Hawking Radiation

A Comparison of Pure-state and Thermal Descriptions

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Outline

Hawking Radiation

Model of Gravitation

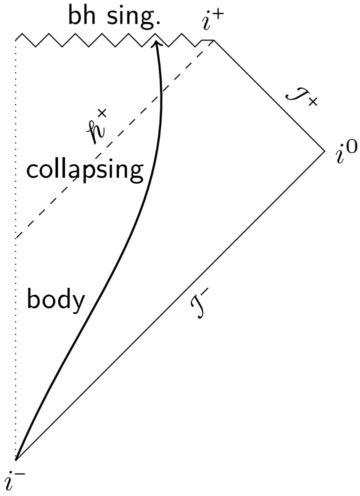
Correlator of Field Strength

Distance between Density Operators

Outline

Background space-time: spherically collapsing body

Schematic conformal (Penrose–Carter) diagram



- Each point represents an S^2
- $i^\mp$: past / future time-like infinities
- i^0: space-like infinity
- $\mathscr{I}^\mp$: past / future null inf.
- $\mathscr{h}^+$: future event horizon
- **Thick** line: boundary of the collapsing body

Hawking Radiation

Results and interpretation [Haw74; Haw75]

- An early-time vacuum on $\mathscr{I}^-$ in collapsing background

$$\hat{a}(p)\,|h\rangle := 0 \quad \Rightarrow \quad \langle h\,|\,\hat{n}_a(p)\,|\,h\rangle =: \langle \hat{n}_a(p)\rangle_h \equiv 0 \tag{1}$$

evolves to a late-time state on $\mathscr{I}^+ \cup \mathscr{h}^+$ with particles on $\mathscr{I}^+$

$$\langle \hat{n}_{\text{on } \mathscr{I}^+}(\omega)\rangle_h =: \langle \hat{n}_b(\omega)\rangle_h \approx \Gamma_\omega \left(\mathrm{e}^{2\pi\omega/\kappa} - 1\right)^{-1}. \tag{2}$$

- Comparing eq. (2) with the Bose–Einstein distribution

$$\langle \hat{n}(\omega)\rangle_{\text{BE}} = \left(\mathrm{e}^{\omega/T} - 1\right)^{-1}, \tag{3}$$

one may conclude that eq. (2) describes a grey-body radiation with the Hawking temperature,

$$T_{\text{H}} := \kappa/2\pi \equiv \hbar/ck \cdot \kappa/2\pi. \tag{4}$$

Hawking Radiation

Tension in the interpretation

- The state $|h\rangle$ or its density operator is pure,

$$\hat{\rho}_h = |h\rangle\langle h|\,, \tag{5}$$

  whilst the Bose–Einstein state of equilibrium bosonic gas

$$\hat{\rho}_{\mathrm{BE}}(T) = Z^{-1}\mathrm{e}^{-\widehat{H}/T} \sim Z^{-1}\sum_E \mathrm{e}^{-E/T}|E\rangle\langle E| \tag{6}$$

  is thermal and mixed.
- How different are they? [Kie01; HR09]

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Classical theory of $(1+1)$d Dilaton Gravity Model I

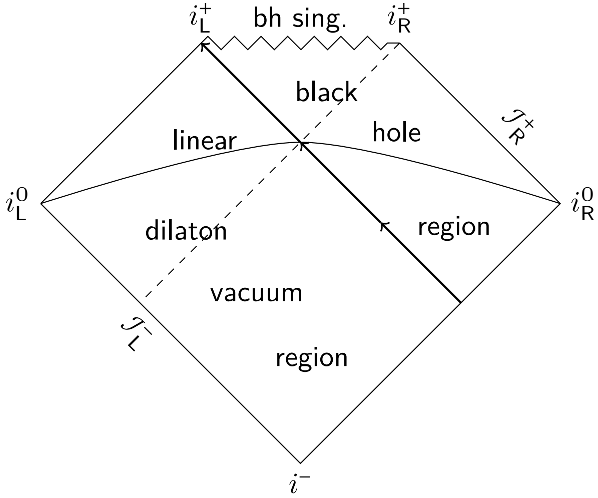
[Cal+92; DK96; APR11]

- The action of the dilaton gravity model reads

$$S = \int \mathrm{d}^2 x \sqrt{-g} \left\{ \frac{\mathrm{e}^{-2\phi}}{G} \left[R + 4(\nabla\phi)^2 + 4\lambda^2 \right] - \frac{1}{2}(\nabla f)^2 \right\}, \tag{7}$$

  where $\phi(x)$ is the dilaton field, $f(x)$ a massless neutral scalar field representing, and $\lambda > 0$ the cosmological constant.
- Has a solution which resembles the collapsing body in $(3+1)$-dimensional Einstein gravitation

Classical theory of $(1+1)$d Dilaton Gravity Model II
[Cal+92; DK96; APR11]



- Each point represents a point
- **Thick** line: null matter shell

Quantum theory of $(1+1)$d Dilaton Gravity Model I
[DK96]

- Constraint system: Schrödinger quantisation does not apply
- (Formally) Dirac quantisation

$$\widehat{\mathcal{H}}_{\parallel}\Psi[g,\phi,f]=0, \qquad \widehat{\mathcal{H}}_{\perp}\Psi[g,\phi,f]=0. \tag{8}$$

- Semi-classical approximation: $\Psi = \mathbb{e}^{\mathbb{i}(G^{-1}S_0+S_1+GS_2+\ldots)}$
  - $\mathrm{O}(G^{-1})$: Hamilton–Jacobi equation for pure gravity
  - $\mathrm{O}(G^0)$: $\Psi = D[g,\phi]\chi[g,\phi,f]$; functional Schrödinger equation for matter $\mathbb{i}\partial_t\chi[f]=\widehat{H}_{\mathrm{m}}\chi[f]$, where

$$\widehat{H}_{\mathrm{m}} = \frac{1}{2}\int_0^{+\infty}\mathbb{d}k\left(-\frac{\boldsymbol{\delta}^2}{\boldsymbol{\delta} f^2(k)}+k^2f^2(k)\right). \tag{9}$$

- A quantum field theory in curved space-time can be derived!

Quantum theory of $(1+1)$d Dilaton Gravity Model II

[DK96]

- At early time, the vacuum wave functional is

$$\chi_0[f_{\mathsf{e}}] \propto \exp\left\{-\frac{1}{2}\int_{\mathbb{R}^+} \mathrm{d}k\, k\, f_{\mathsf{e}}^2(k)\right\} \sim \prod_k \exp\left\{\frac{1}{2}\frac{k}{\Lambda} f_{\mathsf{e}}^2\right\}, \quad (10)$$

while at late time it evolves to

$$\chi_b[f_{\mathsf{l}}] \propto \exp\left\{-\int_{\mathbb{R}} \mathrm{d}p\, p \coth\left(\frac{\pi p}{2\lambda}\right)|f_{\mathsf{l}}(p)|^2\right\} \sim \prod_p \mathrm{e}^{\cdots}, \quad (11)$$

where $f_{\mathsf{e}}(k)$ and $f_{\mathsf{l}}(p)$ are the Fourier transform of the matter field at early and late time, respectively.

- At late time, particle-number expectations are

$$\langle \hat{n}_b(p)\rangle_{\chi_b} = \left(\mathrm{e}^{2\pi|p|/\lambda} - 1\right)^{-1}, \quad (12)$$

leading to a Hawking-like black-body temperature

$$\boxed{T_{\mathsf{HD}} := \lambda/2\pi.} \quad (13)$$

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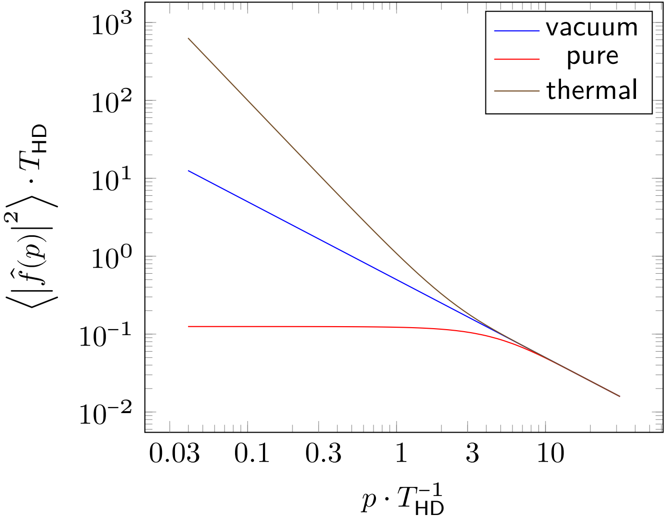
The Fourier-mode correlators of different states can be calculated,

$$\left\langle \hat{f}^\dagger(p_1)\hat{f}(p_2)\right\rangle = \frac{1}{T_{\mathrm{HD}}}\delta(p_1 - p_2)\cdot\begin{cases} \frac{1}{2}\frac{1}{q}, & \text{vacuum;} \\ \frac{1}{8}\frac{\tanh\frac{q}{4}}{\frac{q}{4}}, & \chi_b; \\ \frac{1}{4}\frac{\coth\frac{q}{2}}{\frac{q}{2}}, & \hat{\rho}_{\mathrm{BE}}(T_{\mathrm{HD}}), \end{cases} \tag{14}$$

where $q := p_1/T_{\mathrm{HD}}$.

Correlation of Fourier Modes

Fluctuation of the Fourier modes: diagram in log-log scale



Correlation of Fourier Modes

Fluctuation of the Fourier modes: interpretation

- Vacuum fluctuation

$$\left\langle \left|\hat{f}\right|^2 \right\rangle_{\text{vac}} \sim \mathrm{O}(q^{-1}) \tag{15}$$

- A black hole does not alter the high-energy processes.

$$\left\langle \left|\hat{f}\right|^2 \right\rangle_{\chi_b} \approx \left\langle \left|\hat{f}\right|^2 \right\rangle_{\text{th}} \approx \left\langle \left|\hat{f}\right|^2 \right\rangle_{\text{vac}} \sim \mathrm{O}(q^{-1}), \quad |p| \gg T_{\text{HD}} \tag{16}$$

- A black hole suppresses low-energy fluctuation of the pure state, while enhancing that of the thermal state.

$$\mathrm{O}(1) \sim \left\langle \left|\hat{f}\right|^2 \right\rangle_{\chi_b} \ll \left\langle \left|\hat{f}\right|^2 \right\rangle_{\text{vac}} \ll \left\langle \left|\hat{f}\right|^2 \right\rangle_{\text{th}} \sim \mathrm{O}(q^{-2}), \quad |p| \ll T_{\text{HD}} \tag{17}$$

- Critical scale $|p| \sim T_{\text{HD}}$

Outline

Trace Distance [Wil09, ch. 9]

Definitions

- Trace distance between Hermitian operators $\widehat{M}$ and $\widehat{N}$

$$T\left(\widehat{M},\widehat{N}\right) := \frac{1}{2}\operatorname{tr}\sqrt{\left(\widehat{M}-\widehat{N}\right)^{\dagger}\left(\widehat{M}-\widehat{N}\right)} \tag{18}$$

- For density operators $\hat{\rho}$ and $\hat{\sigma}$,
  - $0 \leq T(\hat{\rho},\widehat{\sigma}) \leq 1.$
  - Controlled by fidelity in [FG99]

$$\boxed{1-F(\hat{\rho},\widehat{\sigma}) \leq T(\hat{\rho},\widehat{\sigma}) \leq \sqrt{1-F^2(\hat{\rho},\widehat{\sigma})},} \tag{19}$$

  where we only need

$$F(|\alpha\rangle,\widehat{\sigma}) := \sqrt{\langle\alpha\,|\,\widehat{\sigma}\,|\,\alpha\rangle}. \tag{20}$$

  - In our application, T is difficult while F can be obtained.

Single-mode Distances between the Density Operators I

Bounds set by fidelity and eq. (19)

- The pure wave functional can sloppily be decomposed $\chi_b[f] \sim \sum_p \chi_b^{(p)}(f_p) \equiv \sum_p \left\langle f_p \,\middle|\, \chi_b^{(p)} \right\rangle$, where $f_p := f(p)$
- So does the thermal density operator $\hat{\rho}_{\text{th}}(T) \sim \bigotimes_p \hat{\rho}_{\text{th}}^{(p)}(T)$
- Fidelity (in eq. (19)) can be factorised as well

$$F \equiv \langle \chi_b \,|\, \hat{\rho}_{\text{th}} \,|\, \chi_b \rangle^{\frac{1}{2}} \sim \prod_p \left\langle \chi_b^{(p)} \,\middle|\, \hat{\rho}_{\text{th}}^{(p)} \,\middle|\, \chi_b^{(p)} \right\rangle^{\frac{1}{2}} =: \prod_p F^{(p)}; \tag{21}$$

- $F^{(p)}$ can be computed in order to find bounds of T

$$F^{(p)}\left(|p\rangle\,, \hat{\rho}_{\text{th}}^{(p)}(T_{\text{HD}})\right) = \frac{\sqrt{u-1}}{\sqrt[4]{u^2+u+1}}, \quad u := \mathrm{e}^{q} \equiv \mathrm{e}^{|p|/T_{\text{HD}}}. \tag{22}$$

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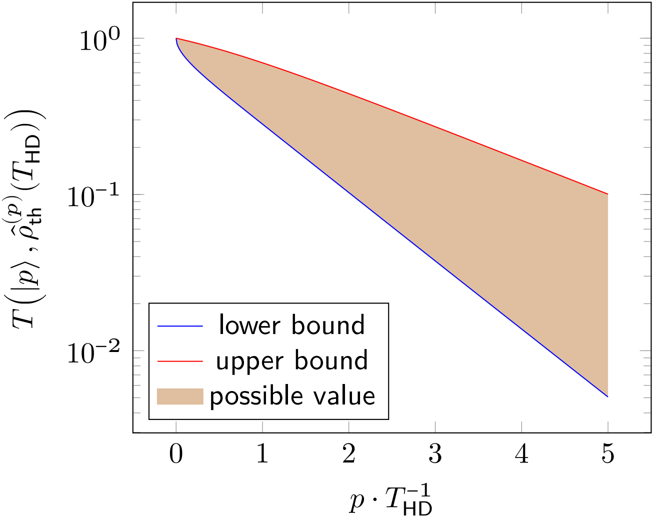
Single-mode Distances between the Density Operators II

Bounds set by fidelity and eq. (19)



All-modes Distances between the Density Operators I

Bounds set by fidelity and eq. (19)

- 'Go to the continuum limit': Λ dimension regulator

$$\sum_p g(p) \to \frac{1}{2\pi\Lambda}\int \mathrm{d}p\, g(p), \tag{23}$$

- Analogously, to regularise a product

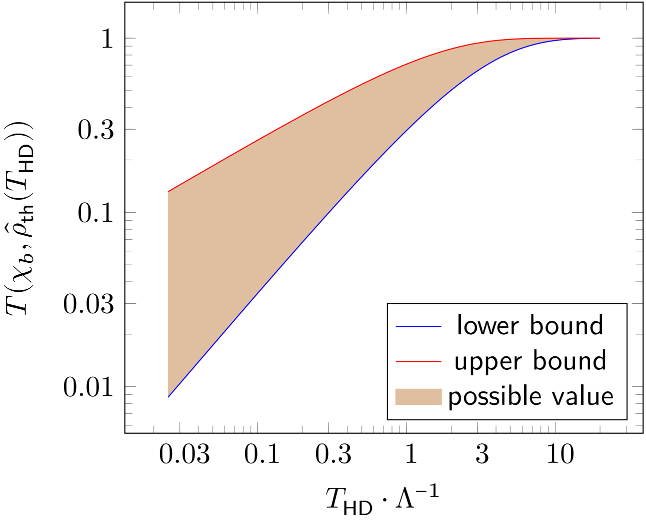
$$\prod_p f(p) \equiv \exp\left\{\sum_p \ln f(p)\right\} \to \exp\left\{\frac{1}{2\pi\Lambda}\int \mathrm{d}p\ \ln f(p)\right\} \tag{24}$$

- Regularised F can be calculated in order to set bounds of T

$$F(\chi_b, \hat{\rho}_{\text{th}}) = \exp\left\{\frac{2}{2\pi\Lambda}\int_0^{+\infty} \mathrm{d}p\ \ln F^{(p)}\right\} = \exp\left(-\frac{\pi}{9}\frac{T_{\text{HD}}}{\Lambda}\right) \tag{25}$$

All-modes Distances between the Density Operators II

Bounds set by fidelity and eq. (19)



Summary

- Compared the pure and the thermal descriptions of the radiation within the solvable dilaton gravity model
- Fourier-mode fluctuation: that of the thermal state diverges faster than the vacuum case does at low energy, while the pure-state fluc. remains finite; at high energies they converge.
- Trace distance: goes exponentially small with black hole temperature going to zero.

- Outlook
  - Understand the Fourier-mode fluctuation
  - Evaluate the real space correlator
  - Understand the regulator in total trace distance
  - Evaluate the exact trace distance

For Further Reading I

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