Spring 2019
HomeWork 4
Due 11:59pm, November 12th

1. (a) Compute the Pigou bound for linear functions of the form $c_e(x) = a_e x + b_e$, $a, b \geq 0$ which form a class $\mathcal{C}$. The Pigou bound is

$$\alpha(\mathcal{C}) = \sup_{c \in \mathcal{C}} \sup_{x,r \geq 0} \frac{rc(r)}{xc(x) + (r-x)c(r)}$$

   (b) Show that the Pigou bound for nondecreasing , nonnegative, concave functions is at most $4/3$.

2. Use the following figure to show that Nash equilibria need not exist in Atomic Congestion Games. Use two players between s and t, with requirement 1 and 2 units.
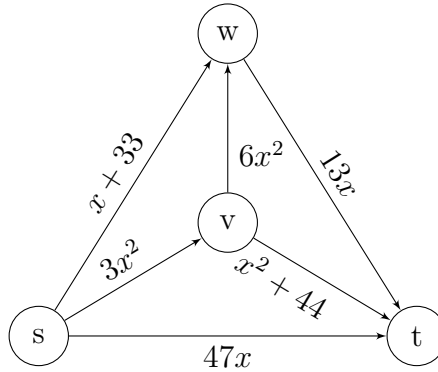
3. Let (G, K, c) be an atomic instance with affine cost functions where G defines the network, K the set of source-sink pairs and c the cost function. Show that (G, K, c) admits at least one equilibrium flow. Use the following potential function

$$\Phi(f) = \sum_{e \in E}(c_e(f_e)f_e + \sum_{i \in P} c_e(r_i)r_i)$$

   where P is the set of players that choose a path that includes e. Note the difference w.r.t. the theorem proved in class. That was specific to all flow requirements being the same value R.