



WORKSHOP IN MATHEMATICAL MODELLING

The Mediterranean Sea clean up

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Abstract

Greenpeace estimates over 12,7 million tonnes of plastic end up each year into our oceans. Due to natural forces, such as ocean currents and meteorological events, plastic is spread through the seas with all the negative consequences this pollution entails. In this context, this paper focuses on the scope of plastic waste through the Mediterranean Sea. First, quantifying the amounts of plastic going to the sea through each part of Mediterranean coastline and forecasting them in the near future. Later, this data will serve as initial input of plastic waste to be tracked in the sea, using meteorological and marine current models. Finally, we should be able to provide an optimal solution to collect plastic in the Mediterranean Sea, given all the previous data and simulations, such as the solution provided by The Ocean Cleanup in the Pacific Ocean.

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1 Introduction

Through the last decades, plastic waste has become a major concern around the world. Plastic Soup Foundation[1] estimates more plastic was produced over the last 10 years than during the entire 20th century and about half of that is single-use plastic thrown away immediately after use. The United Nations Department for Environment[2] determines most of this plastic end up in the rivers which serve as direct conduits of trash from the world's cities to the marine environment. More precisely, they estimate around 90% of the plastic waste come directly from these 10 rivers in the figure below.



Figure 1: UN Environment

In front of such a problem, attention has focused on international action with measurable targets to reduce plastic marine pollution. Initiatives such as SDG 14 supported by the United Nations Environment Assembly, which follows a similar path as initiatives against emissions of polluting gases, are some of the action plans on the topic. However, government initiatives take a lot of time, large majorities and compromises in the long term that tend to be missed.

For these reasons, we were very attracted by The Ocean CleanUp[3], a non-profit organization designing and developing the first feasible method to eradicate the plastics from the world's oceans. Going after the plastic waste in the oceans with vessels and nets could be costly, time-consuming, labour-intense and lead vast amounts of carbon emissions. So this is why they are developing a passive system moving with the currents just like plastic. In their approach, they are focusing on plastic going through the Pacific Ocean focusing on the big island of plastic next to California.

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Inspired by this project, we wanted to develop a similar approach but focusing our scope in the Mediterranean Sea.

In order to apply this idea, we needed to quantify the amounts of plastic going to the sea through each part of Mediterranean coastline. To do so, we needed the amounts of plastic waste going to each of the rivers ending in the Mediterranean Sea what involved countries from Europe and Africa.

Having this data from each of the rivers in a time series format helped us to forecast them in the near future. Later, this data would serve as initial inputs of plastic waste to be tracked in the sea, using meteorological and marine current models. In this part, we used already existing sea current models like Parcels[4] developed by research teams from the Imperial College, Utrecht University and the Grantham Institute.

Finally, with all the data and trajectory simulations made, we should be able to provide an optimal solution to collect plastic in the Mediterranean Sea such as the solution provided in the Pacific Ocean.

2 Data gathering

2.1 Weather Data

Meteorological observations can be used to predict the weather in real time, to do climate studying or to do activities that depend on local weather (airports, harbours, etc.). Even if they are collected for the same purpose the measures have different requirements as of real-time observations, accuracy, format, etc.

Precision scales are closely related to the scale of the phenom that we are studying, for example, short scale predictions require more frequent observations and from a more dense grid in order to detect all the possible features from a low scale that can affect to the development of the physical phenom.

In general, each kind of meteorological application has a different size or temporal scale that averages the variable, density of stations and phenom resolution. In figure 2 we can see the different meteorological size-time scale.

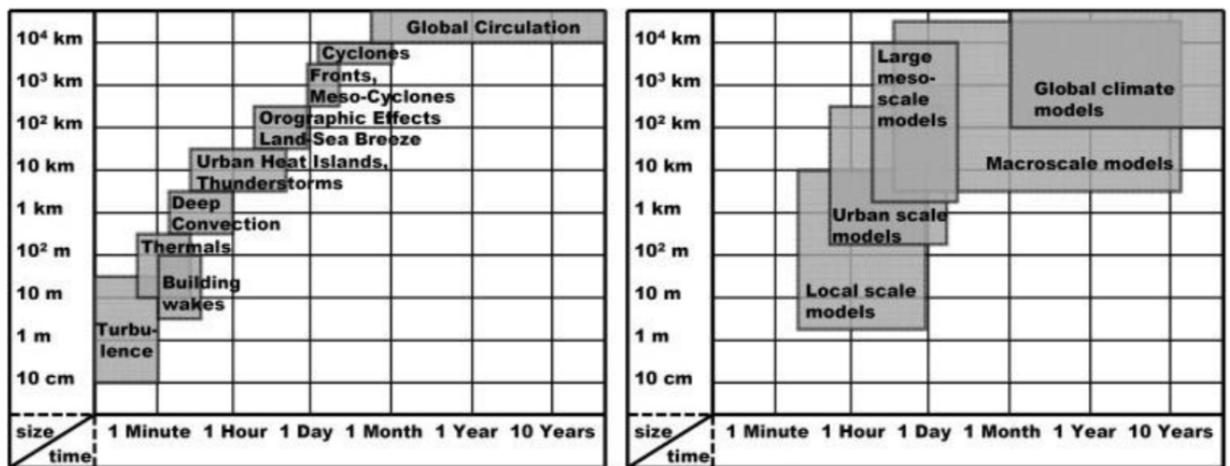


Figure 2: Meteorological Scales

The **meta-data** is especially important for the most sensible variables as precipitations, wind, and temperature. As we know the atmospheric prediction is a deterministic model with chaotic behaviour, so knowing and improving the quality of measurements can be of high importance when doing weather modelling.

Where does the data come from? We can cluster the stations between three possible location places:

- Atmospherics: there is about 1300 station distributed worldwide.

- Oceanographic: mainly by buoys and Voluntary Observing Ship Scheme (VOS), presented in the figure 3.
- Ground: it's the essential measuring point with, more than 11000 measuring stations.



Figure 3: Oceanographic distribution of buoys and VOS

2.2 Mismanaged Plastic Waste Data

Following the assumption from GESAMP[5] that most of the plastic which goes to the sea is through the rivers, the first step is delimiting our geographical work area, in our case all rivers that end in the Mediterranean Sea.

Once we have defined our scope, we need to get the amount of plastic going to each of those rivers. On this problem Lebreton et al. (2017)[6] showed a statistically significant correlation between surface plastic concentrations and human population densities. These shreds of evidence are consistent with the previous estimations in Figure 1 of the top 10 rivers throwing plastic waste into the Oceans because in all of them there are extreme concentrations of populations around.

Maybe the most relevant example of our approach is the high density of people living around the Nile river and the amount of waste through this into the Mediterranean Sea. The same paper showed major micro-plastics concentrations in the rivers after major rain events. This fact is given by seasonality, and they estimated around 75% of the total river plastic input occurs between May and October. Such an important effect is driven by the large inputs of China regulated by the East Asian monsoon. In fact, variations in monthly inputs are not as pronounced for other continents but differentiating two distinct river plastic inputs: the first one occurring between June and October in Africa and North America, and a second one occurring from November to May in Europe, South America and Australia.

All the previous insights were published with a huge data set which has been the starting point in the construction of our input data set. They based their model on the empirical evidence presented before and to determinate the daily mass input from each individual watershed they used the following parametric approach:

$$M_{out} = (k M_{mpw} R)^a \quad (1)$$

where M_{out} is the plastic waste released at the outflow in kilogram per day, M_{mpw} the mass of mismanaged plastic waste (MPW) produced inside the catchment downstream of artificial barriers, and R which is the monthly averaged catchment runoff. k and a are regression parameters and they used values of $k = 1.8510^{-3}$ and $a = 1.52$ because they got a strong coefficient of determination ($R^2 = 0.93$) using them.

In the following part, we are going to decompose this estimation of plastic inputs into the ocean. We divide the task into four main parts: Population density, mismanaged plastic per inhabitant, seasonality and averaged catchment runoff.

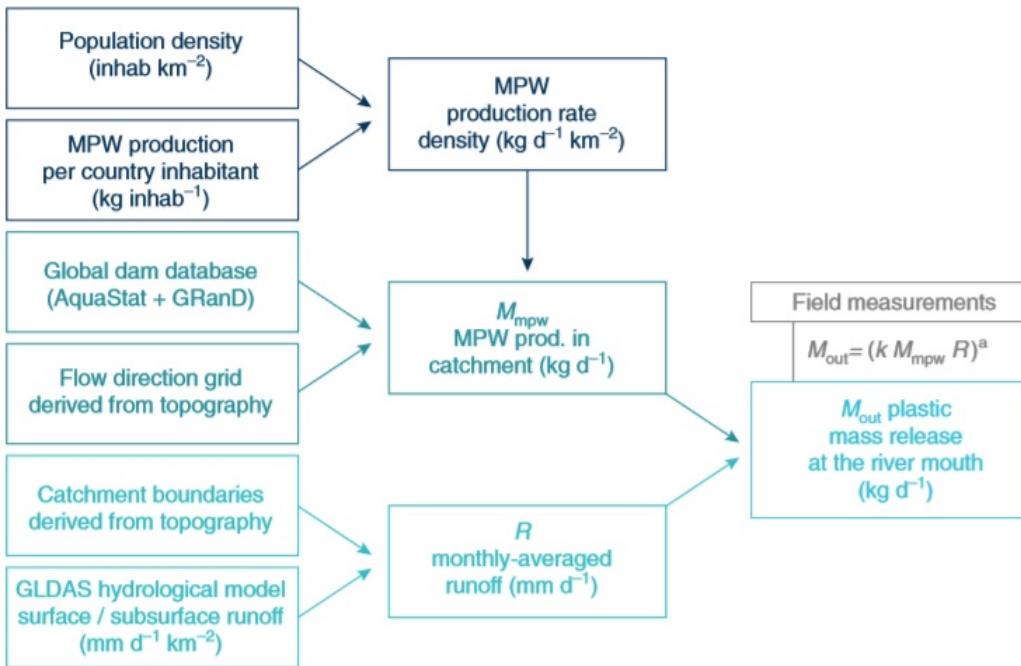


Figure 4: Model framework. Plastic mass production per river catchment

2.2.1 Population density

Population density is the biggest contributor to determine the number of plastic wastes going to the sea in a specific region. The intuition is simple, the more people, the more total wastes and in consequence the bigger amount of plastic wastes.

To do so, the underlying data set used by Lebron et al.[6] it's the Global 15x15 Minute Grids of Population based on the Social Report on Emission Scenarios from 1990 to 2025[7, 8]. These grids are geospatial distributions of the downscaled population per unit area, in other words, population densities.

This data set has been produced by the Columbia University Center and helps to map the population densities and their projections in the future.

2.2.2 Mismanaged plastic per inhabitant

Analyzing the evidence, emphasize the total correlation between population density and the amount of plastic waste going into the sea. Hence, we need to review evidence on the computation of Mismanagement Plastic per inhabitant.

Jambeck et al. (2015) [9] estimated the annual input of plastic to the ocean from waste generated by coastal populations worldwide. In their approach to this concept,

mismanaged waste is a material that is either littered or not adequately disposed of. This mismanaged waste could end in the ocean via inland waterways, wastewater outflows, and transport by wind. Considering this definition, they developed a framework to calculate the amount of mismanaged plastic waste generated annually. For instance, their estimation of total waste generated by people living in the coastline (93% of the global population) in the year 2010 was 1.3 billion millions of tones. From that total, approximately 11% of this waste is plastic (275 millions of tones).

Thus, by focusing on people living near the coastline and taking into account the traceability problems, they estimated an amount of around 100 million tones of plastic waste generated by the populations in the coastlines. Of this, 32 million tones were classified as mismanaged and between 5 to 13 million tones entered the Oceans, between the 2 and 5% of the total plastic waste generation.

2.2.3 Seasonality

Depending on the region and its meteorological conditions, plastic waste going to the river peak in different moments of the year. For example, evidence of peaks from November to March in South East Asia is driven by the summer monsoon.

The model of Lebreton et al. [6] also uses seasonal variations to model river inputs plastics. And focusing on the subset of data specifically related to the Mediterranean Sea, the peak season seems to be between January and April, probably related to spring rainfalls and ice melting as we can see in the following Figure:

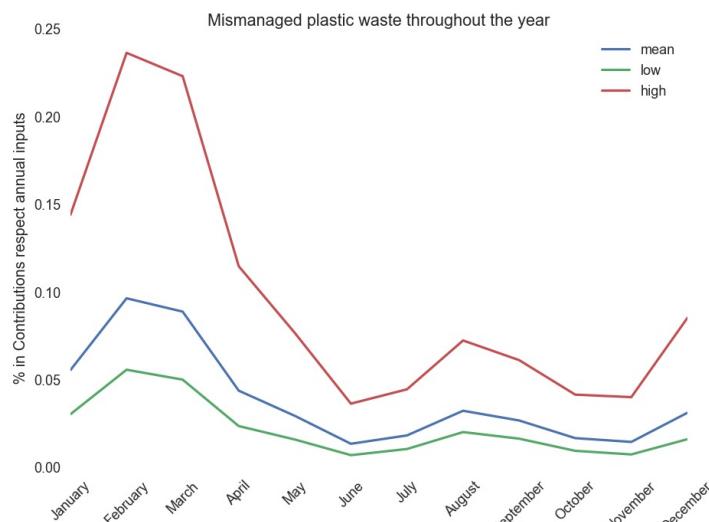


Figure 5: Model framework. Plastic mass production per river catchment

2.2.4 Averaged catchment runoff

Runoff, in hydrology, is the quantity of water discharged in surface streams. Runoff includes not only the waters that travel over the land surface and through channels to reach a stream but also interflow, the water that infiltrates the soil surface and travels by means of gravity toward a stream channel (always above the main groundwater level) and eventually empties into the channel.

Runoff also includes groundwater that is discharged into a stream; streamflow that is composed entirely of groundwater is termed base flow or fair-weather runoff, and it occurs where a stream channel intersects the water table.

The total runoff is equal to the total precipitation minus the losses caused by evapotranspiration (loss to the atmosphere from soil surfaces and plant leaves), storage (as in temporary ponds), and other such abstractions.[10]

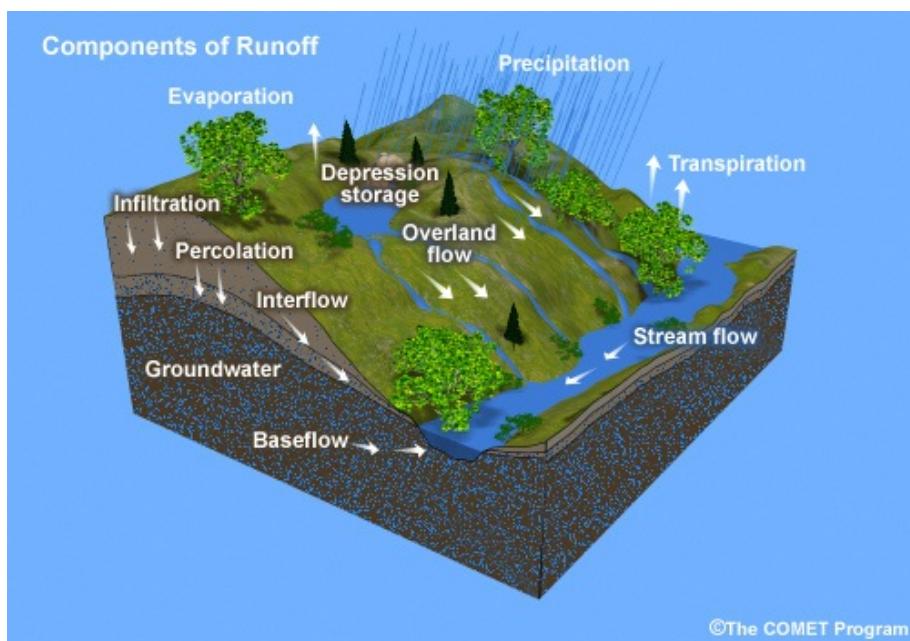


Figure 6: Components of Runoff[11]

GLDAS¹ is a system that has been developed jointly by scientists at NASA², GSFC³, NOAA⁴ and NCEP⁵ in order to perform a wide array of studies in biogeosciences, in

¹Global Land Data Assimilation Systems

²National Aeronautics and Space Administration

³Goddard Space Flight Center

⁴National Oceanic and Atmospheric Administration

⁵National Centers for Environmental Prediction

particular, terrestrial stores of energy and water modulate fluxes between the land and atmosphere and exhibit persistence on diurnal, seasonal, and interannual time scales.

Lebreton et al.[6] also have calculated the monthly average catchment runoff in millimetres per day by using GLDAS driving the NOAH Land Surface Model. This LSM integrates data from advanced ground and space-based observation systems. The model contains land surface parameters for vegetation, soil, elevation and slope. It computes the daily surface and subsurface runoff globally using the forcing data. In figure 5 we can see the GLDAS required force fields and a summary of its output fields

Required forcing fields	Summary of output fields
Precipitation	Soil moisture in each layer
Downward shortwave radiation	Snow depth, fractional coverage, and water equivalent
Downward longwave radiation	Plant canopy surface water storage
Near-surface air temperature	Soil temperature in each layer
Near-surface specific humidity	Average surface temperature
Near-surface <i>U</i> wind	Surface and subsurface runoff
Near-surface <i>V</i> wind	Bare soil, snow, and canopy surface water evaporation
Surface pressure	Canopy transpiration
	Latent, sensible, and ground heat flux
	Snow phase change heat flux
	Snowmelt
	Snowfall and rainfall
	Net surface shortwave and longwave radiation
	Aerodynamic conductance
	Canopy conductance
	Surface albedo

Figure 7: GLDAS required forcing and output fields[12]

3 Mathematical Modelling

Once we have gathered the mismanaged plastic and the meteorological data. We have to predict how will each particle move in the area.

Our goal is to be able to predict how the particle will move given certain meteorological conditions.

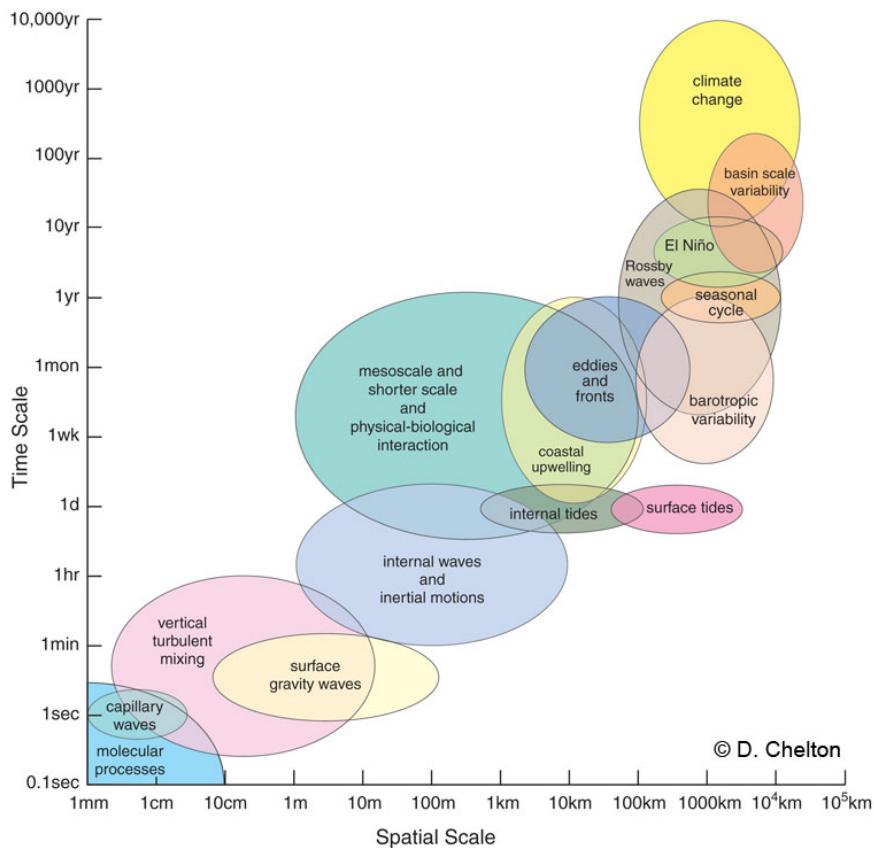


Figure 8: Ocean Movements Scales[15]

In Figure 8 we can see an oceanic version of Figure number 6. It's easy to see there are a lot of different processes happening in the ocean.

Taking into account the scale of the problem it only makes sense to make a dichotomy of all these different modellings and just model two of them: climate and meteorological approach.

3.1 Atmospheric Modelling

In order to simplify the explanation of how a particle moves into the water due to the meteorological influence, we will explain a set of equations that are done with some assumptions [13].

- Over a mass of water the wind produces a linear impulse along the wind direction.
- The mass parcel only feels the Coriolis strength.

The equations for the movement of the parcel of water are the following:

$$\frac{du}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + 2\Omega v \sin \Phi \quad (2)$$

$$\frac{dv}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - 2\Omega u \sin \Phi \quad (3)$$

$$\frac{dw}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + 2\Omega \cos \Phi \quad (4)$$

Where “p” is the pressure, ρ is the density, Ω the earth rotation and Φ the latitude.

Another fact that the model that we have uses is The Ekman spiral[14], which is a structure of currents or winds near a horizontal boundary in which the flow direction rotates as one moves away from the boundary.

3.2 Ocean General Circulation Modelling (OGCM)

Ocean general circulation models (OGCMs) are a particular kind of general circulation model to describe physical and thermodynamic processes in oceans. The oceanic general circulation is defined as the horizontal space scale and time scale larger than mesoscale (of order 100 km and 6 months)[Figure 8].

They depict oceans using a three-dimensional grid that includes active thermodynamics and hence is most directly applicable to climate studies.

They are the most advanced tools currently available for simulating the global ocean system.

The Ocean Circulation in the mesoscale force by the winds, heat and surface water. In the next section, we will briefly introduce them.

3.2.1 Wind

The wind direction and magnitude made from the wind over the ocean surface depends on the velocity. If $\vec{u} = (u, v)$ measures the horizontal wind velocity at a 10 meters height, then the impact of the winds on the ocean is given by the semi-empirical equation (5). Where $\rho_a = 1.2 \frac{\text{kg}}{\text{m}^3}$ is the air density and C_D is a friction coefficient that depends on \vec{u} .

$$\tau = C_D \rho_a |u| \cdot u \quad (5)$$

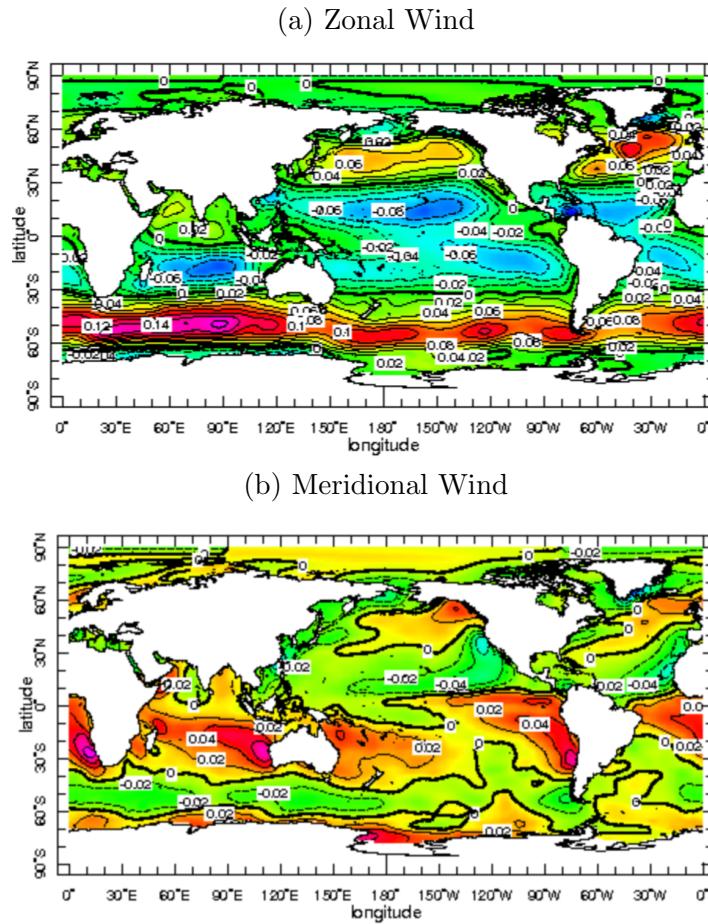


Figure 9: Annual wind average in Pa

In this figure we can see the observed mean annual force produced by winds, separated into two components. The maximum values for the zonal component are around 0.15 Pa and the mean is around 0.05 Pa. The spatial structure of the effort its really similar for the whole year.

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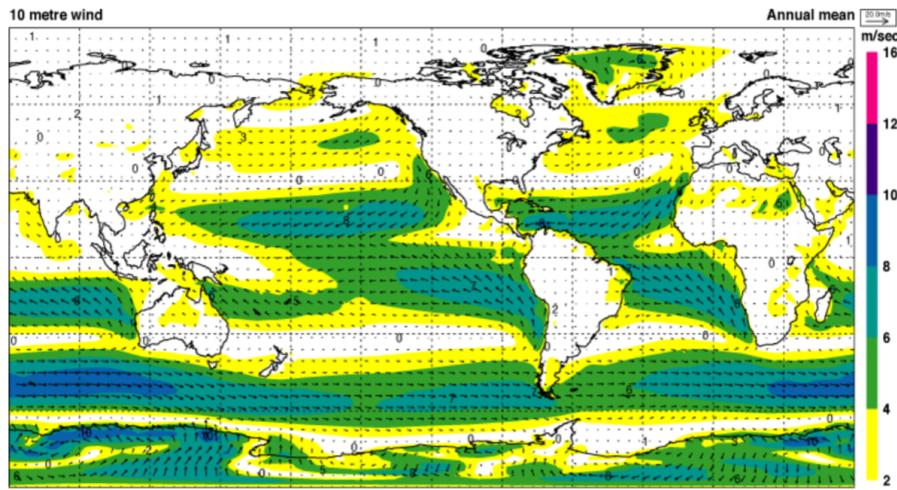


Figure 10: Annual Wind Average at 10m

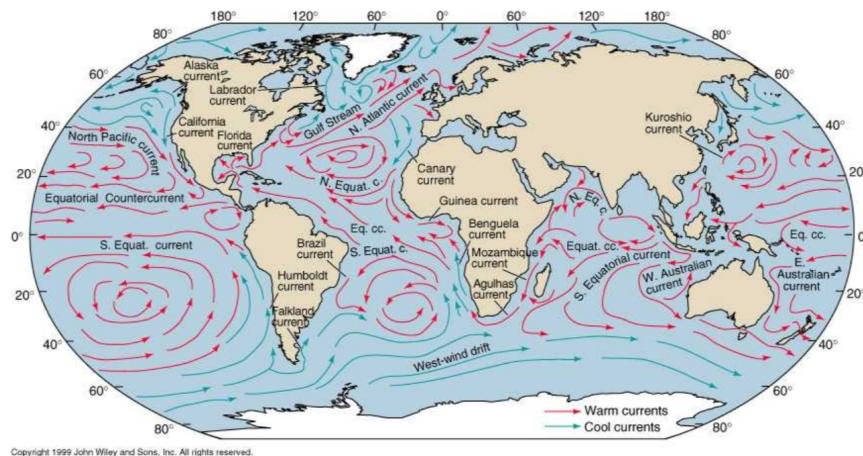


Figure 11: Schema of Currents in the Surface

As faster is the current higher is the volume that it moves. We can calculate it with the following equation.

$$(M_x, M_y) = \int_{-d}^0 \rho \cdot (u, v) dz \quad (6)$$

Where M is the mass of water that goes through a section.

The transportation of volume is the mass transport divided by the density and multiplied by the direction of the transport.

$$(Q_x, Q_y) = \frac{1}{\rho} (Y \cdot M_x, X \cdot M_y) \quad (7)$$

3.2.2 Heat Flux

The net heat flux Q_{oa} [(W/m^2)], positive downward, it the sum of different components:

$$Q_{oa} = Q_H + Q_E + Q_{LW} + Q_{SW} \quad (8)$$

Where Q_H it the sensible heat, Q_E is the latent heat, Q_{LW} is the longwave radiation and Q_{SW} is the short wave radiation.

The heat flux can be estimated with similar formulas to the wind ones. For example, the sensible heat is the glus that occurs close to the interface between the ocean and the atmosphere as a consequence of the turbulent movements of small scale and its proportional to the difference between the Surface temperature T_s and the atmospheric temperature T_a at a height z_a from the surface.

$$Q_H = \rho_a C_{pa} C_H |u| (T_s - (T_a + \gamma_a z_a)) \quad (9)$$

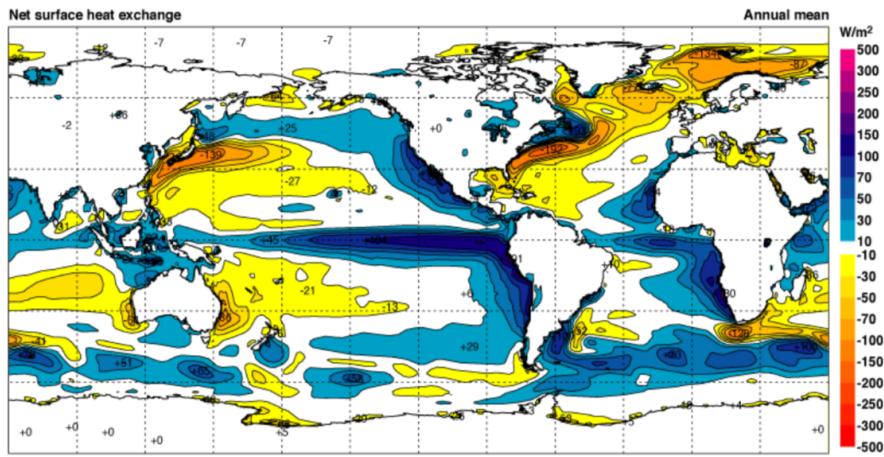


Figure 12: Net surface heat exchange

3.2.3 Fresh Water Flux

The water evaporation $E(m/s)$ over the ocean can be estimated by a formula like:

$$Q_E = \rho_a L_f E \quad (10)$$

Where $L_f = 2.5 \cdot J/kg$ is the latent heat. The precipitation in other hands can be estimated via a satellite or empiric relation between precipitation and the pressure field.

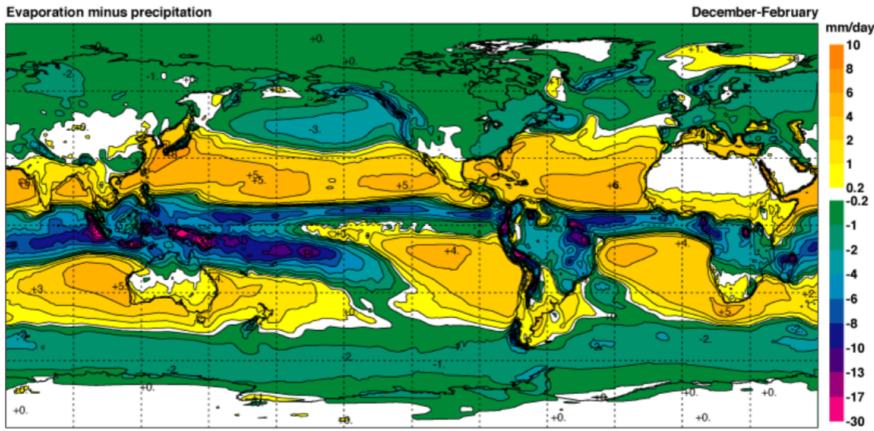


Figure 13: Evaporation minus precipitation in December January and February

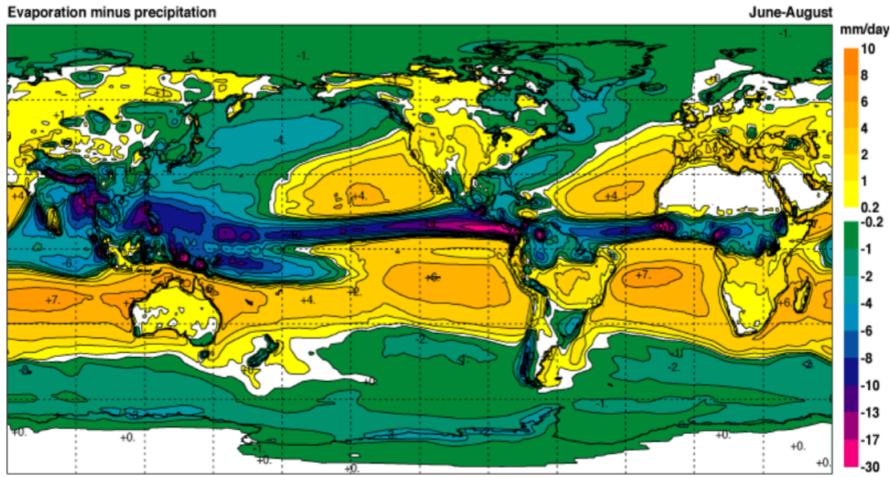


Figure 14: Evaporation minus precipitation in June July and August

In this figures we wan see the temporal average for the evaporation minus the precipitation. The general structure tends to be uniformly in zonas axis. In higher

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latitudes the precipitation is more important than the radiation, this is due to the solar radiation that is less important as you get closer to the poles.

Beside from the rain, there are other freshwater inputs for the oceans, as rivers and groundwater, but they are from minor importance for the oceanic circulation at mesoscale.

4 Parcels

The OceanParcels is a project that develops Parcels (Probably A Really Computationally Efficient Lagrangian Simulator), a set of Python classes and methods to create customized particle tracking simulations using an output from Ocean Circulation models. Parcels can be used to track passive and active particulates such as water, plankton, plastic and fish.

The code from the Ocean Parcels project is licensed under an open source MIT license and developed by research teams from the Imperial College, Utrecht University and the Grantham Institute.

4.1 Lagrangian vs Eulerian approach

There are two ways to track a particle or a parcel. We have used the Lagrangian due to its simplicity and that it makes easier for human sight to track the particle.

The Lagrangian specification of the field is a way of looking at fluid motion where the observer follows an individual fluid parcel as it moves through space and time. Plotting the position of an individual parcel through time gives the path line of the parcel. This can be visualized as sitting in a boat and drifting down a river.

The Eulerian specification of the flow field is a way of looking at the fluid motion that focuses on specific locations in the space through which the fluid flows as time passes.[1][2] This can be visualized by sitting on the bank of a river and watching the water pass the fixed location.

$$\frac{dN}{dt} = \frac{\partial N}{\partial t} + u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} + w \frac{\partial N}{\partial z} \quad (11)$$

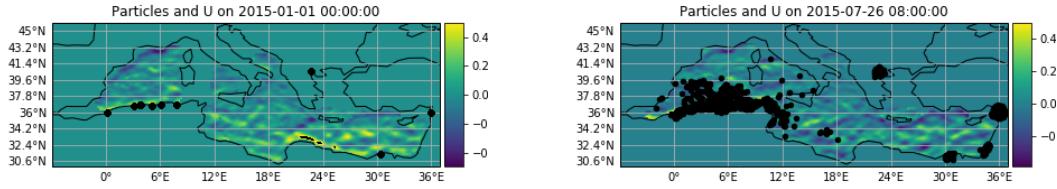
In Equation 1 we can see the total derivative which is a way to change from the Lagrangian reference framework to the Euler one.

5 Results

Once we have the data set of plastic from each of the rivers, we can introduce the required parameters in parcels.

- Plastic particles specified by the river localisation, and the time they are going to be added (seasonality).
- Advection fieldset, which is the superposition between the climatologic data (baseline) and the meteorologic ones (buoys & VOS).

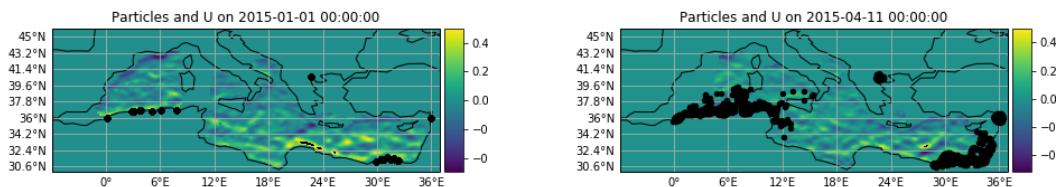
Our first idea was to compute the trajectory of plastic particles coming from the whole Mediterranean sea. However, we realized that simulating 2800 initial points with multiple plastic particles in each of those would take many hours. To avoid this problem, we reduced the scope of the simulations to only the top 20 most polluting Mediterranean rivers.



(a) Particle initial shape.

(b) Particle final shape.

Figure 15: 50 Kg particle packages motion from the top 20 rivers.



(a) Particle initial shape.

(b) Particle final shape.

Figure 16: 1 Kg particle packages motion from the top 20 rivers.

In the figures above, we can observe that most of the plastics inputs come from North Africa. In addition, we can compare the type of propagation taking into account the number of plastics. By taking only huge packages of plastic, we could see that although the Nile river is one of the top 20 global pollutants, huge packages come from Morocco coastline. On the other side, By taking into account small packages of plastics, Nile polluting the Mediterranean is more noticeable. This fact is due to the geographical structure of the Nile river, whose its mouth is not a unique input and it spreads its flux into several ones.

After realizing the potential of our program of estimating particle trajectories, we have reduced our scope of analysis into a smaller and closer problem, in order to take full advantage of the tool. Since we are studying in UAB, it was interesting for us studying trajectories of plastic particles starting in the coastline of Barcelona. Given the complexity of Parcels and the lack of information and tutorial on such an innovative software, we introduced periodically particles in a triangle shape next to Barcelona, simulating sporadic outputs from the city. Doing so, we achieved the following results:

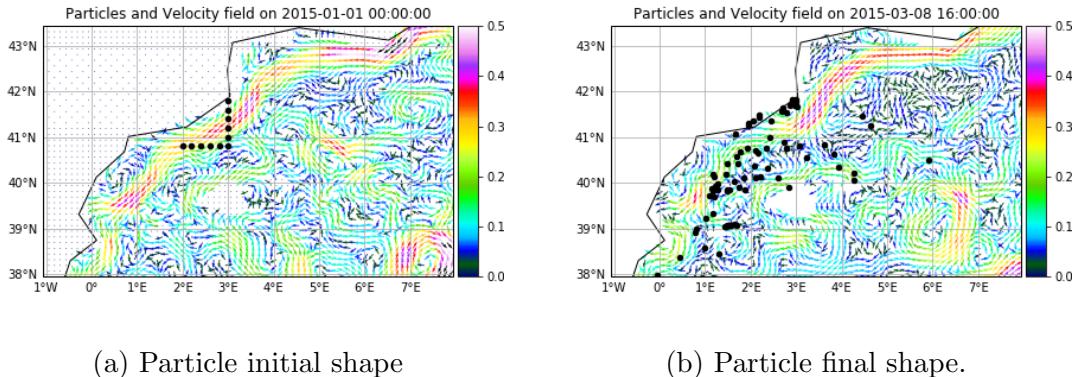


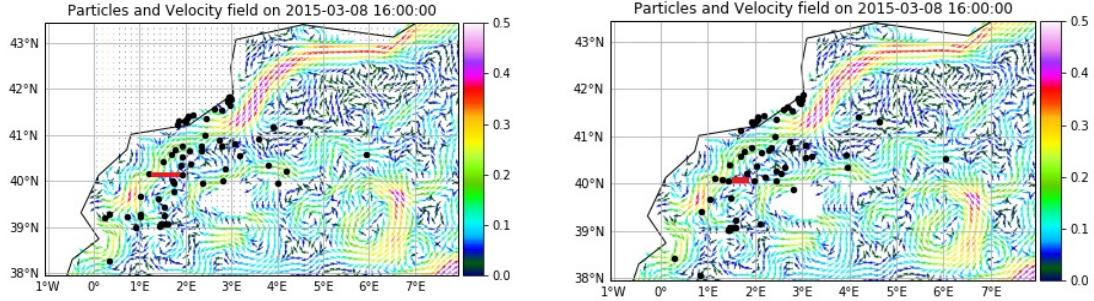
Figure 17: Particle advection from the Barcelona coastline.

Finally, knowing the trajectories with the initial point in Barcelona, in our attempt to create a simulation of a ship collecting the plastics, we have introduced two barriers in two separated simulations. // The first one, a larger barrier with a catching rate (capacity to collect plastic in the sea surface) of 20%, which represent, for instance, a few ships catching plastics in a huge region of the sea.

The second one, a smaller barrier with a catching rate of a 100% representing an optimized collecting plan, for instance, more ships or bigger nets.

This has been done creating two customized kernels, which recreates in some sense the recollecting process. Results are way easier to understand when using the ani-

mated versions of these plots we used in our presentation, but for didactic proposes, and comparison it is enough to see the final distribution of the particle set.



(a) Final shape for a 20% catchment rate. (b) Final shape for a 100% catchment rate.

Figure 18: Final particle distribution for different collecting configurations.

In the above figures, we can see the different final distribution of particles due to the introduction of a specific "barrier". Looking at this result its obvious that different barriers have different effects in the catching process.

6 Conclusions

Between 5 to 15 MT of plastic waste entered to the Oceans worldwide (between 2-5% of the total plastic waste) in 2010, so it's mandatory to face this problem.

The first part of our work has been about gathering data on plastic waste going to the Mediterranean Sea through rivers. We have been working using the data set provided by Lebreton et al.[6] which has been the major contributor on this part. Our main work in this section was understanding how they computed the mismanagement plastic waste through all the parameters explained and subsetting the data set focusing in those rivers ending up in the Mediterranean Sea. Also, we made some data engineering to fit the data set into a more useful and practical structure, Gis Spatial Data Type, compatible with the model requirements.

Once we finished with the data gathering, we focused in simulating trajectories for those plastics in those specific initial points. However, given the high complexity computation of over 2400 initial points, we focused the trajectory simulations for the top 20 most polluting rivers. Later, given the success of those simulations, we focused our scope in the coastline of Catalonia giving as initial points plastic particles leaving the coastline of Barcelona.

After studying the ocean and atmospheric movements, we can say that they are extremely complex. We can say that the ocean movements are a deterministic model where we know the equations but this is a chaotic model that is hard to predict and the partial differential equations are hard to solve. There has been a lot of work in the last centuries to do know try to predict the meteorology but there is still much to do.

Finally, we introduced two simple collecting simulations with different catching rates of plastic emulating ships in a specific area, which is the first step to a real approach like the Ocean Clean Up in the Pacific Ocean. Although it could be a topic of another project, we would like to say that, even if the two final distributions for the barrier simulation is similar, we believe that the one with the smaller "barrier" has a better performance because the particles that has not been gathered are less dispersed than in the one with the huge barrier.

7 Further Work

Once provided the subset of Plastic Waste into the Mediterranean Sea and the simulated trajectories using Parcels, we strongly suggest the idea of developing an optimized collection strategy of plastics in the sea. The idea could range from a theoretical simulation using our output from this project and/or, if dreaming big, simulate the idea of The Cleanup project in the Mediterranean Sea a net connected to a ship.

Also we could try to predict where the movement of the particle with more models. An use Spaghetti Ensemblings a utilized technique between meteorologist to visualize the difference between the models. This way we could have a confidence interval of the movement of water massess in the ocean.

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