

An Introduction to [WZU17]

An Attempt to Avoid the Old Cosmological Constant Problem

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June 15, 2017

Outline

Overview

The Old Cosmological Constant Problem

Matter and space-time models

Solving $a(t, \vec{x})$: parametric oscillator

Solving $a(t, \vec{x})$: adiabatic approximation

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Overview

- ▶ If contributed by vacuum energy, the cosmological constant could **differ 120 order-of-magnitude** from observation.
- ▶ Wang, Zhu, and Unruh consider vacuum energy **density** instead, which is subject to **fluctuation**, and their result moderates the problem.
- ▶ They assumed a localised RW metric and obtained FL-like equations for $a(t, \vec{x})$, whose coefficient contains quantum fluctuation.
- ▶ The equations leads to local Hubble parameters which fluctuate, but a global average could be defined.
- ▶ The solution to the equations is evaluated with the help of theories of parametric oscillator and adiabatic invariance.
- ▶ The result is qualitatively supported by numerical calculation and further quantitative discussions.

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Cosmological constant and vacuum energy

- GR + vacuum QFT

$$G_{\mu\nu} + \lambda_b g_{\mu\nu} = 8\pi G T_{\mu\nu}^{\text{vac}} \quad (1)$$

- In Minkowski space-time, Lorentz invariance requires

$$T_{\mu\nu}^{\text{vac}} = -\rho_{\mu\nu}^{\text{vac}} \eta_{\mu\nu}, \quad (2)$$

which generalises to curved space-time as

$$T_{\mu\nu}^{\text{vac}} = -\rho_{\mu\nu}^{\text{vac}} g_{\mu\nu}. \quad (3)$$

- Effectively, eq. (1) can be written as

$$G_{\mu\nu} + \lambda_{\text{eff}} g_{\mu\nu} = 0, \quad \lambda_{\text{eff}} = \lambda_b + 8\pi G \rho^{\text{vac}}; \quad (4)$$

$$G_{\mu\nu} = -8\pi G \rho_{\text{eff}}^{\text{vac}} g_{\mu\nu}, \quad \rho_{\text{eff}}^{\text{vac}} = \rho^{\text{vac}} + \frac{\lambda_b}{8\pi G}. \quad (5)$$

Hubble parameter and cosmological constant

- Homogeneity and isotropy: Robertson–Walker metric

$$ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j. \quad (6)$$

- Hubble parameter / expansion rate $H := \dot{a}/a$; eqs. (4) and (5) take the corresponding Friedmann–Lemaître form

$$3H^2 = \lambda_{\text{eff}} = 8\pi G\rho_{\text{eff}}^{\text{vac}}, \quad (7)$$

$$3\ddot{a} = \lambda_{\text{eff}}a = 8\pi G\rho_{\text{eff}}^{\text{vac}}a. \quad (8)$$

- Solution to eq. (8)

$$a(t) = a(t_0) e^{H(t-t_0)} \quad (9)$$

The Old Cosmological Problem

- ▶ Contributions to $\rho_{\text{eff}}^{\text{vac}}$ or λ_{eff} : vacuum fluctuation of all quantum fields, Electroweak phase transition, etc.
- ▶ λ_{eff} by vacuum fluctuation evaluated in Minkowski space: taking a scalar field and using sharp-momentum cut-off (also see below),

$$\langle \rho^{\text{vac}} \rangle = \frac{\Lambda^4}{16\pi^2}. \quad (10)$$

Setting $\Lambda = E_{\text{P}}$ results in a surpass of the observed value of λ_{eff} by $\sim 10^{120}$.

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Matter: field and quantisation

► Matter model

$$S_m[\phi] = \int \mathbb{d}^4x \left[-\frac{1}{2} \eta^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi) \right] \quad (11)$$

► Canonical quantisation

$$\phi(t, \vec{x}) = \int \frac{\mathbb{d}^3k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega}} \left(a_{\vec{k}} e^{-i(\omega t - \vec{k} \cdot \vec{x})} + \text{h.c.} \right), \quad (12)$$

so that

$$T_{00} = \frac{1}{2} (\dot{\phi}^2 + (\nabla \phi)^2) \quad (13)$$

$$= \int \mathbb{d}^3k_1 \mathbb{d}^3k_2 f \left(a_{\vec{k}_1} a_{\vec{k}_2}^\dagger, a_{\vec{k}_1}^\dagger a_{\vec{k}_2}, a_{\vec{k}_1} a_{\vec{k}_2}, a_{\vec{k}_1}^\dagger a_{\vec{k}_2}^\dagger \right) \quad (14)$$

Matter: fluctuating vacuum energy density

- $|0\rangle$ is an eigenstate of $H = \int \mathrm{d}^3x T_{00}$, because

$$a_{\vec{k}} |0\rangle := 0, \quad \forall \vec{k} \quad (15)$$

- This is not the case for T_{00} , and

$$\langle (\Delta T_{00})^2 \rangle = \frac{2}{3} \langle T_{00} \rangle^2, \quad \langle T_{00} \rangle = \frac{\Lambda^4}{16\pi^2}. \quad (16)$$

Space-time: localised Robertson–Walker metric

- Inhomogeneity and isotropy

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t, \vec{x}) \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j. \quad (17)$$

- Generalising the proper distance and Hubble parameter

$$L(1 \rightarrow 2; t) = \int_{\vec{x}_1}^{\vec{x}_2} \sqrt{a^2(t, \vec{x})} \mathrm{d}l, \quad H(1 \rightarrow 2; t) = \frac{\dot{L}(1 \rightarrow 2)}{L(1 \rightarrow 2)} \quad (18)$$

- Einstein equations

$$G_{\mu\nu} = 8\pi GT_{\mu\nu}, \quad (19)$$

where $T_{\mu\nu}$'s are viewed as (random c-)numbers (and subject to fluctuation)

Space-time: stochastic oscillator equation for a

- ▶ Taking the trace of eq. (19) leads to

$$\ddot{a} + \Omega^2(t, \vec{x}) a = 0, \quad (20)$$

where

$$\Omega^2 = \frac{4\pi G}{3} g^{\mu\nu} T_{\mu\nu}, \quad (21)$$

which is viewed as a (random c-)number (and subject to fluctuation)

- ▶ $T_{\mu\nu}$ given by the quantised scalar field eqs. (11) and (12); evaluate Hubble parameter by solving eq. (20)

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Parametric oscillator

- ▶ If Ω were definite and periodic in t , eq. (20) would be a Hill's equation, describing **parametric oscillation** (see e.g. [LL76, §27] and Wikipedia)
- ▶ General solution

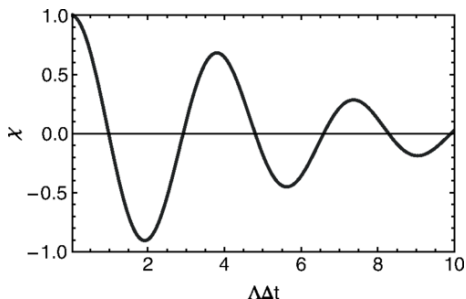
$$a(t, \vec{x}) = c_1 e^{+H_{\vec{x}} t} P_1(t, \vec{x}) + c_2 e^{-H_{\vec{x}} t} P_2(t, \vec{x}) \quad (22)$$

where $H_{\vec{x}} > 0$ (thus second term exponentially suppressed at late time) and P_i 's have the same period as Ω .

- ▶ **Claim:** Ω behaves **quasi-periodically**, because of $\text{Cov}[\Omega^2(t_1, \vec{x}), \Omega^2(t_2, \vec{x})]$ (see below)

Quasi-periodicity

- Covariance of $\Omega^2(t)$, taken from [WZU17]



- The solution is modified (at late time) to be

$$a(t, \vec{x}) \approx e^{\int_0^t H_{\vec{x}}(t') dt'} P(t, \vec{x}), \quad (23)$$

where $P(t, \vec{x})$ behaves also only quasi-periodically (see below).

Proper distance and Hubble parameter revisited

- ▶ The expressions for proper distance and Hubble parameter in terms of eq. (23) are

$$L(1 \rightarrow 2; t) = L(0) e^{Ht}, \quad (24)$$

$$L(1 \rightarrow 2; 0) = \int_{\vec{x}_1}^{\vec{x}_2} \sqrt{P^2(t, \vec{x})} d\vec{x}, \quad (25)$$

$$H = \frac{1}{t} \int_0^t H_{\vec{x}}(t') dt'. \quad (26)$$

- ▶ It looks like that the author assumed averaging $H_{\vec{x}}$ over a long time smooths out the difference between different \vec{x} .
- ▶ P and L can be solved by WKB; the presentation will focus on H

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Review: canonical transformation

- ▶ Introducing the generating function (of the second kind)

$$S = \int p \, \mathrm{d}q - H \, \mathrm{d}t \equiv \int P \, \mathrm{d}Q - K \, \mathrm{d}t + \mathrm{d}(-QP + G_2) \quad (27)$$

where $H = H(q, p, t)$, $K = K(Q, P, t)$, $G_2 = G_2(q, P, t)$, so that

$$\int (p - \partial_q G_2) \, \mathrm{d}q + (Q - \partial_P G_2) \, \mathrm{d}P - (H - \partial_t G_2 - K) \, \mathrm{d}t \equiv 0. \quad (28)$$

- ▶ Arbitrariness of the track in phase space requires

$$p = \partial_q G_2, \quad Q = \partial_P G_2, \quad K = H - \partial_t G_2. \quad (29)$$

- ▶ It can be shown that G_2 generates a Poisson-bracket-keeping transformation from $(q, p; H)$ to $(Q, P; K)$

Action-angle variables

- ▶ Consider a $G_2(q, J)$ which brings $(q, p; H)$ to $(\varphi, I; K(I))$
- ▶ Dynamics in the new variables reads

$$\dot{\varphi} = \partial_I K, \quad \dot{I} = 0; \quad (30)$$

$$\varphi = \partial_I K t + \varphi_0, \quad I = \text{const.} \quad (31)$$

Summary

- ▶ Compared the **pure** and the **thermal** descriptions of the radiation within the solvable dilaton gravity model
- ▶ Fourier-mode fluctuation: that of the thermal state **diverges** faster than the vacuum case does at low energy, while the pure-state fluc. remains **finite**; at high energies they **converge**.
- ▶ Trace distance: goes exponentially **small** with black hole **temperature** going to zero.
- ▶ Outlook in the proposed PhD study
 - ▶ Further discussion within full CGHS, BTZ, etc.
 - ▶ Make use of the decoherence theory
 - ▶ Nature of the microscopic degrees of freedom for BH entropy
 - ▶ Breakdown of the semi-classical approximations in QG

For Further Reading I



L.D. Landau and E.M. Lifshitz. *Mechanics*. 3rd. Vol. 1. Course of Theoretical Physics. Elsevier, 1976.



Qingdi Wang, Zhen Zhu, and William G. Unruh. “How the huge energy of quantum vacuum gravitates to drive the slow accelerating expansion of the Universe”. In: *Physical Review D* 95.10 (May 2017).

More on Trace Distance

Interpretation

- ▶ Maximal probability-difference obtainable

$$T(\hat{\rho}, \hat{\sigma}) = \max_{0 \leq \hat{\Lambda} \leq \hat{1}} \text{tr}\{\hat{\Lambda}(\hat{\rho} - \hat{\sigma})\}, \quad (32)$$

where all eigenvalues of $\hat{\Lambda}$ are in the range $[0, 1]$

- ▶ E.g. $\hat{\Lambda} := |\alpha\rangle\langle\alpha|$, $|\alpha\rangle$ eigenstate of \hat{A} with eigenvalue α
 - ▶ $\text{tr}\{\hat{\Lambda}\hat{\rho}\}$: the probability of getting α in measuring \hat{A}
 - ▶ $\text{tr}\{\hat{\Lambda}(\hat{\rho} - \hat{\sigma})\}$: the difference of the probability above
 - ▶ $T(\hat{\rho}, \hat{\sigma})$: the maximal value of the difference above

More on Fidelity

► General definition

$$F(|\alpha\rangle, |\beta\rangle) = |\langle\alpha|\beta\rangle| \quad (33)$$

$$F(|\alpha\rangle, \hat{\sigma}) = \sqrt{\langle\alpha|\hat{\sigma}|\alpha\rangle} \quad (?? \text{ rev.})$$

$$F(\hat{\rho}, \hat{\sigma}) = \text{tr} \sqrt{\hat{\rho}^{\frac{1}{2}} \hat{\sigma} \hat{\rho}^{\frac{1}{2}}} \quad (34)$$

► Interpretation: faithfulness

$$F(|\alpha\rangle, |\alpha\rangle) = 1 \quad (35)$$

Another New Foundation of Statistical Physics

Specific and easy version of the construction

- ▶ Total isolated system U with **energy** constraint $\langle \widehat{H}_U \rangle := E_U$, divided into a **(sub)system** S and an environment E
- ▶ Hilbert spaces $\mathcal{H}_R \supseteq \mathcal{H}_U = \mathcal{H}_S \otimes \mathcal{H}_E$; $\hat{1}_R$ identity on \mathcal{H}_R , dimension $d_R := \dim \mathcal{H}_R < +\infty$
- ▶ Equiprobable / maximal-ignorant state of U

$$\hat{\mathcal{E}}_R := d_R^{-1} \hat{1}_R \in \mathcal{H}_R \quad (36)$$

- ▶ Hamiltonians $\widehat{H}_U = \widehat{H}_S + \widehat{H}_E + \widehat{H}_{\text{int}}$
- ▶ **Canonical state** of S with energy constraint

$$\widehat{\Omega}_S^{(\text{E})} := \text{tr}_E \hat{\mathcal{E}}_R \propto \exp(-\widehat{H}_S/T_{\text{th}}) \quad (37)$$

- ▶ **Theorem:** $\forall |\phi\rangle \in \mathcal{H}_R$, the reduced state of S

$$\text{tr}_E |\phi\rangle \langle \phi| =: \hat{\rho}_S(\phi) \approx \widehat{\Omega}_S^{(\text{E})}.$$

Another New Foundation of Statistical Physics

Generic and exact version of the construction

- ▶ **Arbitrary** constraint R ; study the **trace distance** T between $\hat{\rho}_S(\phi)$ and $\hat{\Omega}_S$
- ▶ Lemma: **average distance** is small w.r.t. d_S/d_E^{eff}

$$\langle T(\hat{\rho}_S(\phi), \hat{\Omega}_S) \rangle \leq \frac{1}{2} \sqrt{d_S/d_E^{\text{eff}}} \quad (39)$$

- ▶ Theorem: **probability of large deviation** is exponentially small w.r.t. the distance; an easy version

$$\frac{V\left[\left\{|\phi\rangle \mid T(\hat{\rho}_S(\phi), \hat{\Omega}_S) \geq d_R^{-\frac{1}{3}}\right\}\right]}{V[\{|\phi\rangle\}]} \leq 4 \exp\left(-\frac{2d_R^{\frac{1}{3}}}{9\mathfrak{M}^3}\right) \quad (40)$$

- ▶ Effective dimension of E : setting $\hat{\Omega}_E = \text{tr}_S \hat{\mathcal{E}}_R$,

$$d_U/d_S \equiv d_E \geq d_E^{\text{eff}} := \left(\text{tr} \hat{\Omega}_E^2\right)^{-1} \geq d_R/d_S.$$