if they happen to share a path. Suppose players have weights $w_i \geq 0$ and the benefit from sharing a path is proportional to the weights of the players, i.e. the gain from a path for player i is w_i/W where $W = \sum_j w_j$. Does Nash equilibrium exist? Can you design a potential function for this problem?