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CS 330: Homework 3

1) 260 days/year
How many people to get 0.5 probability that 2 have the same birthday?

$|u| = 260^n$

$|u - B| = \frac{260!}{(260-n)!}$

$P(\text{no one shares a b-day}) = \frac{260!}{(260-n)!\,260^n}$

$P(2 \text{ ppl share a b-day}) = 1 - \frac{260!}{(260-n)!\,260^n}$

$0.5 = 1 - \frac{260!}{(260-n)!\,260^n}$

$0.5 = \frac{260!}{(260-n)!\,260^n} \longrightarrow \frac{260!}{(260-19)!\,260^{19}} = 0.5097$

$\boxed{\sim 19 \text{ people}}$

2) Find probability that a random string of length 10 <u>does</u> <u>not</u> contain a 0 if bits are independent,

a) and a 0 bit + 1 bit are equally likely

u = all possible outcomes

$|u| = 2^{10}$ possible combinations

$$\boxed{P(\text{no zeros}) = \frac{|\text{no zeros}|}{|u|} = \frac{1}{2^{10}}}$$

b) and $P(\text{each bit} = 1) = 0.6$

$P(\text{each bit} = 0) = 1 - 0.6 = 0.4$

$$\boxed{P(\text{every bit} = 1) = P(\text{no zeros}) = 0.6^{10}}$$

c) and $P(i^{th} \text{ bit} = 1) = 1/2^i$ for $i = 1, 2, 3 \ldots 10$

$$1 + 2 + \ldots + n = \frac{n(n+1)}{2}$$

$$P(\text{no zeros}) = P(\text{all bits} = 1) = 1/2^1 + 1/2^2 + \ldots + 1/2^{10}$$

$$= \frac{1}{2^{1+2+\ldots 10}} = \frac{1}{2^{\frac{10(11)}{2}}} = \frac{1}{2^{55}}$$

$$\boxed{P(\text{no zeros}) = \frac{1}{2^{55}}}$$

3) s spam messages arrive
h non-spam messages arrive

a) Estimate $p(s)$ and $p(\bar{s})$

$u = s + h$

$$\boxed{P(s) = \frac{|s|}{|s+h|}} \quad \boxed{P(\bar{s}) = \frac{|h|}{|s+h|}}$$

b) Use Bayes' theorem to estimate the probability that an incoming message containing word w is spam, where $p(w)$ is the probability the word occurs in a spam message + $q(w)$ is the probability that w occurs in non-spam.

$$P(s|w) = \frac{p(s)\, p(w|s)}{p(w)} \quad p(w|s) = p(w) \qquad P(\bar{s}|w) = \frac{p(\bar{s})\, p(w|\bar{s})}{p(w)} \quad p(w|\bar{s}) = q(w)$$

$$P(s|w) = \frac{p(w|s)\, p(s)}{p(w|s)\, p(s) + p(w|\bar{s})\, p(\bar{s})}$$

$$= \frac{p(w) \frac{s}{s+h}}{p(w)\frac{s}{s+h} + q(w)\frac{h}{s+h}} \quad (s+h)$$

$$\boxed{P(s|w) = \frac{p(w)\, s}{p(w)\, s + q(w)\, h}}$$

4) Player chooses 6 numbers from 1-49 without replacement. Order is irrelevant.

a) How many different combinations of 6 numbers can the player pick?

$\boxed{{}_{49}C_6 = 13,983,816 \text{ combinations}}$

b) Find the probability that the same numbers are picked twice in a row.

$$P(\text{same}) = \frac{1 \leftarrow \text{one correct combination}}{13,983,816} = \boxed{7.1511 E-8}$$

all possible combinations

c) How many draws does it take before the probability is 0.5 that a single winning number is repeated?

$|u| = 49$ $\quad P(\text{repeat}) = \frac{6n}{49}$

$|r| = 6n$

$0.5(49) = 6n \rightarrow 6n = 24.5 = 4.08\overline{33}$

$\boxed{n \approx 4 \text{ draws}}$

d) Over how many drawings does the probability become 0.5 that two successive picks will be the same?

$|u| = 49$ $\quad P(\text{repeat}) = \frac{6n}{49(6n-1)}$

$|r| = \frac{6n}{6n-1}$

?

5) n students, k rounds

a) $|U| = 2^{n-1}$ ← number of ways all other students' coins could land

$|R| = n$

↑ any one of the students could ~~have the odd toss~~

$$P(\text{one odd}) = \frac{n}{2^{n-1}}$$

$$p(k^{th}\text{ round odd}) = (1-p)^{k-1}\,p$$ ← probability of odd toss

↑ probability that all previous rounds were NOT an odd toss

What is the expected number of rounds a student must toss?

$$\left(1-\frac{n}{2^{n-1}}\right)^{k-1}\left(\frac{n}{2^{n-1}}\right) \xrightarrow{\text{Derive}} \boxed{\frac{2^{n-1}}{n} = k}$$

2^{n-1} → # ways each coin could land

n → choose odd coin

$$k = \frac{|U|}{|R|}$$

b) Explain how one student using a loaded coin (p(heads) > p(tails)) would effect the number of times the student must toss.

This loaded coin makes it more likely for his to land on heads and everyone else's to land on tails than it is that another student will be the odd toss; this decreases the number of expected tosses since this scenario is more likely.

6) a) $p = p(\text{boy})$ $q = p - 1 = p(\text{girl})$

Find the expected # of children if they want one of each.

$$\boxed{k = \frac{pq + 1}{pq} = \frac{p^2 - p + 1}{p(1-p)}} = \frac{|U|}{|R|}$$

probability of one boy AND one girl

OR this kid is same as first one

What if $p = 0, 1$?

If $p = 0$ or 1, the term is undefined, meaning this situation is impossible.

b) Find the expected # of children if they want 2 same

$$\boxed{k = \frac{p^2 + 1}{p^2}}$$

probability of two same OR two different

probability of two same

$$k = \frac{|U|}{|R|}$$

What if $p = 0, 1$?

If $p = 0$, the term is undefined, telling us this is an impossible situation. However, if $p = 1$, $k = 2$, since they must only have 2 children if they are guaranteed boys