My understanding of [WZU17]

An Attempt to Avoid the Old Λ Problem

Yi-Fan Wang (王 一帆)

Institut für Theoretische Physik, Universität zu Köln

June 13, 2017

Outline

Overview

Hawking Radiation

Model of Gravitation

Correlator of Field Strength

Distance between Density Operators

Outline

Overview

- **Topic**: to compare the pure state of the radiation field used by , and the thermal equilibrium state of the field, if it were at Hawking temperature
- **Importance**: to understand Hawking temperature from a new perspective, which plays a central role in BH physics and QG
  - information loss, origin of BH entropy, etc.
- **Methods**: based on a dilaton gravity model, which is more solvable than Einstein–Hilbert; using methods in traditional QFT and QIT
- **Results**: the difference exists and is quantitatively visualised

Outline

Universität zu Köln

Background space-time: spherically collapsing body

Schematic conformal (Penrose–Carter) diagram

- QFT on fixed background
- Each point represents an S^2
- $i^{\mp}$: past / future time-like infinities
- i^0: space-like infinity
- $\mathscr{I}^{\mp}$: past / future null inf.
- $\hslash^+$: future event horizon
- **Thick** line: boundary of the collapsing body

Hawking Radiation

Results and interpretation

- An early-time vacuum on $\mathscr{I}^-$ in collapsing background

$$\hat{a}(p)\,|h\rangle := 0 \quad \Rightarrow \quad \langle h\,|\,\hat{n}_a(p)\,|\,h\rangle =: \langle \hat{n}_a(p)\rangle_h \equiv 0 \tag{1}$$

evolves to a late-time state on $\mathscr{I}^+ \cup \mathscr{h}^+$ with particles on $\mathscr{I}^+$

$$\langle \hat{n}_{\text{on } \mathscr{I}^+}(\omega)\rangle_h =: \langle \hat{n}_b(\omega)\rangle_h \approx \Gamma_\omega\left(\mathrm{e}^{2\pi\omega/\kappa} - 1\right)^{-1}. \tag{2}$$

- Comparing eq. (2) with the Bose–Einstein distribution

$$\langle \hat{n}(\omega)\rangle_{\text{BE}} = \left(\mathrm{e}^{\omega/T} - 1\right)^{-1}, \tag{3}$$

one may conclude that eq. (2) describes a grey-body radiation with the Hawking temperature,

$$T_{\text{H}} := \kappa/2\pi \equiv \hbar/ck \cdot \kappa/2\pi. \tag{4}$$

Universität zu Köln

Hawking Radiation

Tension in the interpretation

- The state $|h\rangle$ or its density operator is pure,

$$\hat{\rho}_h = |h\rangle \langle h| \,, \tag{5}$$

whilst the of equilibrium bosonic ideal gas

$$\hat{\rho}_{\mathrm{BE}}(T) = Z^{-1} \mathbb{e}^{-\widehat{H}/T} \sim Z^{-1} \sum_E \mathbb{e}^{-E/T} |E\rangle \langle E| \tag{6}$$

is thermal and mixed.

- How different are they?

Outline

Classical theory of $(1+1)$d Dilaton Gravity Model I

- The action of the dilaton gravity model reads

$$S = \int \mathrm{d}^2 x \sqrt{-g} \left\{ \frac{\mathrm{e}^{-2\phi}}{G} \left[R + 4(\nabla\phi)^2 + 4\lambda^2 \right] - \frac{1}{2}(\nabla f)^2 \right\}, \tag{7}$$

  - $\phi(x)$ the dilaton field, without which topological
  - $f(x)$ a massless neutral scalar field representing matter
  - $\lambda > 0$ the cosmological constant

- Has a solution which resembles the collapsing body in $(3+1)$-dimensional Einstein gravitation

- Each point represents a point
- **Thick** line: null matter shell

Quantum theory of $(1+1)$d Dilaton Gravity Model I

- Constraint system: Schrödinger quantisation does not apply
- (Formally) Dirac quantisation

$$\widehat{\mathcal{H}}_{\|}\Psi[g,\phi,f]=0, \qquad \widehat{\mathcal{H}}_{\perp}\Psi[g,\phi,f]=0. \tag{8}$$

- Semi-classical approximation: $\Psi = \mathbb{e}^{\mathbb{i}(G^{-1}S_0+S_1+GS_2+\dots)}$
  - $\mathrm{O}(G^{-1})$: Hamilton–Jacobi equation for pure gravity
  - $\mathrm{O}(G^{0})$: $\Psi = D[g,\phi]\chi[g,\phi,f]$; functional Schrödinger equation for matter $\mathbb{i}\partial_t\chi[f]=\widehat{H}_{\mathrm{m}}\chi[f]$, where

$$\widehat{H}_{\mathrm{m}}=\frac{1}{2}\int_0^{+\infty}\mathbb{d}k\left(-\frac{\boldsymbol{\delta}^2}{\boldsymbol{\delta}f^2(k)}+k^2f^2(k)\right). \tag{9}$$

- A quantum field theory in curved space-time can be derived!

Quantum theory of $(1+1)$d Dilaton Gravity Model II

- At early time, the vacuum wave functional is

$$\chi_0[f_\mathsf{e}] \propto \exp\left\{-\frac{1}{2}\int_{\mathbb{R}^+} \mathrm{d}k\, k\, f_\mathsf{e}^2(k)\right\} \sim \prod_k \exp\left\{\frac{1}{2}\frac{k}{\Lambda} f_\mathsf{e}^2\right\}, \quad (10)$$

while at late time it evolves to

$$\chi_b[f_\mathsf{l}] \propto \exp\left\{-\int_{\mathbb{R}} \mathrm{d}p\, p \coth\left(\frac{\pi p}{2\lambda}\right)|f_\mathsf{l}(p)|^2\right\} \sim \prod_p \mathrm{e}^{\cdots}, \quad (11)$$

where $f_\mathsf{e}(k)$ and $f_\mathsf{l}(p)$ are the Fourier transform of the matter field at early and late time, respectively.

- At late time, particle-number expectations are

$$\langle \hat{n}_b(p)\rangle_{\chi_b} = \left(\mathrm{e}^{2\pi|p|/\lambda} - 1\right)^{-1}, \quad (12)$$

leading to a Hawking-like black-body temperature

$$\boxed{T_{\mathsf{HD}} := \lambda/2\pi.} \quad (13)$$

Universität zu Köln

Outline

Correlation of Fourier Modes

The discrepancy

- The Fourier-mode correlators can be calculated,

$$\left\langle \hat{f}^\dagger(p_1)\hat{f}(p_2)\right\rangle = \frac{1}{T_{\mathrm{HD}}}\delta(p_1 - p_2)\cdot\begin{cases}\frac{1}{2}\frac{1}{q}, & \text{vacuum};\\ \frac{1}{8}\frac{\tanh\frac{q}{4}}{\frac{q}{4}}, & \chi_b;\\ \frac{1}{4}\frac{\coth\frac{q}{2}}{\frac{q}{2}}, & \hat{\rho}_{\mathrm{BE}}(T_{\mathrm{HD}}),\end{cases} \tag{14}$$

where $q := p_1/T_{\mathrm{HD}}$

- Diagonal elements (fluctuations) are plotted

Correlation of Fourier Modes

Fluctuation of the Fourier modes: diagram in log-log scale

Correlation of Fourier Modes

Fluctuation of the Fourier modes: interpretation

- Vacuum fluctuation

$$\left\langle \left|\hat{f}\right|^2 \right\rangle_{\text{vac}} = \mathrm{O}(q^{-1}) \tag{15}$$

- A black hole does not alter the high-energy processes

$$\left\langle \left|\hat{f}\right|^2 \right\rangle_{\chi_b} \approx \left\langle \left|\hat{f}\right|^2 \right\rangle_{\text{th}} \approx \left\langle \left|\hat{f}\right|^2 \right\rangle_{\text{vac}} = \mathrm{O}(q^{-1}), \quad |p| \gg T_{\text{HD}} \tag{16}$$

- A black hole suppresses low-energy fluctuation of the pure state, while enhancing that of the thermal state

$$\mathrm{O}(1) \sim \left\langle \left|\hat{f}\right|^2 \right\rangle_{\chi_b} \ll \left\langle \left|\hat{f}\right|^2 \right\rangle_{\text{vac}} \ll \left\langle \left|\hat{f}\right|^2 \right\rangle_{\text{th}} \sim \mathrm{O}(q^{-2}), \quad |p| \ll T_{\text{HD}} \tag{17}$$

- Critical scale $|p| \sim T_{\text{HD}}$

Outline

Universität zu Köln

Trace Distance

Definitions

- Trace distance between Hermitian operators $\widehat{M}$ and $\widehat{N}$

$$T\left(\widehat{M}, \widehat{N}\right) := \frac{1}{2} \operatorname{tr} \sqrt{\left(\widehat{M} - \widehat{N}\right)^{\dagger} \left(\widehat{M} - \widehat{N}\right)} \tag{18}$$

- For density operators $\hat{\rho}$ and $\hat{\sigma}$,
  - $0 \leq T(\hat{\rho}, \widehat{\sigma}) \leq 1$;
  - Controlled by fidelity in

$$\boxed{1 - F(\hat{\rho}, \widehat{\sigma}) \leq T(\hat{\rho}, \widehat{\sigma}) \leq \sqrt{1 - F^2(\hat{\rho}, \widehat{\sigma})},} \tag{19}$$

    where we only need

$$F(|\alpha\rangle, \widehat{\sigma}) := \langle \alpha \,|\, \widehat{\sigma} \,|\, \alpha \rangle^{\frac{1}{2}} . \tag{20}$$

  - In our application, T is difficult while F can be obtained.

Single-mode Distances between the Density Operators I

Bounds set by fidelity and eq. (19)

- The pure state can be decomposed upon discretisation $\chi_b[f] \sim \sum_p \chi_b^{(p)}(f_p) \equiv \sum_p \left\langle f_p \,\middle|\, \chi_b^{(p)} \right\rangle$, where $f_p := f(p)$
- So is the thermal density operator $\hat{\rho}_{\text{th}}(T) \sim \bigotimes_p \hat{\rho}_{\text{th}}^{(p)}(T)$
- Fidelity (in eq. (19)) can be factorised as well

$$F \equiv \langle \chi_b \,|\, \hat{\rho}_{\text{th}} \,|\, \chi_b \rangle^{\frac{1}{2}} \sim \prod_p \left\langle \chi_b^{(p)} \,\middle|\, \hat{\rho}_{\text{th}}^{(p)} \,\middle|\, \chi_b^{(p)} \right\rangle^{\frac{1}{2}} =: \prod_p F^{(p)}; \tag{21}$$

- $F^{(p)}$ can be computed in order to find bounds of T

$$F^{(p)}\left(|p\rangle\,, \hat{\rho}_{\text{th}}^{(p)}(T_{\text{HD}})\right) = \frac{\sqrt{u-1}}{\sqrt[4]{u^2+u+1}}, \quad u := \mathbb{e}^{q} \equiv \mathbb{e}^{|p|/T_{\text{HD}}}. \tag{22}$$

All-modes Distances between the Density Operators I

Bounds set by fidelity and eq. (19)

- ‘Go to the continuum limit’: Λ dimension regulator

$$\sum_p g(p) \to \frac{1}{2\pi\Lambda} \int \mathrm{d}p\, g(p), \tag{23}$$

- Analogously, to regularise a product

$$\prod_p f(p) \equiv \exp\left\{ \sum_p \ln f(p) \right\} \to \exp\left\{ \frac{1}{2\pi\Lambda} \int \mathrm{d}p \, \ln f(p) \right\} \tag{24}$$

- Regularised F can be calculated in order to set bounds of T

$$F(\chi_b, \hat{\rho}_{\text{th}}) = \exp\left\{ \frac{2}{2\pi\Lambda} \int_0^{+\infty} \mathrm{d}p \, \ln F^{(p)} \right\} = \exp\left(-\frac{\pi}{9} \frac{T_{\text{HD}}}{\Lambda} \right) \tag{25}$$

Summary

- Compared the pure and the thermal descriptions of the radiation within the solvable dilaton gravity model
- Fourier-mode fluctuation: that of the thermal state diverges faster than the vacuum case does at low energy, while the pure-state fluc. remains finite; at high energies they converge.
- Trace distance: goes exponentially small with black hole temperature going to zero.

- Outlook in the proposed PhD study
  - Further discussion within full CGHS, BTZ, etc.
  - Make use of the decoherence theory

  - Nature of the microscopic degrees of freedom for BH entropy
  - Breakdown of the semi-classical approximations in QG

Universität zu Köln

For Further Reading I

Qingdi Wang, Zhen Zhu, and William G. Unruh. "How the huge energy of quantum vacuum gravitates to drive the slow accelerating expansion of the Universe". In: *Physical Review D* 95.10 (May 2017).

More on Trace Distance

Interpretation

- Maximal probability-difference obtainable

$$T(\hat{\rho}, \hat{\sigma}) = \max_{0 \leq \widehat{\Lambda} \leq \widehat{1}} \mathrm{tr}\left\{\widehat{\Lambda}(\hat{\rho} - \hat{\sigma})\right\}, \tag{26}$$

  where all eigenvalues of $\widehat{\Lambda}$ are in the range $[0, 1]$

- E.g. $\widehat{\Lambda} := |\alpha\rangle\langle\alpha|$, $|\alpha\rangle$ eigenstate of $\widehat{\mathrm{A}}$ with eigenvalue α
  - $\mathrm{tr}\left\{\widehat{\Lambda}\hat{\rho}\right\}$: the probability of getting α in measuring $\widehat{\mathrm{A}}$
  - $\mathrm{tr}\left\{\widehat{\Lambda}(\hat{\rho} - \widehat{\sigma})\right\}$: the difference of the probability above
  - $T(\hat{\rho}, \widehat{\sigma})$: the maximal value of the difference above

More on Fidelity

- General definition

$$F(|\alpha\rangle, |\beta\rangle) = |\langle\alpha\,|\,\beta\rangle| \tag{27}$$

$$F(|\alpha\rangle, \hat{\sigma}) = \sqrt{\langle\alpha\,|\,\hat{\sigma}\,|\,\alpha\rangle} \tag{20 rev.}$$

$$F(\hat{\rho}, \hat{\sigma}) = \mathrm{tr}\,\sqrt{\hat{\rho}^{\frac{1}{2}}\hat{\sigma}\hat{\rho}^{\frac{1}{2}}} \tag{28}$$

- Intepretation: faithfulness

$$F(|\alpha\rangle, |\alpha\rangle) = 1 \tag{29}$$

Another New Foundation of Statistical Physics

Specific and easy version of the construction

- Total isolated system U with energy constraint $\left\langle \widehat{H}_U \right\rangle := E_U$, divided into a (sub)system S and an environment E
- Hilbert spaces $\mathcal{H}_R \supseteq \mathcal{H}_U = \mathcal{H}_S \otimes \mathcal{H}_E$; $\hat{1}_R$ identity on $\mathcal{H}_R$, dimension $d_R := \dim \mathcal{H}_R < +\infty$
- Equiprobable / maximal-ignorant state of U

$$\hat{\mathcal{E}}_R := d_R^{-1} \hat{1}_R \in \mathcal{H}_R \tag{30}$$

- Hamiltonians $\widehat{H}_U = \widehat{H}_S + \widehat{H}_E + \widehat{H}_{\text{int}}$
- Canonical state of S with energy constraint

$$\widehat{\Omega}_S^{(\mathsf{E})} := \operatorname{tr}_E \hat{\mathcal{E}}_R \propto \exp\left(-\widehat{H}_S / T_{\text{th}}\right) \tag{31}$$

- Theorem: $\forall \, |\phi\rangle \in \mathcal{H}_R$, the reduced state of S

$$\boxed{\operatorname{tr}_E |\phi\rangle \langle\phi| =: \hat{\rho}_S(\phi) \approx \widehat{\Omega}_S^{(\mathsf{E})}.} \tag{32}$$

Universität zu Köln

Another New Foundation of Statistical Physics

Generic and exact version of the construction

- Arbitrary constraint R; study the trace distance T between $\hat{\rho}_S(\phi)$ and $\widehat{\Omega}_S$
- Lemma: average distance is small w.r.t. d_S/d_E^{eff}

$$\left\langle T\left(\hat{\rho}_S(\phi), \widehat{\Omega}_S\right)\right\rangle \leq \tfrac{1}{2}\sqrt{d_S/d_E^{\text{eff}}} \tag{33}$$

- Theorem: probability of large deviation is exponentially small w.r.t. the distance; an easy version

$$\frac{V\left[\left\{|\phi\rangle \;\middle|\; T\left(\hat{\rho}_S(\phi), \widehat{\Omega}_S\right) \geq d_R^{-\frac{1}{3}}\right\}\right]}{V[\{|\phi\rangle\}]} \leq 4\exp\left(-\frac{2d_R^{\frac{1}{3}}}{9\pi^3}\right) \tag{34}$$

- Effective dimension of E: setting $\widehat{\Omega}_E = \operatorname{tr}_S \hat{\mathcal{E}}_R$,

$$d_U/d_S \equiv d_E \geq d_E^{\text{eff}} := \left(\operatorname{tr}\widehat{\Omega}_E^2\right)^{-1} \geq d_R/d_S. \tag{35}$$