Friction Force Simulator

Chares Moustakas

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1 Introduction

The current software is attempting to simulate the vehicle's potential slide under constant acceleration on a turn. It is important to underline that the aforementioned software is just a fun, non-optimized project and as a consequence the reader can feel free to use, modify, improve and play with it. It is notable to mention that this software was developed in C++ using relatively low level APIs and Libraries such as modern OpenGL, custom fragment-vertex shaders through GLSL, Dear ImGUI, openAssimp and CMake, while the "handmade" Objects-meshes were constructed through Blender. I would also like to thank: https://free3d.com/3d-models/ for the great ready-implemented objects.

Figure 1: Vehicle Object in Blender

2 Mathematics

The concept of this simulation is to calculate the static friction Force that the asphalt exterts to the vehicle's tires and exploring the boundary conditions that potentially can make the vehicle to slide. To begin with, it is critical to make the approximations clear, the vehicle is considered as a point object and not as a solid one, in order not to calculate expensive tasks such as center mass calculations, rotation axis of each scenario and complex torques. Let's hope on Maths and under the guidance of the equations the capable and necessary conditions will appear. The force that makes possible for the vehicle to turn is the static friction:

$$\overrightarrow{\mathrm{T}_{\mathrm{s}}} = m\frac{|\overrightarrow{\mathrm{u}^{2}}|}{R}\hat{r}$$

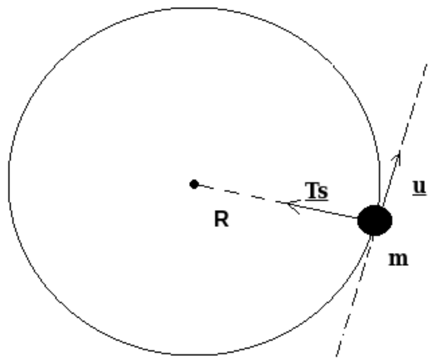

Figure 2: Vehicle Object in Blender

Where **m** is the mass of the vehicle, **u** the velocity, **R** the radius of convergence of the orbit and **r** the unit vector that points to the center of the orbit.

2.1 Boundary Condition

The primary condition for the vehicle in order to have a steady turn is:

$$|\overrightarrow{\mathrm{T_s}}| < \mu|\overrightarrow{\mathrm{N}}|$$

Where the right side of the inequality is equal to the sliding friction force wich is equal to the multiplication of the sliding friction coefficient and the vertical reaction. So, for bounary condition the aforementioned expression is equal to:

$$m\frac{|\overrightarrow{\mathrm{u}^2}|}{R} = \mu|\overrightarrow{\mathrm{N}}|$$

$$m\frac{|\overrightarrow{\mathrm{u}^2}|}{R} = \mu m|\overrightarrow{\mathrm{g}}|$$

$$\frac{|\overrightarrow{\mathrm{u}^2}|}{R} = \mu|\overrightarrow{\mathrm{g}}|$$

$$|\overrightarrow{\mathrm{u}^2}| = \mu R|\overrightarrow{\mathrm{g}}|$$

$$|\overrightarrow{\mathrm{u}}| = \sqrt{\mu R|\overrightarrow{\mathrm{g}}|} \tag{1}$$

The equation above underline that the sliping condition we are looking for is not related to the mass of the object! Consequently the expression (1) operates as a threshold value for the specific attributes given the fact that velocity is calculatable due to the fact that acceleration is known.

2.2 Arithmetic Example

Lets consider the following example. A turn of a road has sliding friction coefficient value (related to the tier) 0.8, the radius of convergence is 200m and the gravitational acceleration is 10 m/s^2.The threshold value provided by (1) will be equal to:

$$|\overrightarrow{\mathrm{u}}| = \sqrt{0.8200(m)10(m/s^2)}$$

$$|\overrightarrow{\mathrm{u}}| = \sqrt{1600(m^2/s^2)}$$

$$|\overrightarrow{\mathrm{u}}| = 40(m/s)$$

$$|\overrightarrow{\mathrm{u}}| = 40(3600km/1000h)$$

$$|\overrightarrow{\mathrm{u}}| = 40 \cdot 3.6(km/h)$$

$$|\overrightarrow{\mathrm{u}}| = 144(km/h)$$

Therefore, if velocity's magnitude is greater than the value above, under the specific conditions, the vehicle is about to slip.