An Introduction to [WZU17]

An Attempt to Avoid the Old Problem of Cosmological Constant

Yi-Fan Wang (王 一帆)

Institut für Theoretische Physik
Universität zu Köln

June 13, 2017

Outline

Outline

Overview

- A vacuum-energy-contributed effective cosmological constant could differ 120 order-of-magnitude from observation, which needs an extreme fine-tuning by the bare cosmological constant.

- Wang, Zhu, and Unruh consider vacuum energy density instead, which is subject to fluctuation, and their result moderates the problem.

- They assumed a localised RW metric and obtained a FL-like equation for $a(t, \vec{x})$, whose coefficient contains quantum fluctuation. The solution to the equation is evaluated with the help of theories of parametric oscillations and adiabatic invariances.

- The result is qualitatively supported by numerical calculation and further quantitative discussions.

Outline

Cosmological Constant and Vacuum energy

- GR + vacuum QFT

$$G_{\mu\nu} + \lambda_{\mathsf{b}} g_{\mu\nu} = 8\pi G T_{\mu\nu}^{\mathsf{vac}} \tag{1}$$

- In Minkowski space-time, Lorentz invariance requires

$$T_{\mu\nu}^{\mathsf{vac}} = -\rho_{\mu\nu}^{\mathsf{vac}} \eta_{\mu\nu}, \tag{2}$$

  which generalises to curved space-time as

$$T_{\mu\nu}^{\mathsf{vac}} = -\rho_{\mu\nu}^{\mathsf{vac}} g_{\mu\nu}. \tag{3}$$

- Effectively, eq. (1) can be written as

$$G_{\mu\nu} + \lambda_{\mathsf{eff}} g_{\mu\nu} = 0, \qquad \lambda_{\mathsf{eff}} = \lambda_{\mathsf{b}} + 8\pi G \rho^{\mathsf{vac}}; \tag{4}$$

$$G_{\mu\nu} = -8\pi G \rho_{\mathsf{eff}}^{\mathsf{vac}} g_{\mu\nu}, \qquad \rho_{\mathsf{eff}}^{\mathsf{vac}} = \rho^{\mathsf{vac}} + \frac{\lambda_{\mathsf{b}}}{8\pi G}. \tag{5}$$

Hawking Radiation
Results and interpretation

▶ An early-time vacuum on $\mathcal{I}^-$ in collapsing background

$$\hat{a}(p)\,|h\rangle := 0 \quad \Rightarrow \quad \langle h\,|\,\hat{n}_a(p)\,|\,h\rangle =: \langle \hat{n}_a(p)\rangle_h \equiv 0 \qquad (6)$$

evolves to a late-time state on $\mathcal{I}^+ \cup \hslash^+$ with particles on $\mathcal{I}^+$

$$\langle \hat{n}_{\text{on } \mathcal{I}^+}(\omega)\rangle_h =: \langle \hat{n}_b(\omega)\rangle_h \approx \Gamma_\omega \big(e^{2\pi\omega/\kappa} - 1\big)^{-1}. \qquad (7)$$

▶ Comparing eq. (7) with the Bose–Einstein distribution

$$\langle \hat{n}(\omega)\rangle_{\text{BE}} = \big(e^{\omega/T} - 1\big)^{-1}, \qquad (8)$$

one may conclude that eq. (7) describes a grey-body radiation with the Hawking temperature,

$$T_{\text{H}} := \kappa/2\pi \equiv \hbar/ck \cdot \kappa/2\pi. \qquad (9)$$

Hawking Radiation

Tension in the interpretation

- The state $|h\rangle$ or its density operator is pure,

$$\hat{\rho}_h = |h\rangle \langle h|, \qquad (10)$$

whilst the of equilibrium bosonic ideal gas

$$\hat{\rho}_{\mathsf{BE}}(T) = Z^{-1} \mathbb{e}^{-\widehat{H}/T} \sim Z^{-1} \sum_E \mathbb{e}^{-E/T} |E\rangle \langle E| \qquad (11)$$

is thermal and mixed.

- How different are they?

Outline

Classical theory of $(1+1)$d Dilaton Gravity Model I

- The action of the dilaton gravity model reads

$$S = \int \mathrm{d}^2 x \sqrt{-g} \left\{ \frac{\mathrm{e}^{-2\phi}}{G} \left[R + 4(\nabla\phi)^2 + 4\lambda^2 \right] - \frac{1}{2}(\nabla f)^2 \right\}, \tag{12}$$

  - $\phi(x)$ the dilaton field, without which topological
  - $f(x)$ a massless neutral scalar field representing matter
  - $\lambda > 0$ the cosmological constant

- Has a solution which resembles the collapsing body in $(3+1)$-dimensional Einstein gravitation

  - Each point represents a point

  - **Thick** line: null matter shell

Quantum theory of $(1+1)$d Dilaton Gravity Model I

- Constraint system: Schrödinger quantisation does not apply
- (Formally) Dirac quantisation

$$\widehat{\mathcal{H}}_{\parallel}\Psi[g,\phi,f] = 0, \qquad \widehat{\mathcal{H}}_{\perp}\Psi[g,\phi,f] = 0. \qquad (13)$$

- Semi-classical approximation: $\Psi = e^{\frac{i}{\hbar}(G^{-1}S_0 + S_1 + GS_2 + \dots)}$
  - $O(G^{-1})$: Hamilton–Jacobi equation for pure gravity
  - $O(G^0)$: $\Psi = D[g,\phi]\chi[g,\phi,f]$; functional Schrödinger equation for matter $i\hbar\partial_t\chi[f] = \widehat{H}_{\mathsf{m}}\chi[f]$, where

$$\widehat{H}_{\mathsf{m}} = \frac{1}{2}\int_0^{+\infty}\mathrm{d}k\left(-\frac{\boldsymbol{\delta}^2}{\boldsymbol{\delta}f^2(k)} + k^2 f^2(k)\right). \qquad (14)$$

- A quantum field theory in curved space-time can be derived!

Quantum theory of $(1+1)$d Dilaton Gravity Model II

▶ At early time, the vacuum wave functional is

$$\chi_0[f_{\mathsf{e}}] \propto \exp\left\{-\frac{1}{2}\int_{\mathbb{R}^+}\mathrm{d}k\,k\,f_{\mathsf{e}}^2(k)\right\} \sim \prod_k \exp\left\{\frac{1}{2}\frac{k}{\Lambda}\,f_{\mathsf{e}}^2\right\}, \quad (15)$$

while at late time it evolves to

$$\chi_b[f_{\mathsf{l}}] \propto \exp\left\{-\int_{\mathbb{R}}\mathrm{d}p\,p\coth\left(\frac{\pi p}{2\lambda}\right)|f_{\mathsf{l}}(p)|^2\right\} \sim \prod_p \mathbb{e}^{\cdots}, \quad (16)$$

where $f_{\mathsf{e}}(k)$ and $f_{\mathsf{l}}(p)$ are the Fourier transform of the matter field at early and late time, respectively.

▶ At late time, particle-number expectations are

$$\langle \hat{n}_b(p) \rangle_{\chi_b} = \left(\mathbb{e}^{2\pi|p|/\lambda} - 1\right)^{-1}, \quad (17)$$

leading to a Hawking-like black-body temperature

$$\boxed{T_{\mathsf{HD}} := \lambda/2\pi.} \quad (18)$$

Outline

Correlation of Fourier Modes

The discrepancy

▶ The Fourier-mode correlators can be calculated,

$$\left\langle \hat{f}^{\dagger}(p_1)\hat{f}(p_2)\right\rangle = \frac{1}{T_{\mathsf{HD}}}\delta(p_1 - p_2)\cdot\begin{cases}\frac{1}{2}\frac{1}{q}, & \text{vacuum};\\ \frac{1}{8}\frac{\tanh\frac{q}{4}}{\frac{q}{4}}, & \chi_b;\\ \frac{1}{4}\frac{\coth\frac{q}{2}}{\frac{q}{2}}, & \hat{\rho}_{\mathsf{BE}}(T_{\mathsf{HD}}),\end{cases}$$

$$(19)$$

where $q \coloneqq p_1/T_{\mathsf{HD}}$

▶ Diagonal elements (fluctuations) are plotted

Correlation of Fourier Modes

Fluctuation of the Fourier modes: diagram in log-log scale

Correlation of Fourier Modes

Fluctuation of the Fourier modes: interpretation

- Vacuum fluctuation

$$\left\langle \left| \hat{f} \right|^2 \right\rangle_{\text{vac}} = \mathrm{O}(q^{-1}) \tag{20}$$

- A black hole does not alter the high-energy processes

$$\left\langle \left| \hat{f} \right|^2 \right\rangle_{\chi_b} \approx \left\langle \left| \hat{f} \right|^2 \right\rangle_{\text{th}} \approx \left\langle \left| \hat{f} \right|^2 \right\rangle_{\text{vac}} = \mathrm{O}(q^{-1}), \quad |p| \gg T_{\text{HD}} \tag{21}$$

- A black hole suppresses low-energy fluctuation of the pure state, while enhancing that of the thermal state

$$\mathrm{O}(1) \sim \left\langle \left| \hat{f} \right|^2 \right\rangle_{\chi_b} \ll \left\langle \left| \hat{f} \right|^2 \right\rangle_{\text{vac}} \ll \left\langle \left| \hat{f} \right|^2 \right\rangle_{\text{th}} \sim \mathrm{O}(q^{-2}), \quad |p| \ll T_{\text{HD}} \tag{22}$$

- Critical scale $|p| \sim T_{\text{HD}}$

Outline

Trace Distance

Definitions

▶ Trace distance between Hermitian operators $\widehat{M}$ and $\widehat{N}$

$$T\big(\widehat{M}, \widehat{N}\big) := \frac{1}{2} \operatorname{tr} \sqrt{\big(\widehat{M} - \widehat{N}\big)^{\dagger}\big(\widehat{M} - \widehat{N}\big)} \qquad (23)$$

▶ For density operators $\hat{\rho}$ and $\hat{\sigma}$,
  ▶ $0 \leq T(\hat{\rho}, \hat{\sigma}) \leq 1$;
  ▶ Controlled by fidelity in

$$\boxed{1 - F(\hat{\rho}, \hat{\sigma}) \leq T(\hat{\rho}, \hat{\sigma}) \leq \sqrt{1 - F^2(\hat{\rho}, \hat{\sigma})},} \qquad (24)$$

where we only need

$$F(|\alpha\rangle, \hat{\sigma}) := \langle \alpha | \hat{\sigma} | \alpha \rangle^{\frac{1}{2}}. \qquad (25)$$

▶ In our application, T is difficult while F can be obtained.

Single-mode Distances between the Density Operators I

▶ The pure state can be decomposed upon discretisation
$\chi_b[f] \sim \sum_p \chi_b^{(p)}(f_p) \equiv \sum_p \left\langle f_p \,\middle|\, \chi_b^{(p)} \right\rangle$, where $f_p := f(p)$

▶ So is the thermal density operator $\hat{\rho}_{\mathsf{th}}(T) \sim \bigotimes_p \hat{\rho}_{\mathsf{th}}^{(p)}(T)$

▶ Fidelity (in eq. (24)) can be factorised as well

$$F \equiv \left\langle \chi_b \,\middle|\, \hat{\rho}_{\mathsf{th}} \,\middle|\, \chi_b \right\rangle^{\frac{1}{2}} \sim \prod_p \left\langle \chi_b^{(p)} \,\middle|\, \hat{\rho}_{\mathsf{th}}^{(p)} \,\middle|\, \chi_b^{(p)} \right\rangle^{\frac{1}{2}} =: \prod_p F^{(p)};$$

$$(26)$$

▶ $F^{(p)}$ can be computed in order to find bounds of T

$$F^{(p)}\left(|p\rangle, \hat{\rho}_{\mathsf{th}}^{(p)}(T_{\mathsf{HD}})\right) = \frac{\sqrt{u-1}}{\sqrt[4]{u^2+u+1}}, \quad u := \mathbb{e}^q \equiv \mathbb{e}^{|p|/T_{\mathsf{HD}}}.$$

$$(27)$$

All-modes Distances between the Density Operators I

- 'Go to the continuum limit': Λ dimension regulator

$$\sum_p g(p) \to \frac{1}{2\pi\Lambda} \int \mathrm{d}p \, g(p), \qquad (28)$$

- Analogously, to regularise a product

$$\prod_p f(p) \equiv \exp\left\{\sum_p \ln f(p)\right\} \to \exp\left\{\frac{1}{2\pi\Lambda} \int \mathrm{d}p \, \ln f(p)\right\}$$
$$(29)$$

- Regularised F can be calculated in order to set bounds of T

$$F(\chi_b, \hat{\rho}_{\mathrm{th}}) = \exp\left\{\frac{2}{2\pi\Lambda} \int_0^{+\infty} \mathrm{d}p \, \ln F^{(p)}\right\} = \exp\left(-\frac{\pi}{9}\frac{T_{\mathrm{HD}}}{\Lambda}\right)$$
$$(30)$$

Summary

▶ Compared the pure and the thermal descriptions of the radiation within the solvable dilaton gravity model

▶ Fourier-mode fluctuation: that of the thermal state diverges faster than the vacuum case does at low energy, while the pure-state fluc. remains finite; at high energies they converge.

▶ Trace distance: goes exponentially small with black hole temperature going to zero.

▶ Outlook in the proposed PhD study
  ▶ Further discussion within full CGHS, BTZ, etc.
  ▶ Make use of the decoherence theory

  ▶ Nature of the microscopic degrees of freedom for BH entropy
  ▶ Breakdown of the semi-classical approximations in QG

For Further Reading I

📄 Qingdi Wang, Zhen Zhu, and William G. Unruh. "How the huge energy of quantum vacuum gravitates to drive the slow accelerating expansion of the Universe". In: *Physical Review D* 95.10 (May 2017).

▶ Maximal probability-difference obtainable

$$T(\hat{\rho}, \hat{\sigma}) = \max_{0 \leq \widehat{\Lambda} \leq \widehat{1}} \mathrm{tr}\left\{\widehat{\Lambda}(\hat{\rho} - \hat{\sigma})\right\}, \qquad (31)$$

where all eigenvalues of $\widehat{\Lambda}$ are in the range $[0, 1]$

▶ E.g. $\widehat{\Lambda} := |\alpha\rangle \langle \alpha|$, $|\alpha\rangle$ eigenstate of $\widehat{A}$ with eigenvalue α

    ▶ $\mathrm{tr}\left\{\widehat{\Lambda}\hat{\rho}\right\}$: the probability of getting α in measuring $\widehat{A}$

    ▶ $\mathrm{tr}\left\{\widehat{\Lambda}(\hat{\rho} - \hat{\sigma})\right\}$: the difference of the probability above

    ▶ $T(\hat{\rho}, \hat{\sigma})$: the maximal value of the difference above

More on Fidelity

- General definition

$$F(|\alpha\rangle, |\beta\rangle) = |\langle \alpha \,|\, \beta \rangle| \tag{32}$$

$$F(|\alpha\rangle, \hat{\sigma}) = \sqrt{\langle \alpha \,|\, \hat{\sigma} \,|\, \alpha \rangle} \tag{25 rev.}$$

$$F(\hat{\rho}, \hat{\sigma}) = \operatorname{tr} \sqrt{\hat{\rho}^{\frac{1}{2}} \hat{\sigma} \hat{\rho}^{\frac{1}{2}}} \tag{33}$$

- Intepretation: faithfulness

$$F(|\alpha\rangle, |\alpha\rangle) = 1 \tag{34}$$

Another New Foundation of Statistical Physics

Specific and easy version of the construction

▶ Total isolated system U with energy constraint $\left\langle \widehat{H}_U \right\rangle := E_U$, divided into a (sub)system S and an environment E

▶ Hilbert spaces $\mathcal{H}_R \supseteq \mathcal{H}_U = \mathcal{H}_S \otimes \mathcal{H}_E$; $\hat{1}_R$ identity on $\mathcal{H}_R$, dimension $d_R := \dim \mathcal{H}_R < +\infty$

▶ Equiprobable / maximal-ignorant state of U

$$\hat{\mathcal{E}}_R := d_R^{-1} \hat{1}_R \in \mathcal{H}_R \tag{35}$$

▶ Hamiltonians $\widehat{H}_U = \widehat{H}_S + \widehat{H}_E + \widehat{H}_{\text{int}}$

▶ Canonical state of S with energy constraint

$$\widehat{\Omega}_S^{(\mathsf{E})} := \operatorname{tr}_E \hat{\mathcal{E}}_R \propto \exp\left(-\widehat{H}_S / T_{\text{th}}\right) \tag{36}$$

▶ Theorem: $\forall \, |\phi\rangle \in \mathcal{H}_R$, the reduced state of S

$$\boxed{\operatorname{tr}_E |\phi\rangle\langle\phi| =: \hat{\rho}_S(\phi) \approx \widehat{\Omega}_S^{(\mathsf{E})}.} \tag{37}$$

Another New Foundation of Statistical Physics

Generic and exact version of the construction

▶ Arbitrary constraint R; study the trace distance T between $\hat{\rho}_S(\phi)$ and $\widehat{\Omega}_S$

▶ Lemma: average distance is small w.r.t. d_S/d_E^{eff}

$$\left\langle T\left(\hat{\rho}_S(\phi), \widehat{\Omega}_S\right)\right\rangle \leq \tfrac{1}{2}\sqrt{d_S/d_E^{\text{eff}}} \tag{38}$$

▶ Theorem: probability of large deviation is exponentially small w.r.t. the distance; an easy version

$$\frac{V\left[\left\{|\phi\rangle \;\middle|\; T\left(\hat{\rho}_S(\phi), \widehat{\Omega}_S\right) \geq d_R^{-\frac{1}{3}}\right\}\right]}{V[\{|\phi\rangle\}]} \leq 4\exp\left(-\frac{2d_R^{\frac{1}{3}}}{9\mathbb{π}^3}\right) \tag{39}$$

▶ Effective dimension of E: setting $\widehat{\Omega}_E = \text{tr}_S\,\hat{\mathcal{E}}_R$,

$$d_U/d_S \equiv d_E \geq d_E^{\text{eff}} := \left(\text{tr}\,\widehat{\Omega}_E^2\right)^{-1} \geq d_R/d_S. \tag{40}$$