Hawking Radiation

A Comparison of Pure-state and Thermal Description

Yi-Fan Wang 王一帆

Institut für Theoretische Physik
Universität zu Köln

DPG-Frühjahrstagung 2017, Bremen

Outline

Hawking Radiation

Model of Gravitation

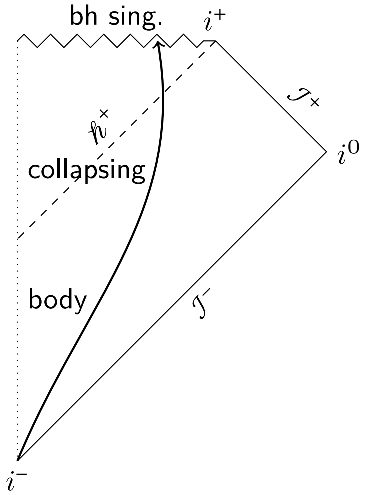
Correlator of Field Strength

Distance between Density Operators

Outline

Hawking Radiation

Background space-time: sperically collapsing body



Hawking Radiation

Result and interpretation

- An early-time vacuum on $\mathscr{I}^-$ in collapsing background

$$\hat{a}(p)\,|h\rangle := 0 \quad \Rightarrow \quad \langle \hat{n}_a(p) \rangle_h \equiv 0 \tag{1}$$

evolves to a late-time state on $\mathscr{I}^+ \cup \mathscr{h}^+$ with particles [Haw74]

$$\langle \hat{n}_b(\omega) \rangle_h \approx \Gamma_\omega \left(e^{2\pi\omega/\kappa} - 1 \right)^{-1}. \tag{2}$$

- Comparing eq. (2) with the Bose–Einstein distribution

$$\langle \hat{n}(\omega) \rangle_{\mathsf{BE}} = \left(e^{\omega/T} - 1 \right)^{-1}, \tag{3}$$

one may conclude that eq. (2) describes a grey-body radiation with the Hawking temperature [Haw75],

$$T_{\mathsf{H}} := \kappa/2\pi. \tag{4}$$

- The state $|h\rangle$ or its density operator is pure,

$$\hat{\rho}_h = |h\rangle \langle h| \,, \tag{5}$$

whilst the Bose–Einstein state of equilibrium bosonic gas

$$\hat{\rho}_{\mathsf{BE}} = Z^{-1} \mathrm{e}^{-\widehat{H}/T} \sim Z^{-1} \sum_E \mathrm{e}^{-E/T} |E\rangle \langle E| \tag{6}$$

is thermal and mixed.

- How different are they? [Kie01; HR09]

Outline

$(1+1)$-dimensional Dilaton Gravity Model

Classical theory 1/2 [Cal+92; DK96; APR11]

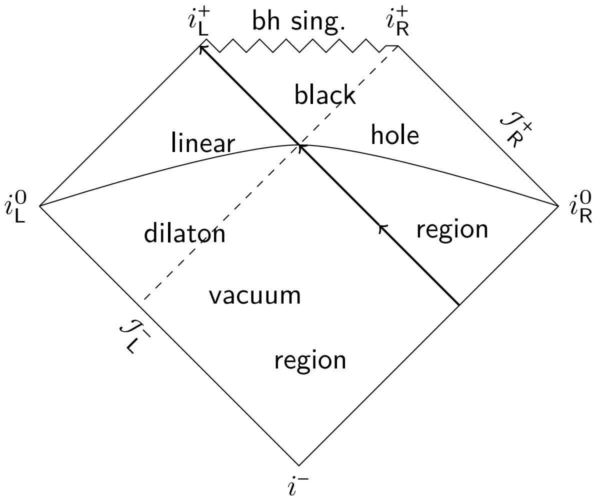
- The action of the dilaton gravity model reads

$$
\begin{aligned}
S &= \int \mathrm{d}^2 x \sqrt{-\overline{g}} \left\{ \frac{\mathrm{e}^{-2\overline{\phi}}}{G} \left[\overline{R} + 4\left(\nabla\overline{\phi}\right)^2 + 4\lambda^2 \right] - \frac{1}{2}(\nabla f)^2 \right\} \\
&= \int \mathrm{d}^2 x \sqrt{-g} \left\{ \frac{1}{G}\left[R\phi + 4\lambda^2\right] - \frac{1}{2}(\nabla f)^2 \right\},
\end{aligned} \tag{7}
$$

  where in eq. (7), the kinetic term of the dilaton field ϕ is eliminated by substituting $\phi = \mathrm{e}^{-2\overline{\phi}}$ and $g_{\alpha\beta} = \mathrm{e}^{-2\overline{\phi}}\overline{g}_{\alpha\beta}$.
- Has a solution which resembles the collapsing body in $(3+1)$d Einstein gravitation

$(1+1)$-dimensional Dilaton Gravity Model

Classical theory 2/2 [Cal+92; DK96; APR11]



$(1+1)$-dimensional Dilaton Gravity Model

Quantum theory 1/2 [DK96]

- Can formally be canonically quantised as a constraint system
- Insert the ansatz $\Psi[g,\phi,f] = \mathbb{e}^{\mathbb{i}(G^{-1}S_0+S_1+GS_2+\dots)}$
  - Order G^{-1}: Hamilton–Jacobi equation for pure gravity
  - Order G^0: functional Schrödinger equation for matter

$$\mathbb{i}\frac{\partial\chi}{\partial t} = \widehat{H}_{\mathrm{m}}\chi, \tag{7}$$

$$\widehat{H}_{\mathrm{m}} \equiv \frac{1}{2}\int_0^{+\infty} \mathbb{d}k \left(-\frac{\boldsymbol{\delta}^2}{\boldsymbol{\delta} f^2(k)} + k^2 f^2(k)\right). \tag{8}$$

$(1+1)$-dimensional Dilaton Gravity Model

Quantum theory 2/2 [DK96]

- At early time, the ground-state solution to eq. (7) is

$$\chi_0[f_\mathsf{e}] \propto \exp\left\{-\frac{1}{2}\int_0^{+\infty} \mathrm{d}k\, k\, f_\mathsf{e}(k)^2\right\}, \tag{7}$$

  while at late time it evolves to

$$\chi_b[f_\mathsf{l}] \propto \exp\left\{-\int_{-\infty}^{+\infty} \mathrm{d}p\, p \coth\left(\frac{\pi p}{2\lambda}\right)|f_\mathsf{l}(p)|^2\right\}, \tag{8}$$

  where $f_\mathsf{e}(k)$ and $f_\mathsf{l}(p)$ are the Fourier transform of the matter field at early and late time, respectively.
- At late time, particle-number expectations are

$$\langle \hat{n}_b(p)\rangle_{\chi_b} = \left(\mathrm{e}^{2\pi|p|/\lambda} - 1\right)^{-1}, \tag{9}$$

  leading to a Hawking-like black-body temperature

$$\boxed{T_{\mathsf{HD}} := \lambda/2\pi.} \tag{10}$$

Outline

Correlation of Fourier Modes

The discrepancy

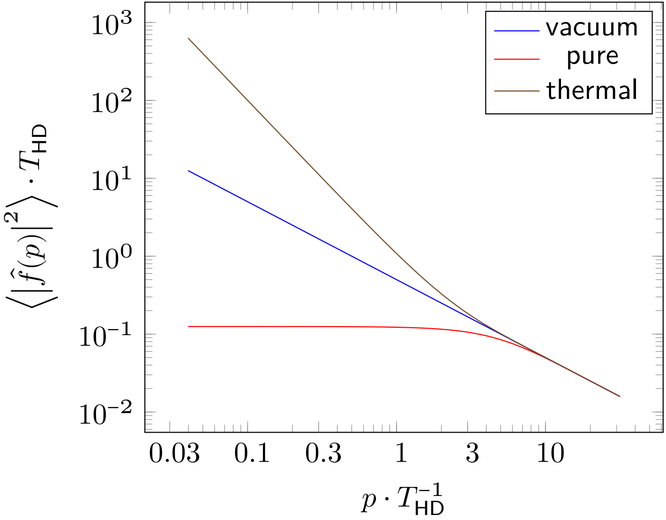
The Fourier-mode correlators of different states can be calculated,

$$\left\langle \hat{f}^{\dagger}(p_1)\hat{f}(p_2)\right\rangle = \frac{1}{T_{\mathsf{HD}}}\delta(p_1 - p_2)\cdot\begin{cases} \frac{1}{2}\frac{1}{q}, & \text{vacuum;} \\ \frac{1}{8}\frac{\tanh\frac{q}{4}}{\frac{q}{4}}, & \chi_b; \\ \frac{1}{4}\frac{\coth\frac{q}{2}}{\frac{q}{2}}, & \hat{\rho}_{\mathsf{BE}}(T_{\mathsf{HD}}), \end{cases} \tag{11}$$

where $q := p_1/T_{\mathsf{HD}}$.

Correlation of Fourier Modes

Fluctuation of the Fourier modes: diagram in log-log scale



Correlation of Fourier Modes

Fluctuation of the Fourier modes: interpretation

- Vacuum fluctuation

$$\left\langle \left|\hat{f}\right|^2 \right\rangle_{\text{vac}} \sim \mathrm{O}(q^{-1}) \tag{12}$$

- A black hole does not alter the high-energy processes.

$$\left\langle \left|\hat{f}\right|^2 \right\rangle_{\chi_b} \approx \left\langle \left|\hat{f}\right|^2 \right\rangle_{\text{th}} \approx \left\langle \left|\hat{f}\right|^2 \right\rangle_{\text{vac}} \sim \mathrm{O}(q^{-1}), \quad |p| \gg T_{\text{HD}} \tag{13}$$

- A black hole suppresses low-energy fluctuation of the pure state, while enhancing that of the thermal state.

$$\mathrm{O}(1) \sim \left\langle \left|\hat{f}\right|^2 \right\rangle_{\chi_b} \ll \left\langle \left|\hat{f}\right|^2 \right\rangle_{\text{vac}} \ll \left\langle \left|\hat{f}\right|^2 \right\rangle_{\text{th}} \sim \mathrm{O}(q^{-2}), \quad |p| \ll T_{\text{HD}} \tag{14}$$

- Critical scale $|p| \sim T_{\text{HD}}$

Outline

Trace Distance [Wil09, ch. 9]

Definitions

- Trace distance between Hermitian operators $\widehat{M}$ and $\widehat{N}$

$$T\left(\widehat{M}, \widehat{N}\right) := \frac{1}{2} \operatorname{tr} \sqrt{\left(\widehat{M} - \widehat{N}\right)^{\dagger} \left(\widehat{M} - \widehat{N}\right)} \tag{15}$$

- For density operators $\hat{\rho}$ and $\hat{\sigma}$,
  - $0 \leq T(\hat{\rho}, \widehat{\sigma}) \leq 1.$
  - Controlled by fidelity in [FG99]

$$1 - F(\hat{\rho}, \widehat{\sigma}) \leq T(\hat{\rho}, \widehat{\sigma}) \leq \sqrt{1 - F^2(\hat{\rho}, \widehat{\sigma})}, \tag{16}$$

where we only need

$$F(|\alpha\rangle, \widehat{\sigma}) := \sqrt{\langle \alpha \,|\, \widehat{\sigma} \,|\, \alpha \rangle}. \tag{17}$$

  - In our application, T is difficult while F can be obtained.

Distances between the Density Operators

Single-mode trace distances

- The pure wave functional can sloppily be decomposed $\chi_b[g] \sim \sum_p \chi_b^{(p)}(g_p) \equiv \sum_p \left\langle g_p \,\middle|\, \chi_b^{(p)} \right\rangle$, where $g_p := g(p)$
- So does the thermal density operator $\hat{\rho}_{\mathsf{th}}(T) \sim \bigotimes_p \hat{\rho}_{\mathsf{th}}^{(p)}(T)$
- Fidelity in eq. (16) can be factorised as well

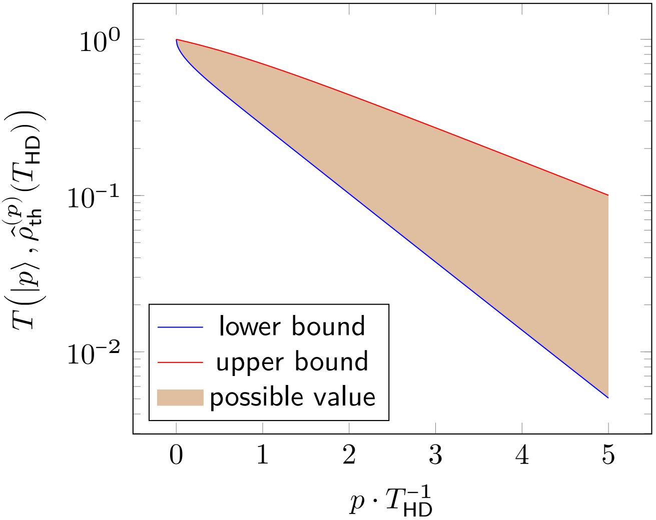
$$F \equiv \langle \chi_b \,|\, \hat{\rho}_{\mathsf{th}} \,|\, \chi_b \rangle^{\frac{1}{2}} \sim \prod_p \left\langle \chi_b^{(p)} \,\middle|\, \hat{\rho}_{\mathsf{th}}^{(p)} \,\middle|\, \chi_b^{(p)} \right\rangle^{\frac{1}{2}} =: \prod_p F^{(p)}; \tag{18}$$

- $F^{(p)}$ can be computed in order to find bounds of T

$$F^{(p)}\left(|p\rangle \,, \hat{\rho}_{\mathsf{th}}^{(p)}(T_{\mathsf{HD}})\right) = \frac{\sqrt{u-1}}{\sqrt[4]{u^2+u+1}}, \quad u := \mathbb{e}^{q} \equiv \mathbb{e}^{|p|/T_{\mathsf{HD}}}. \tag{19}$$

Distances between the Density Operators

Single-mode trace distances, bounds set by fidelity and eq. (16)



Distances between the Density Operators

All-mode trace distance

- 'Go to the continuum limit': Λ dimension regulator

$$\sum_p g(p) \to \frac{1}{2\pi\Lambda}\int \mathrm{d}p\, g(p), \tag{18}$$

- Analogously, to regularise a product

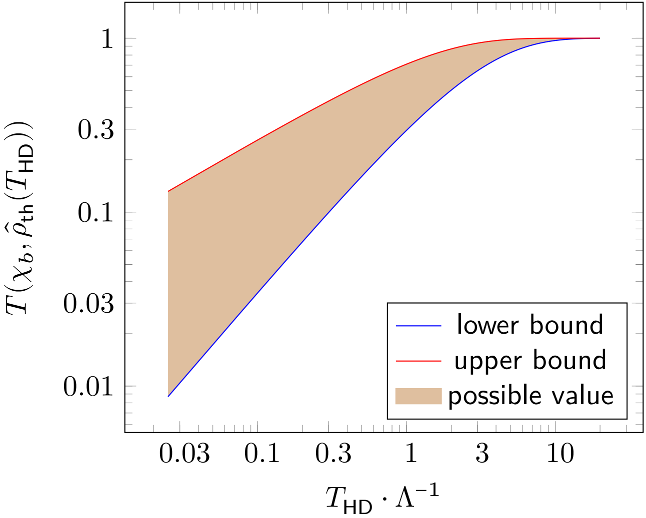
$$\prod_p f(p) \equiv \exp\left\{\sum_p \ln f(p)\right\} \to \exp\left\{\frac{1}{2\pi\Lambda}\int \mathrm{d}p\ \ln f(p)\right\} \tag{19}$$

- Regularised F can be calculated in order to set bounds of T

$$F(\chi_b, \hat{\rho}_{\text{th}}) = \exp\left\{\frac{2}{2\pi\Lambda}\int_0^{+\infty} \mathrm{d}p\ \ln F^{(p)}\right\} = \exp\left(-\frac{\pi}{9}\frac{T_{\text{HD}}}{\Lambda}\right) \tag{20}$$

Distances between the Density Operators

All-mode trace distance, bounds set by fidelity and eq. (16)



Summary

- Compared the pure and the thermal descriptions of the radiation within the solvable dilaton gravity model
- Fourier-mode fluctuation: that of the thermal state diverges faster than the vacuum case does at low energy, while the pure-state fluc. remains finite; at high energies they converge.
- Trace distance: goes exponentially small with black hole temperature going to zero.

- Outlook
  - Understand the Fourier-mode fluctuation
  - Evaluate the real space correlator
  - Understand the regulator in total trace distance
  - Evaluate the exact trace distance

For Further Reading I

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