Aerobic Training and its Influence on Long-Distance Running Performance

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MA389: United States Military Academy

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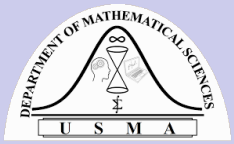
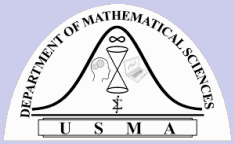
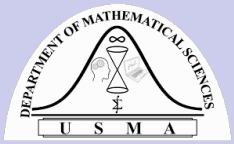


Agenda



Motivation

- Popularity of marathon and long-distance running



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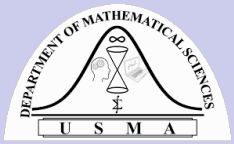
- Popularity of marathon and long-distance running
- Well-established predictors of performance (VO_2 max, lactate threshold, etc.)



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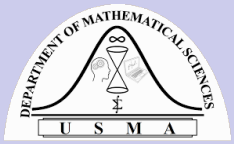
- Expensive, time consuming, and difficult tests



Motivation

- Popularity of marathon and long-distance running
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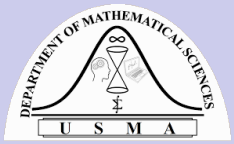
- Expensive, time consuming, and difficult tests
- **Training indices** bridge gap (distance, pace, training sessions, etc.)



Statistical Models

- Slovic (1977) incorporated **training indices** into model. [4]
- Tanda (2011) found relationship between **average weekly distance** (K) and **average pace** (P) on race pace (P_m). [5]

$$P_m = 17.1 + 140e^{-0.0053K} + 0.55P$$

- Doherty et al's (2020) meta analysis of 85 articles in the field. [1]
  - Results help inform endurance training

Tanda's Model

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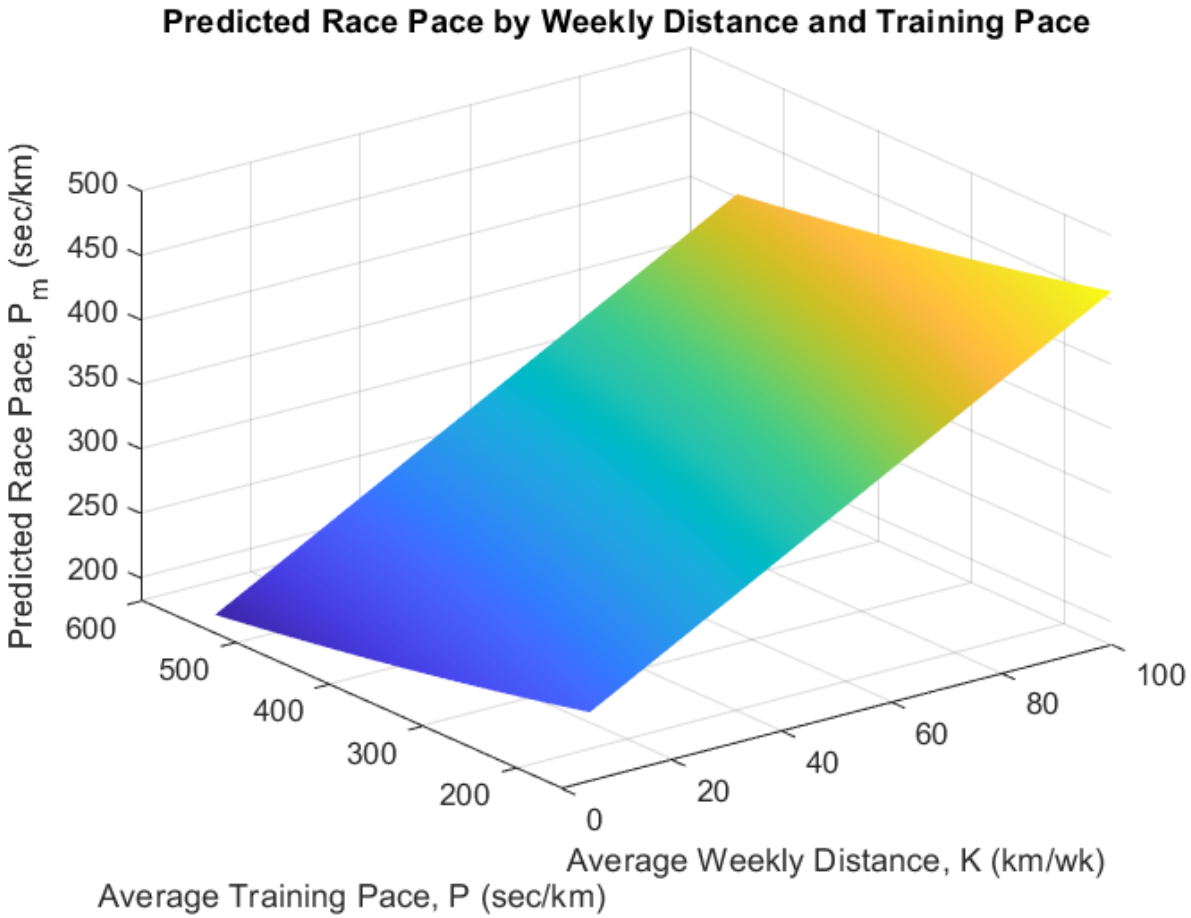
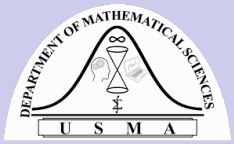




Mechanistic Models

- Keller's (1973, 1974) foundational work [2, 3]
- Woodside (1991) extended the model [6]

Keller's Problem

Given the following:

$$\frac{dv}{dt} + \frac{1}{\tau}v = f(t), \quad f(t) \leq F$$

$$\frac{dE}{dt} = \sigma - f(t)v(t), \quad E(0) = E_0$$

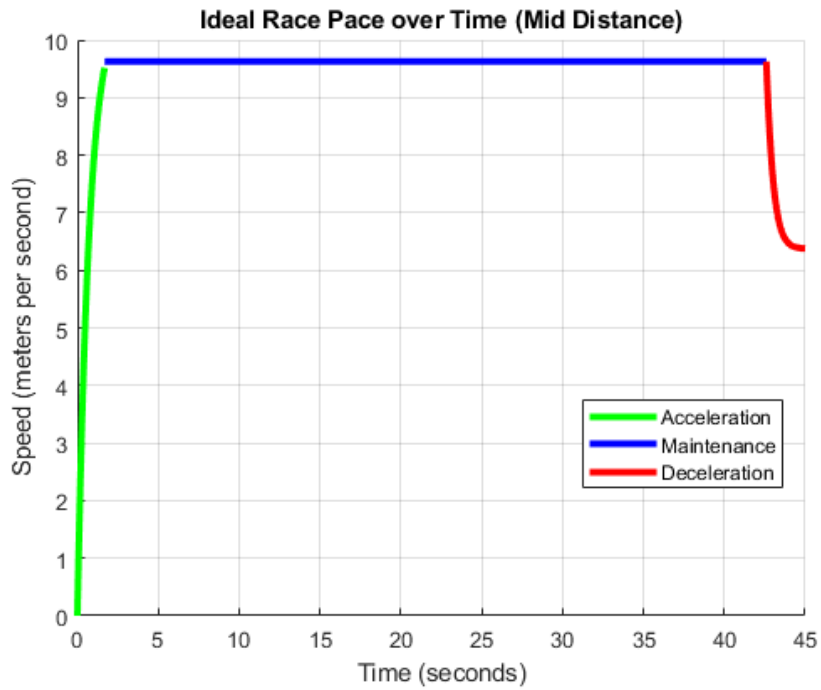
$$T, \tau, \sigma$$

Maximize:

$$D = \int_0^T v(t)\, dt$$

Keller's Model

$$v(t) = \begin{cases} F\tau(1 - e^{-t/\tau}), & 0 \le t \le t_1 \\ \tau/\lambda, & t_1 \le t \le t_2 \\ \sqrt{\sigma\tau + [v^2(t_2) - \sigma\tau]e^{-2(t_2-t)/\tau}}, & t_2 \le t \le T \end{cases}$$



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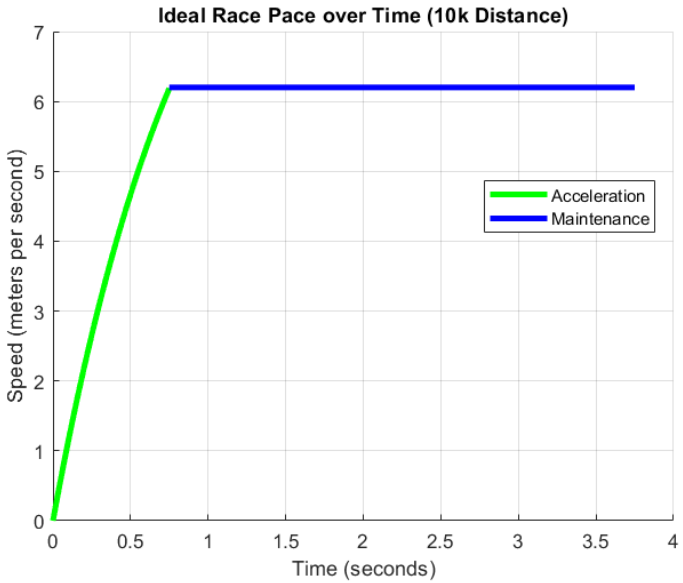
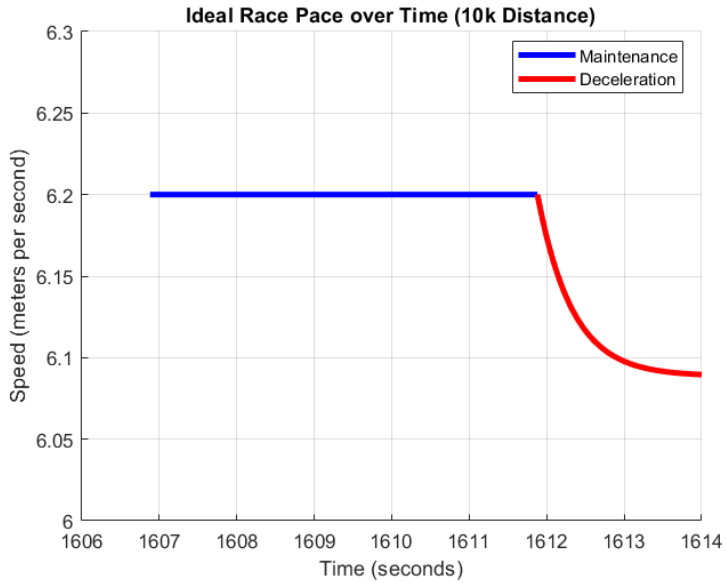
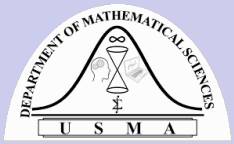
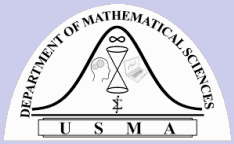






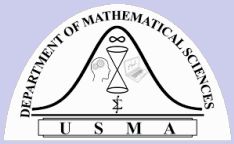
Way Ahead

- Identify links between approaches



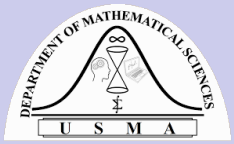
Way Ahead

- Identify links between approaches
- Determine important training indices



Way Ahead

- Identify links between approaches
- Determine important training indices
- Communicate effectively



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[1] Cailbhe Doherty, Alison Keogh, James Davenport, Aonghus Lawlor, Barry Smyth, and Brian Caulfield, *An evaluation of the training determinants of marathon performance: A meta-analysis with meta-regression*, Journal of Science and Medicine in Sport **23** (2020), no. 2, 182–188.

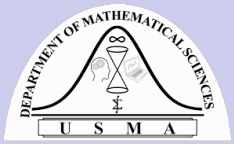
[2] Joseph B. Keller, *A theory of competitive running*, Physics Today **26** (1973), 43–47.

[3] _____, *Optimal velocity in a race*, The American Mathematical Monthly **81** (1974), no. 5, 474–480.

[4] Paul Slovic, *Empirical study of training and performance in the marathon*, Research Quarterly. American Alliance for Health, Physical Education and Recreation **48** (1977), no. 4, 769–777, available at `https://doi.org/10.1080/10671315.1977.10615491`.

[5] Giovanni Tanda, *Prediction of marathon performance time on the basis of training indices*, Journal of Human Sport and Exercise **Volume 6** (201109), 521–520.

[6] William Woodside, *The optimal strategy for running a race (a mathematical model for world records from 50 m to 275 km)*, Mathematical and Computer Modelling **15** (1991), no. 10, 1–12.



Appendix A – Doherty

Marathon finish time	Average weekly distance	Weekly training hours	Peak' week	Longest training run	N runs >32km	Average training pace	N weekly runs
250	**38.2**	**3.9**	50.0	**23.5**		**90.9**	2.4
249	**38.7**	**3.9**	51.3	**23.7**		**91.6**	2.4
248	**39.3**	**4.0**	52.6	**23.8**		**92.2**	2.5
247	**39.9**	**4.1**	53.9	**24.0**		**92.8**	2.6
246	**40.5**	**4.1**	55.2	**24.2**		**93.4**	2.7
245	**41.1**	**4.2**	56.5	**24.4**		**94.0**	2.8
244	**41.7**	**4.2**	**57.8**	**24.5**		**94.6**	2.8
243	**42.3**	**4.3**	**59.1**	**24.7**		**95.2**	2.9
242	**43.0**	**4.4**	**60.4**	**24.9**		**95.8**	3.0
241	**43.6**	**4.4**	**61.7**	**25.1**		**96.4**	3.1
240	**44.2**	**4.5**	**63.0**	**25.2**		**97.1**	3.2

Appendix B – Tanda 2D