An Introduction to [WZU17]

An Attempt to Avoid the Old Cosmological Constant Problem

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Outline

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Overview

- If contributed by vacuum energy, the cosmological constant could differ 120 order-of-magnitude from observation.
- Wang, Zhu, and Unruh consider vacuum energy density instead, which is subject to fluctuation, and their result moderates the problem.
- They assumed a localised RW metric and obtained FL-like equations for $a(t, \vec{x})$, whose coefficient contains quantum fluctuation.
- The equations leads to local Hubble parameters which fluctuate, but a global average could be defined.
- The solution to the equations is evaluated with the help of theories of parametric oscillator and adiabatic invariance.
- The result is qualitatively supported by numerical calculation and further quantitative discussions.

Outline

Cosmological constant and vacuum energy

- GR + vacuum QFT

$$G_{\mu\nu} + \lambda_{\mathrm{b}} g_{\mu\nu} = 8\pi G T^{\mathrm{vac}}_{\mu\nu} \tag{1}$$

- In Minkowski space-time, Lorentz invariance requires

$$T^{\mathrm{vac}}_{\mu\nu} = -\rho^{\mathrm{vac}}_{\mu\nu} \eta_{\mu\nu}, \tag{2}$$

which generalises to curved space-time as

$$T^{\mathrm{vac}}_{\mu\nu} = -\rho^{\mathrm{vac}}_{\mu\nu} g_{\mu\nu}. \tag{3}$$

- Effectively, eq. (1) can be written as

$$G_{\mu\nu} + \lambda_{\mathrm{eff}} g_{\mu\nu} = 0, \qquad \lambda_{\mathrm{eff}} = \lambda_{\mathrm{b}} + 8\pi G \rho^{\mathrm{vac}}; \tag{4}$$

$$G_{\mu\nu} = -8\pi G \rho^{\mathrm{vac}}_{\mathrm{eff}} g_{\mu\nu}, \qquad \rho^{\mathrm{vac}}_{\mathrm{eff}} = \rho^{\mathrm{vac}} + \frac{\lambda_{\mathrm{b}}}{8\pi G}. \tag{5}$$

Hubble parameter and cosmological constant

- Homogeneity and isotropy: Robertson–Walker metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j. \tag{6}$$

- Hubble parameter / expansion rate $H := \dot{a}/a$; eqs. (4) and (5) take the corresponding Friedmann–Lemaître form

$$3H^2 = \lambda_{\text{eff}} = 8\pi G\rho_{\text{eff}}^{\text{vac}}, \tag{7}$$

$$3\ddot{a} = \lambda_{\text{eff}} a = 8\pi G\rho_{\text{eff}}^{\text{vac}} a. \tag{8}$$

- Solution to eq. (8)

$$a(t) = a(t_0)\,\mathrm{e}^{H(t-t_0)} \tag{9}$$

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The Old Cosmological Problem

- Contributions to $\rho_{\text{eff}}^{\text{vac}}$ or λ_{eff}: vacuum fluctuation of all quantum fields, Electroweak phase transition, etc.
- λ_{eff} by vacuum fluctuation evaluated in Minkowski space: taking a scalar field and using sharp-momentum cut-off (also see below),

$$\langle \rho^{\text{vac}} \rangle = \frac{\Lambda^4}{16\pi^2}. \tag{10}$$

Setting $\Lambda = E_{\text{P}}$ results in a surpass of the observed value of λ_{eff} by $\sim 10^{120}$.

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Matter: field and quantisation

- Matter model

$$S_{\mathsf{m}}[\phi] = \int \mathrm{d}^4 x \left[-\frac{1}{2} \eta^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) \right] \tag{11}$$

- Canonical quantisation

$$\phi(t, \vec{x}) = \int \frac{\mathrm{d}^3 k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega}} \left(a_{\vec{k}}\, \mathrm{e}^{-\mathrm{i}\left(\omega t - \vec{k}\cdot\vec{x}\right)} + \text{h.c.} \right), \tag{12}$$

so that

$$T_{00} = \frac{1}{2}\left(\dot{\phi}^2 + (\nabla\phi)^2\right) \tag{13}$$

$$= \int \mathrm{d}^3 k_1\, \mathrm{d}^3 k_2\, f\left(a_{\vec{k}_1} a^\dagger_{\vec{k}_2}, a^\dagger_{\vec{k}_1} a_{\vec{k}_2}, a_{\vec{k}_1} a_{\vec{k}_2}, a^\dagger_{\vec{k}_1} a^\dagger_{\vec{k}_2} \right) \tag{14}$$

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Matter: fluctuating vacuum energy density

- $|0\rangle$ is an eigenstate of $H = \int \mathrm{d}^3 x T_{00}$, because

$$a_{\vec{k}} |0\rangle := 0, \quad \forall \vec{k} \tag{15}$$

- This is not the case for T_{00}, and

$$\left\langle (\Delta T_{00})^2 \right\rangle = \frac{2}{3} \langle T_{00} \rangle^2, \qquad \langle T_{00} \rangle = \frac{\Lambda^4}{16\pi^2}. \tag{16}$$

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Space-time: localised Robertson–Walker metric

- Inhomogeneity and isotropy

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t,\vec{x})\,\delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j. \tag{17}$$

- Generalising the proper distance

$$L(1\to 2;t) = \int_{\vec{x}_1}^{\vec{x}_2} \sqrt{a^2(t,\vec{x})}\,\mathrm{d}l \tag{18}$$

and Hubble parameter

$$H(1\to 2;t) = \frac{\dot{L}(1\to 2)}{L(1\to 2)} \tag{19}$$

- Einstein equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{20}$$

where $T_{\mu\nu}$'s subject to fluctuation.

Space-time: stochastic oscillator equation for a

- Taking the trace of **??** leads to

  $$\ddot{a} + \Omega^2(t, \vec{x})\, a = 0, \tag{21}$$

  where

  $$\Omega^2 = \frac{4\pi G}{3} g^{\mu\nu} T_{\mu\nu}, \tag{22}$$

  which subjects to fluctuation.
- $T_{\mu\nu}$ given by KG in Minkowski; evaluate H by solving **??**

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Quantum theory of $(1+1)$d Dilaton Gravity Model I

Outline

- The Fourier-mode correlators can be calculated,

$$\left\langle \hat{f}^\dagger(p_1)\hat{f}(p_2)\right\rangle = \frac{1}{T_{\mathsf{HD}}}\delta(p_1 - p_2)\cdot\begin{cases}\frac{1}{2}\frac{1}{q}, & \text{vacuum;}\\ \frac{1}{8}\frac{\tanh\frac{q}{4}}{\frac{q}{4}}, & \chi_b;\\ \frac{1}{4}\frac{\coth\frac{q}{2}}{\frac{q}{2}}, & \hat{\rho}_{\mathsf{BE}}(T_{\mathsf{HD}}),\end{cases} \tag{23}$$

where $q := p_1/T_{\mathsf{HD}}$

- Diagonal elements (fluctuations) are plotted

Correlation of Fourier Modes

Fluctuation of the Fourier modes: diagram in log-log scale

Correlation of Fourier Modes

Fluctuation of the Fourier modes: interpretation

- Vacuum fluctuation

$$\left\langle \left| \hat{f} \right|^2 \right\rangle_{\text{vac}} = \mathrm{O}(q^{-1}) \tag{24}$$

- A black hole does not alter the high-energy processes

$$\left\langle \left| \hat{f} \right|^2 \right\rangle_{\chi_b} \approx \left\langle \left| \hat{f} \right|^2 \right\rangle_{\text{th}} \approx \left\langle \left| \hat{f} \right|^2 \right\rangle_{\text{vac}} = \mathrm{O}(q^{-1}), \quad |p| \gg T_{\text{HD}} \tag{25}$$

- A black hole suppresses low-energy fluctuation of the pure state, while enhancing that of the thermal state

$$\mathrm{O}(1) \sim \left\langle \left| \hat{f} \right|^2 \right\rangle_{\chi_b} \ll \left\langle \left| \hat{f} \right|^2 \right\rangle_{\text{vac}} \ll \left\langle \left| \hat{f} \right|^2 \right\rangle_{\text{th}} \sim \mathrm{O}(q^{-2}), \quad |p| \ll T_{\text{HD}} \tag{26}$$

- Critical scale $|p| \sim T_{\text{HD}}$

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Trace Distance

Definitions

- Trace distance between Hermitian operators $\widehat{M}$ and $\widehat{N}$

$$T\left(\widehat{M}, \widehat{N}\right) := \frac{1}{2} \operatorname{tr} \sqrt{\left(\widehat{M} - \widehat{N}\right)^{\dagger} \left(\widehat{M} - \widehat{N}\right)} \tag{27}$$

- For density operators $\hat{\rho}$ and $\hat{\sigma}$,
  - $0 \leq T(\hat{\rho}, \widehat{\sigma}) \leq 1$;
  - Controlled by fidelity in

$$\boxed{1 - F(\hat{\rho}, \widehat{\sigma}) \leq T(\hat{\rho}, \widehat{\sigma}) \leq \sqrt{1 - F^2(\hat{\rho}, \widehat{\sigma})},} \tag{28}$$

  where we only need

$$F(|\alpha\rangle, \widehat{\sigma}) := \langle \alpha \,|\, \widehat{\sigma} \,|\, \alpha \rangle^{\frac{1}{2}} . \tag{29}$$

  - In our application, T is difficult while F can be obtained.

Single-mode Distances between the Density Operators I

Bounds set by fidelity and eq. (26)

- The pure state can be decomposed upon discretisation $\chi_b[f] \sim \sum_p \chi_b^{(p)}(f_p) \equiv \sum_p \left\langle f_p \,\middle|\, \chi_b^{(p)} \right\rangle$, where $f_p := f(p)$
- So is the thermal density operator $\hat{\rho}_{\text{th}}(T) \sim \bigotimes_p \hat{\rho}_{\text{th}}^{(p)}(T)$
- Fidelity (in eq. (26)) can be factorised as well

$$F \equiv \langle \chi_b \,|\, \hat{\rho}_{\text{th}} \,|\, \chi_b \rangle^{\frac{1}{2}} \sim \prod_p \left\langle \chi_b^{(p)} \,\middle|\, \hat{\rho}_{\text{th}}^{(p)} \,\middle|\, \chi_b^{(p)} \right\rangle^{\frac{1}{2}} =: \prod_p F^{(p)}; \tag{30}$$

- $F^{(p)}$ can be computed in order to find bounds of T

$$F^{(p)}\left(|p\rangle \,, \hat{\rho}_{\text{th}}^{(p)}(T_{\text{HD}})\right) = \frac{\sqrt{u-1}}{\sqrt[4]{u^2+u+1}}, \quad u := \mathbb{e}^{q} \equiv \mathbb{e}^{|p|/T_{\text{HD}}}. \tag{31}$$

All-modes Distances between the Density Operators I

Bounds set by fidelity and eq. (26)

- 'Go to the continuum limit': Λ dimension regulator

$$\sum_p g(p) \to \frac{1}{2\pi\Lambda} \int \mathrm{d}p\, g(p), \tag{32}$$

- Analogously, to regularise a product

$$\prod_p f(p) \equiv \exp\left\{ \sum_p \ln f(p) \right\} \to \exp\left\{ \frac{1}{2\pi\Lambda} \int \mathrm{d}p \, \ln f(p) \right\} \tag{33}$$

- Regularised F can be calculated in order to set bounds of T

$$F(\chi_b, \hat{\rho}_{\mathrm{th}}) = \exp\left\{ \frac{2}{2\pi\Lambda} \int_0^{+\infty} \mathrm{d}p \, \ln F^{(p)} \right\} = \exp\left(-\frac{\pi}{9} \frac{T_{\mathrm{HD}}}{\Lambda} \right) \tag{34}$$

Summary

- Compared the pure and the thermal descriptions of the radiation within the solvable dilaton gravity model
- Fourier-mode fluctuation: that of the thermal state diverges faster than the vacuum case does at low energy, while the pure-state fluc. remains finite; at high energies they converge.
- Trace distance: goes exponentially small with black hole temperature going to zero.

- Outlook in the proposed PhD study
  - Further discussion within full CGHS, BTZ, etc.
  - Make use of the decoherence theory

  - Nature of the microscopic degrees of freedom for BH entropy
  - Breakdown of the semi-classical approximations in QG

For Further Reading I

Qingdi Wang, Zhen Zhu, and William G. Unruh. "How the huge energy of quantum vacuum gravitates to drive the slow accelerating expansion of the Universe". In: *Physical Review D* 95.10 (May 2017).

More on Trace Distance

Interpretation

- Maximal probability-difference obtainable

$$T(\hat{\rho},\hat{\sigma}) = \max_{0\leq\widehat{\Lambda}\leq\widehat{1}} \mathrm{tr}\left\{\widehat{\Lambda}(\hat{\rho}-\hat{\sigma})\right\}, \tag{35}$$

  where all eigenvalues of $\widehat{\Lambda}$ are in the range $[0,1]$

- E.g. $\widehat{\Lambda} := |\alpha\rangle\langle\alpha|$, $|\alpha\rangle$ eigenstate of $\widehat{\mathrm{A}}$ with eigenvalue α
  - $\mathrm{tr}\left\{\widehat{\Lambda}\hat{\rho}\right\}$: the probability of getting α in measuring $\widehat{\mathrm{A}}$
  - $\mathrm{tr}\left\{\widehat{\Lambda}(\hat{\rho}-\widehat{\sigma})\right\}$: the difference of the probability above
  - $T(\hat{\rho},\widehat{\sigma})$: the maximal value of the difference above

More on Fidelity

- General definition

$$F(|\alpha\rangle, |\beta\rangle) = |\langle\alpha\,|\,\beta\rangle| \tag{36}$$

$$F(|\alpha\rangle, \hat{\sigma}) = \sqrt{\langle\alpha\,|\,\hat{\sigma}\,|\,\alpha\rangle} \tag{27 rev.}$$

$$F(\hat{\rho}, \hat{\sigma}) = \operatorname{tr}\sqrt{\hat{\rho}^{\frac{1}{2}}\hat{\sigma}\hat{\rho}^{\frac{1}{2}}} \tag{37}$$

- Intepretation: faithfulness

$$F(|\alpha\rangle, |\alpha\rangle) = 1 \tag{38}$$

Another New Foundation of Statistical Physics

Specific and easy version of the construction

- Total isolated system U with energy constraint $\left\langle \widehat{H}_U \right\rangle := E_U$, divided into a (sub)system S and an environment E
- Hilbert spaces $\mathcal{H}_R \supseteq \mathcal{H}_U = \mathcal{H}_S \otimes \mathcal{H}_E$; $\hat{1}_R$ identity on $\mathcal{H}_R$, dimension $d_R := \dim \mathcal{H}_R < +\infty$
- Equiprobable / maximal-ignorant state of U

$$\hat{\mathcal{E}}_R := d_R^{-1} \hat{1}_R \in \mathcal{H}_R \tag{39}$$

- Hamiltonians $\widehat{H}_U = \widehat{H}_S + \widehat{H}_E + \widehat{H}_{\text{int}}$
- Canonical state of S with energy constraint

$$\widehat{\Omega}_S^{(\mathsf{E})} := \operatorname{tr}_E \hat{\mathcal{E}}_R \propto \exp\left(-\widehat{H}_S / T_{\text{th}}\right) \tag{40}$$

- Theorem: $\forall\, |\phi\rangle \in \mathcal{H}_R$, the reduced state of S

$$\boxed{\operatorname{tr}_E |\phi\rangle \langle\phi| =: \hat{\rho}_S(\phi) \approx \widehat{\Omega}_S^{(\mathsf{E})}.} \tag{41}$$

Another New Foundation of Statistical Physics

Generic and exact version of the construction

- Arbitrary constraint R; study the trace distance T between $\hat{\rho}_S(\phi)$ and $\widehat{\Omega}_S$
- Lemma: average distance is small w.r.t. d_S/d_E^{eff}

$$\left\langle T\left(\hat{\rho}_S(\phi), \widehat{\Omega}_S\right)\right\rangle \leq \tfrac{1}{2}\sqrt{d_S/d_E^{\mathrm{eff}}} \tag{42}$$

- Theorem: probability of large deviation is exponentially small w.r.t. the distance; an easy version

$$\frac{V\left[\left\{|\phi\rangle \;\middle|\; T\left(\hat{\rho}_S(\phi), \widehat{\Omega}_S\right) \geq d_R^{-\frac{1}{3}}\right\}\right]}{V[\{|\phi\rangle\}]} \leq 4\exp\left(-\frac{2d_R^{\frac{1}{3}}}{9\pi^3}\right) \tag{43}$$

- Effective dimension of E: setting $\widehat{\Omega}_E = \operatorname{tr}_S \hat{\mathcal{E}}_R$,

$$d_U/d_S \equiv d_E \geq d_E^{\mathrm{eff}} := \left(\operatorname{tr}\widehat{\Omega}_E^2\right)^{-1} \geq d_R/d_S. \tag{44}$$

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