CS 539

HW 2

Christopher Morcom
A20385764

① 2 players: choose $[1, k]$ → creates a $k \times k$ decision matrix

$$M_{k \times k} = \left\{ \begin{array}{lcl} x_2 < x_1 \leq 3x_2 & \xrightarrow{\text{payoff}} & (1, -1) \\ x_1 < x_2 \leq 3x_1 & \rightarrow & (-1, 1) \\ x_1 = x_2 & \rightarrow & (0, 0) \end{array} \right\}$$

$\boxed{\text{We assume payout } = (0,0) \text{ if } \left\{ \begin{array}{l} x_1 > 3x_2 \text{ or} \\ x_2 > 3x_1 \end{array} \right\}}$

Matrix:

Payout P_1 Choice

P_2 Choice	1	2	3	...	k
1	0,0	1,-1	1,-1	0,0	0,0
2	-1,1	0,0	1,-1	...	0,0
3	-1,1	-1,1	0,0	...	0,0
⋮	0,0	...	...	0,0	...
k	0,0	0,0	0,0	...	0,0

"..." denotes unknown due to dependence on matrix index (see below).

According to our matrix, there will be a Pure N.E. when $x_1 = x_2$ since neither player can increase utility.

For non-diagnols, the matrix is symmetric s.t. $A = (-1 \cdot A)^T$.
entries are dependent on the row/col order where:

$$\forall a_{ij} \in A : a_{ij} = \left\{ \begin{array}{ll} (0,0) & \text{if } x_1 = x_2 \\ (0,0) & \text{if } x_1 > 3x_2 \text{ or } x_2 > 3x_1 \\ (1,-1) & \text{if } x_2 < x_1 \leq 3x_2 \\ (-1,1) & \text{if } x_1 < x_2 \leq 3x_1 \end{array} \right\}$$

Mixed NE:
Using the support set theorem, a dominant strategy for P_1 & P_2 each are:

P_1: when x_1 chooses $(x_2, 3x_2]$

P_2: when x_2 chooses $(x_1, 3x_1]$

Since the probability distributions are uniform across these dependent choices that discourage numbers that are too high or too low —

② How does c change equilibria?

let c = cost of effort;

Payoff	Slack	Effort
Slack	$(0,0)$	$(0,-c)$
Effort	$(-c,0)$	$(1-c),(1-c)$

Basic cases:

-> When $c = 0$, Pure NE exists at (E,E)

-> when $c = 1$, mixed NE exists at: (S,S) or (E,E)

Note: $(S,S) = P_1$ Slack, P_2 Slack

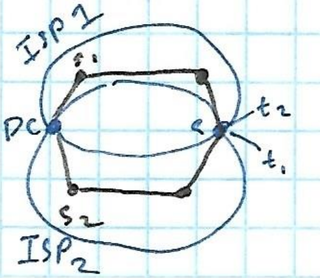
$(E,E) = P_1$ Effort, P_2 Effort

Advanced cases:

-> when $c < 0$: there exists a pure NE at (E,E)

-> when $c > 1$: there is mixed Nash Equilibrium at (S,S) & (E,E) since either player would move to (E,E) if their choices are different, but either player would not be incentivized to change if they both decide to slack off

③ Find Mixed NE for Hot-Potato Routing Game:



Cost Matrix

ISP2 \ ISP 1:	DC	C
DC	-4, -4	-5, -1
C	-1, -5	-2, -2

Note: $\begin{Bmatrix} \pi_1(DC) + \pi_1(C) = 1 \\ \pi_2(DC) + \pi_2(C) = 1 \end{Bmatrix}$

Using the support set thm:

$$u_1(DC) = -4\,\pi_2(DC) - 1\,\pi_2(C)$$
$$= u_1(C) = -5\,\pi_2(DC) - 2\,\pi_2(C)$$

$$\Rightarrow 0 = (\underbrace{-4 - 5}_{-9})\,\pi_2(DC) + (\underbrace{2 - 1}_{1})\,\pi_2(C)$$

$$\Rightarrow \pi_2(DC) = -\pi_2(C)$$

$$\pi_2(DC) + \pi_2(C) = 0 \neq 1 \text{ violating}$$

Hence: There is <u>NO mixed NE</u> since each player has a probability $\pi = 0$ for one of their strategies.

<u>Pure NE exists</u> because of the dominant strategy $(-4, -4)$

④ Find Mixed NE for Rock, Paper, Scissors:

Payoff (P2 \ P1)	R	P	S
R	0, 0	10, -10	-10, 10
P	-10, 10	0, 0	10, -10
S	10, -10	-10, 10	0, 0

Note: $A = (-1 \cdot A)^T$.

Note: game is symmetric: $\pi_1(A) = \pi_2(A)$

Using support set thm:

$$\pi_2(R) + \pi_2(P) + \pi_2(S) = 1$$

$$u_1(R) = u_1(P) = u_1(S)$$

$$\Rightarrow -\pi_2(P) + \pi_2(S) = \pi_2(R) - \pi_2(S) = -\pi_2(R) - \pi_2(P)$$

$$\hookrightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \pi_2(R) \\ \pi_2(P) \\ \pi_2(S) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \therefore$$

$$\boxed{\begin{array}{l} \pi_2(R) = \pi_2(P) = \pi_2(S) = \frac{1}{3} \\ \text{by symmetry:} \\ \pi_1(R) = \pi_1(P) = \pi_1(S) = \frac{1}{3} \end{array}}$$

⑤ Find MNE for Payoff.

		P2 Intercepts		
	Payoff	4	5	6
P1 ships	$A=\{4,5\}$	5, 4	4, 5	9, 0
	$B=\{4,6\}$	6, 4	10, 0	4, 6
	$C=\{5,6\}$	11, 0	6, 5	5, 6

Note:

$\pi_2(4) + \pi_2(5) + \pi_2(6) = 1$

$u_2(\pi) = \sum_{\tilde{A}} u_2(\tilde{A})\pi(\tilde{A})$

$$\begin{cases} u_2(4) = 4\pi_1(A) + 4\pi_1(B) + 0 & \leftarrow E_1 \\ u_2(5) \cdot 5\pi_1(A) + 0 \quad + 5\pi_1(C) & \leftarrow E_2 \\ u_2(6) = 0 + 6\pi_1(B) + 6\pi_1(C) & \leftarrow E_3 \end{cases}$$

$4\pi_1(A) + 4\pi_1(B) = 5\pi_1(A) + 5\pi_1(C) = 6\pi_1(B) + 6\pi_1(C)$

Let $\pi_1(A) = \alpha$, $\pi_2(B) = \beta$, $\pi_3(C) = 1-\alpha-\beta$ since A, B, C are pairwise independent

$E_1 = E_3$ & expansion gives us: $\left[\begin{array}{cc|c} 4 & -5 & -9 \\ 6 & 1 & -5 \end{array}\right]$

so $\alpha = -17/13$

$\beta = 37/13$

hence: $\boxed{\begin{aligned} \pi_1(A) &= -17/13 \\ \pi_1(B) &= 37/13 \\ \pi_1(C) &= -7/13 \end{aligned}}$

Ran out of time for solving due to late policy.