1 Foundations of Cosmology

1.1 A brief history of cosmology

Cosmology is the science of origin and evolution of the Universe as a whole.

cosmogenesis: part of cosmology, dealing with origin and evolution of celestial bodies.

κόσμος, kósmos: universe; order decoration, embellishment (cosmetics!)

Begin of 'rational' cosmology with ancient philosophers:

- Anaximander (Ἀναξίμανδρος Anaxímandros; ⋆c. 610, †c. 545 BC)
- Empedokles (Ἐμπεδοκλῆς Empedoklḗs; ⋆c. 485, †c. 425 BC)
- Aristoteles (Ἀριστοτέλης Aristotélēs; ⋆384, †322 BC)
- Galileo Galilei (⋆1564, †1642)
- Johannes Kepler (⋆1571, †1630)
- Isaac Newton (⋆1642, †1727)
- Immanuel Kant (⋆1724, †1804)
- Harlow Shapley (⋆1885, †1972)
- Vesto Slipher (⋆1875, †1969)
- Albert Einstein (⋆1879, †1955)
- Willem de Sitter (⋆1872, †1934)
- Edwin Hubble (⋆1889, †1953)
- Carl Wilhelm Wirtz (⋆1876, †1939)
- Alexander Friedmann (⋆1888, †1925)
- Georges Edouard Lemaître (⋆1894, †1966)
- Maarten Schmidt (⋆1929) SCHMIDT 1963
- Alpher, Bethe, Gamov (1948)
- Penzias and Wilson (1965) Dicke et al. 1965; Penzias and Wilson 1965a,b
- COBE satellite (1989)
- Further satellite missions
- Supernova observations
- Important theoretical developments

1.2 Structures in the Universe and the Cosmological Principle

We observe *structures* in the Universe. Appropriate scale: *Megaparsec* (Mpc),

$$1\,\mathrm{Mpc} = 10^6\,\mathrm{pc} \approx 3.09 \times 10^{24}\,\mathrm{cm}. \tag{1.1}$$

Up to $\sim 100\,\mathrm{Mpc}$ galaxies ($\sim 30\,\mathrm{kpc}$), clusters of galaxies ($\sim 5\,\mathrm{Mpc}$), superclusters ($\sim 50\,\mathrm{Mpc}$) and voids ($\sim 100\,\mathrm{Mpc}$ to $200\,\mathrm{Mpc}$)

diagram (Milky way) (mainly from red-shift surveys)

dispute about structures beyond

$\gtrsim 100\,\mathrm{Mpc}$: mainly repetition of these structures

observable Universe $\sim$ *Hubble length*

$$cH_0^{-1} \approx 3h^{-1}\,\mathrm{Gpc}, \tag{1.2}$$

where

$$H_0 =: 100h\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \tag{1.3}$$

Planck Collaboration et al. 2014:

$$hH_0 = (67.4 \pm 1.4)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}, \tag{1.4}$$

so that $h \approx 0.67$, and

$$cH_0^{-1} \approx 4.4\,\mathrm{Gpc}. \tag{1.5}$$

$M_{\text{Milky Way}} \sim 1.4 \times 10^{11}\,M_\odot$; we observe about 10^{11} galaxies.

Milky Way belongs to the cluster 'Local group' and to the super-cluster 'Virgo' whose centre in about $20\,\mathrm{Mpc}$ distance.

Volume filled by galaxies $\lesssim 10^{-6}$ total volume, so that treat galaxies in cosmological models as 'points'.

There is also *Dark Matter*.

diagram Zwicky 1933

If matter were only in stars: $v_{\mathrm{rad}} \propto r^{-1/2}$ outside visible part,

$$v = \sqrt{\frac{GM(r)}{r}} \tag{1.6}$$

where $M(r)$ mass from centre to r. For large r, $M(r) \propto r$ fits curves. See III et al. 2005 for a cartography of our universe.

CMB highly isotropic (up to $\mathrm{O}(10^{-5})$); Universe also looks roughly the same in every direction.

Cosmological Principle dates back to Einstein 1917; in his paper "Zum kosmologischen Problem der allgemeinen Relativitätstheorie" he wrote:

> Alle Stellen des Universums sind gleichwertig; im speziellen soll also auch die örtlich gemittelte Dichte der Sternmaterie überall gleich sein.

("All places in the Universe are alike") (name *cosmological principle* due to Edward Milne 1933 **cannot find citation**)

Harrison compares it to Rudyard Kipling's "The Cat That Walked by Himself": "I am the Cat who walks by himself, and all places are alike to me."

In 1930, Einstein mentions the following second principle of his 1917 paper, which he now gives up in view of Hubble's observation:

> Räumliche Struktur und Dichte sollen zeitlich konstant sein.

(Later called “perfect cosmological principle” and revived in the theory of the steady-state universe Bondi and Gold 1948; Hoyle 1948)

Somewhat different from the cosmological principle is the *Copernican principle* (Bondi 1960 **cannot find citation**) (Copernicus: Sunis at the centre, but we are not there),

> We are not at the centre of the Universe.

where “We” refers to “us” and seems to perpetuate the belief that a centre exists (but we are not there), so that better to talk about the cosmological principle only!

Important consequences:

Isotropy around our position (CMB, etc.). Combined with cosmological principle one has isotropy about every point; according to a theorem Schur 1886 (see also section 1.3), this leads also to homogeneity, or spaces of constant curvature; see section 1.3.

Only with assumed homogeneity can one draw conclusions such as the following:

diagram

Statement “our Galaxy is at $t = t_2$ similarly structured than the galaxy (of the same class) that we see today as it was at $t = t_2$” can be drawn only if homogeneity holds.

1.3 Spaces of constant curvature

Isotropy around one point At the origin of a Riemann Normal Coordinate System, the Riemann tensor must be a tensor invariant under arbitrary rotations

(we talk about space here; in space-time, this would refer to Lorentz transformations, which would only be relevant for the perfect cosmological principle)

$$R_{iklm} = a\delta_{ik}\delta_{lm} + b\delta_{il}\delta_{km} + c\delta_{im}\delta_{kl} + d\epsilon_{iklm}, \tag{1.7}$$

where ϵ_{iklm} is a pseudo-tensor, so we set $d = 0$.

Antisymmetry properties of R_{iklm} entail

$$a = 0, \qquad b = -c, \tag{1.8}$$

so that

$$R_{iklm} = b(\delta_{il}\delta_{km} - \delta_{im}\delta_{kl}) \tag{1.9}$$

at origin of the Riemann Normal Coordinate System. In an arbitrary coordinate system,

$$R_{iklm} = b(x)(g_{il}g_{km} - g_{im}g_{kl}). \tag{1.10}$$

But if we had $b(x)$, then the gradient $\to \nabla b(x)$ would specify a *distinguished direction*, in contradiction to the cosmological principle[1], so that

$$b(x) = b = \text{const.}, \tag{1.11}$$

leading to a *space of constant curvature* (ibid.: isotropy everywhere leads to homogeneity).

[1]Result also follows from $G^i{}_{k;i} = 0$.

Line element? section 5.9: Two metrics are conformal to each other if

$$\overline{g}_{ab}(x) = \Omega^2(x) g_{ab}(x). \tag{1.12}$$

If this is the case, the Weyl tensor $C^{\cdot}{}_{\dots}$ coincide; conformally flat means $g_{ab}(x) = \Omega^{-2}(x)\delta_{ab}$, where the Weyl tensor for the metric δ_{ab} vanishes, so that $C^{\cdot}{}_{\dots} = 0$ for $g_{ab}(x)$ as well.

Special case of $n = 3$: $C^{\cdot}{}_{\dots} \equiv 0$; necessary and sufficient for conformal flatness is the vanishing of the *Cotton tensor* (Emile Cotton ($\star$1982, †1950) [**unrecognisable**]

$$C_{ijk} = R_{ij;k} - R_{ik;j} + \frac{1}{4}\big(g_{ik}R_{,j} - g_{ij}R_{,k}\big). \tag{1.13}$$

Use $R_{iklm}(x) = b(g_{il}g_{km} - g_{im}g_{kl})$, $b = \text{const.}$ [space of a constant curvature] to show that in this case $C^{\cdot}{}_{\dots} = 0$ ($n = 3$: $C_{\dots} = 0$).

So that *every space of constant curvature is conformally flat.*

1.4 Robertson-Walker line element

We have seen:

1.5 Friedmann-Lemaître equations

To derive the Friedmann-Lemaître equation, we write

1.6 Distance measure

Observation:

1.7 Temporal evolution of the scalar factor ($\Lambda = 0$)

We had

$$\Omega + \Omega_\Lambda + \Omega_k = 1 \tag{1.14}$$

1.8 Horizons

Study of *causal structure*

1.9 Cosmological constant and dark energy

We had

$$\Omega + \Omega_\Lambda + \Omega_k = 1 \tag{1.15}$$

1.10 de Sitter and anti-de Sitter space

Exact vacuum solution of Einstein equation for $\Lambda > 0$.