# An Introduction to [WZU17]

## An Attempt to Avoid the Old Cosmological Constant Problem


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# Outline

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# Overview

- If contributed by vacuum energy, the cosmological constant could differ 120 order-of-magnitude from observation.
- Wang, Zhu, and Unruh consider vacuum energy density instead, which is subject to fluctuation, and their result moderates the problem.
- They assumed a localised RW metric and obtained FL-like equations for $a(t, \vec{x})$, whose coefficient contains quantum fluctuation.
- The equations leads to local Hubble parameters which fluctuate, but a global average could be defined.
- The solution to the equations is evaluated with the help of theories of parametric oscillator and adiabatic invariance.
- The result is qualitatively supported by numerical calculation and further quantitative discussions.

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# Cosmological constant and vacuum energy

- GR + vacuum QFT

$$G_{\mu\nu} + \lambda_{\mathrm{b}} g_{\mu\nu} = 8\pi G T^{\mathrm{vac}}_{\mu\nu} \tag{1}$$

- In Minkowski space-time, Lorentz invariance requires

$$T^{\mathrm{vac}}_{\mu\nu} = -\rho^{\mathrm{vac}}_{\mu\nu} \eta_{\mu\nu}, \tag{2}$$

which generalises to curved space-time as

$$T^{\mathrm{vac}}_{\mu\nu} = -\rho^{\mathrm{vac}}_{\mu\nu} g_{\mu\nu}. \tag{3}$$

- Effectively, eq. (1) can be written as

$$G_{\mu\nu} + \lambda_{\mathrm{eff}} g_{\mu\nu} = 0, \qquad \lambda_{\mathrm{eff}} = \lambda_{\mathrm{b}} + 8\pi G \rho^{\mathrm{vac}}; \tag{4}$$

$$G_{\mu\nu} = -8\pi G \rho^{\mathrm{vac}}_{\mathrm{eff}} g_{\mu\nu}, \qquad \rho^{\mathrm{vac}}_{\mathrm{eff}} = \rho^{\mathrm{vac}} + \frac{\lambda_{\mathrm{b}}}{8\pi G}. \tag{5}$$

# Hubble parameter and cosmological constant

- Homogeneity and isotropy: Robertson–Walker metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j. \tag{6}$$

- Hubble parameter / expansion rate $H := \dot{a}/a$; eqs. (4) and (5) take the corresponding Friedmann–Lemaître form

$$3H^2 = \lambda_{\text{eff}} = 8\pi G\rho_{\text{eff}}^{\text{vac}}, \tag{7}$$

$$3\ddot{a} = \lambda_{\text{eff}} a = 8\pi G\rho_{\text{eff}}^{\text{vac}} a. \tag{8}$$

- Solution to eq. (8)

$$a(t) = a(t_0)\,\mathrm{e}^{H(t-t_0)} \tag{9}$$

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# The Old Cosmological Problem

- Contributions to $\rho_{\text{eff}}^{\text{vac}}$ or $\lambda_{\text{eff}}$: vacuum fluctuation of all quantum fields, Electroweak phase transition, etc.
- $\lambda_{\text{eff}}$ by vacuum fluctuation evaluated in Minkowski space: taking a scalar field and using sharp-momentum cut-off (also see below),

$$\langle \rho^{\text{vac}} \rangle = \frac{\Lambda^4}{16\pi^2}. \tag{10}$$

  Setting $\Lambda = E_{\text{P}}$ results in a surpass of the observed value of $\lambda_{\text{eff}}$ by $\sim 10^{120}$.

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# Matter: field and quantisation

- Matter model

$$S_{\mathsf{m}}[\phi] = \int \mathbb{d}^4 x \left[ -\frac{1}{2} \eta^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) \right] \tag{11}$$

- Canonical quantisation

$$\phi(t, \vec{x}) = \int \frac{\mathbb{d}^3 k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega}} \left( a_{\vec{k}}\, \mathbb{e}^{-\mathbb{i}(\omega t - \vec{k}\cdot\vec{x})} + \text{h.c.} \right), \tag{12}$$

so that

$$T_{00} = \frac{1}{2}\left(\dot{\phi}^2 + (\nabla\phi)^2\right) \tag{13}$$

$$= \int \mathbb{d}^3 k_1\, \mathbb{d}^3 k_2\, f\left( a_{\vec{k}_1} a^\dagger_{\vec{k}_2}, a^\dagger_{\vec{k}_1} a_{\vec{k}_2}, a_{\vec{k}_1} a_{\vec{k}_2}, a^\dagger_{\vec{k}_1} a^\dagger_{\vec{k}_2} \right) \tag{14}$$

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# Matter: fluctuating vacuum energy density

- $|0\rangle$ is an eigenstate of $H = \int \mathrm{d}^3 x T_{00}$, because

$$a_{\vec{k}} |0\rangle := 0, \quad \forall \vec{k} \tag{15}$$

- This is not the case for $T_{00}$, and

$$\left\langle (\Delta T_{00})^2 \right\rangle = \frac{2}{3} \langle T_{00} \rangle^2, \qquad \langle T_{00} \rangle = \frac{\Lambda^4}{16\pi^2}. \tag{16}$$

# Space-time: localised Robertson–Walker metric

- Inhomogeneity and isotropy

$$\mathbb{d}s^2 = -\mathbb{d}t^2 + a(t, \vec{x})\, \delta_{ij}\, \mathbb{d}x^i\, \mathbb{d}x^j. \tag{17}$$

- Generalising the proper distance and Hubble parameter

$$L(1 \to 2; t) = \int_{\vec{x}_1}^{\vec{x}_2} \sqrt{a^2(t, \vec{x})}\, \mathbb{d}l, \quad H(1 \to 2; t) = \frac{\dot{L}(1 \to 2)}{L(1 \to 2)} \tag{18}$$

- Einstein equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{19}$$

where $T_{\mu\nu}$'s are viewed as (random c-)numbers (and subject to fluctuation)

# Space-time: stochastic oscillator equation for $a$

- Taking the trace of eq. (19) leads to

$$\ddot{a} + \Omega^2(t, \vec{x})\, a = 0, \tag{20}$$

where

$$\Omega^2 = \frac{4\pi G}{3} g^{\mu\nu} T_{\mu\nu}, \tag{21}$$

which is viewed as a (random c-)number (and subject to fluctuation)
- $T_{\mu\nu}$ given by the quantised scalar field eqs. (11) and (12); evaluate Hubble parameter by solving eq. (20)

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## Parametric oscillator

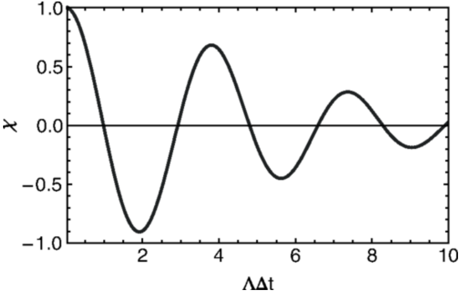
- If $\Omega$ were definite and periodic in $t$, eq. (20) would be a Hill's equation, describing parametric oscillation (see e.g. [LL76, §27] and Wikipedia)
- General solution

$$a(t, \vec{x}) = c_1 \mathrm{e}^{+H_{\vec{x}} t} P_1(t, \vec{x}) + c_2 \mathrm{e}^{-H_{\vec{x}} t} P_2(t, \vec{x}) \tag{22}$$

  where $H_{\vec{x}} > 0$ (thus second term exponentially suppressed at late time) and $P_i$'s have the same period as $\Omega$.
- **Claim**: $\Omega$ behaves quasi-periodically, because of $\mathrm{Cov}[\Omega^2(t_1, \vec{x}), \Omega^2(t_2, \vec{x})]$ (see below)

# Quasi-periodicity

- Covariance of $\Omega^2(t)$, taken from [WZU17]




- The solution is modified (at late time) to be

$$a(t, \vec{x}) \approx \mathrm{e}^{\int_0^t H_{\vec{x}}(t')\,\mathrm{d}t'} P(t, \vec{x}), \tag{23}$$

where $P(t, \vec{x})$ behaves also only quasi-periodically (see below).

# Proper distance and Hubble parameter revisited

- The expressions for proper distance and Hubble parameter in terms of eq. (23) are

$$L(1 \to 2; t) = L(0)\, \mathbb{e}^{Ht}, \tag{24}$$

$$L(1 \to 2; 0) = \int_{\vec{x}_1}^{\vec{x}_2} \sqrt{P^2(t, \vec{x})}\, \mathbb{d}t, \tag{25}$$

$$H = \frac{1}{t} \int_0^t H_{\vec{x}}(t')\, \mathbb{d}t'. \tag{26}$$

- It looks like that the authors assumed averaging $H_{\vec{x}}$ over a long time smooths out the difference between different $\vec{x}$.
- $P$ and $L$ can be solved by WKB; the presentation will focus on $H$

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# Review: canonical transformation

- Introducing the (type-2) generating function $G_2(q, P, t)$

$$S = \int p \, \mathrm{d}q - H \, \mathrm{d}t \equiv \int P \, \mathrm{d}Q - K \, \mathrm{d}t + \mathrm{d}(-QP + G_2) \quad (27)$$

where $H = H(q, p, t)$, $K = K(Q, P, t)$, so that

$$\int (p - \partial_q G_2) \, \mathrm{d}q + (Q - \partial_P G_2) \, \mathrm{d}P + (K - H + \partial_t G_2) \, \mathrm{d}t \equiv 0. \quad (28)$$

- Arbitrariness of the track in phase space requires

$$p = \partial_q G_2, \quad Q = \partial_P G_2, \quad K = H - \partial_t G_2. \quad (29)$$

- It can be shown that $G_2$ generates a Possion-bracket-keeping transformation from $(q, p; H)$ to $(Q, P; K)$.

# Action-angle variables

- Consider a $G_2(q, J)$ which brings $(q, p; H(q, p))$ to $(\varphi, I; K(I))$; note the time-independence
- Dynamics in the new variables reads

$$\dot{\varphi} = \partial_I K, \qquad \dot{I} = 0; \tag{30}$$

$$\Rightarrow \varphi = (\partial_I K)\, t + \varphi_0, \quad I \equiv I_0 \tag{31}$$

- $G_2$ can be solved by the Hamilton–Jacobi-like equation

$$H(q, \partial_q G_2) = K(J). \tag{32}$$

- If the motion has a period $T$ and it requires $\Delta\varphi := (\partial_I K)\, T \equiv 2\pi$ (angle variable), it can be shown that

$$I = \frac{1}{2\pi} \oint p \, \mathrm{d}q, \tag{33}$$

thus the name action-angle variable comes.

# Adiabatic approximation

- The variables above can be extended to $H = H(q, p; \lambda)$; where a perturbative expansion is made in terms of $\epsilon = \mathrm{d}\lambda/\mathrm{d}t$.
- It can be shown that the leading-order expansion gives

$$|I(t) - I(0)| \leq C\epsilon, \quad 0 \leq t \leq \epsilon^{-1} \tag{34}$$

- Let $\omega$ be the angular frequency of the unperturbed system. If

$$\lambda^{-1}\epsilon \ll \omega, \tag{35}$$

then $I$ varies only slightly during one quasi-period $2\pi/\omega$ and is thus a good adiabatic invariance.

# Summary

- Compared the pure and the thermal descriptions of the radiation within the solvable dilaton gravity model
- Fourier-mode fluctuation: that of the thermal state diverges faster than the vacuum case does at low energy, while the pure-state fluc. remains finite; at high energies they converge.
- Trace distance: goes exponentially small with black hole temperature going to zero.

- Outlook in the proposed PhD study
  - Further discussion within full CGHS, BTZ, etc.
  - Make use of the decoherence theory

  - Nature of the microscopic degrees of freedom for BH entropy
  - Breakdown of the semi-classical approximations in QG

# For Further Reading I


L.D. Landau and E.M. Lifshitz. *Mechanics*. 3rd. Vol. 1. Course of Theoretical Physics. Elsevier, 1976.

Qingdi Wang, Zhen Zhu, and William G. Unruh. "How the huge energy of quantum vacuum gravitates to drive the slow accelerating expansion of the Universe". In: *Physical Review D* 95.10 (May 2017).