

# Hawking Radiation

## A Comparison of Pure-state and Thermal Description

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# Outline

Hawking Radiation

Model of Gravitation

Correlator of Field Strength

Distance between Density Operators

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Hawking Radiation

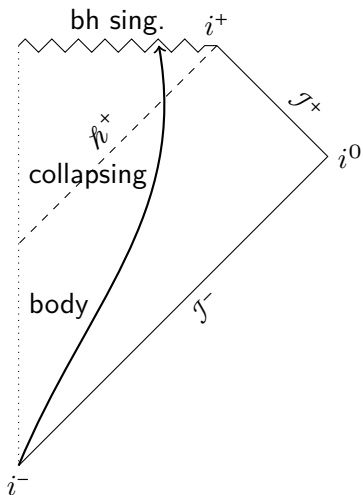
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# Hawking Radiation

Background space-time: spherically collapsing body



# Hawking Radiation

## Result and interpretation

- ▶ An early-time **vacuum** on  $\mathcal{I}^-$  in collapsing background

$$\hat{a}(p) |h\rangle := 0 \quad \Rightarrow \quad \langle \hat{n}_a(p) \rangle_h \equiv 0 \quad (1)$$

evolves to a late-time state on  $\mathcal{I}^+ \cup \mathcal{H}^+$  **with particles** [Haw74]

$$\langle \hat{n}_b(\omega) \rangle_h \approx \Gamma_\omega (e^{2\pi\omega/\kappa} - 1)^{-1}. \quad (2)$$

- ▶ Comparing eq. (2) with the Bose–Einstein distribution

$$\langle \hat{n}(\omega) \rangle_{\text{BE}} = (e^{\omega/T} - 1)^{-1}, \quad (3)$$

one may conclude that eq. (2) describes a grey-body radiation with the **Hawking temperature** [Haw75],

$$T_H := \kappa/2\pi.$$



# Hawking Radiation

## Tension in the interpretation

- ▶ The state  $|h\rangle$  or its density operator is **pure**,

$$\hat{\rho}_h = |h\rangle \langle h|, \quad (5)$$

whilst the Bose–Einstein state of equilibrium bosonic gas

$$\hat{\rho}_{\text{BE}} = Z^{-1} e^{-\hat{H}/T} \sim Z^{-1} \sum_E e^{-E/T} |E\rangle \langle E| \quad (6)$$

is **thermal** and **mixed**.

- ▶ How different are they? [Kie01; HR09]

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# (1 + 1)-dimensional Dilaton Gravity Model

Classical theory 1/2 [Cal+92; DK96; APR11]

- The action of the dilaton gravity model reads

$$\begin{aligned} S &= \int \mathbb{D}^2 x \sqrt{-\bar{g}} \left\{ \frac{e^{-2\bar{\phi}}}{G} \left[ \bar{R} + 4(\nabla \bar{\phi})^2 + 4\lambda^2 \right] - \frac{1}{2}(\nabla f)^2 \right\} \\ &= \int \mathbb{D}^2 x \sqrt{-g} \left\{ \frac{1}{G} [R\phi + 4\lambda^2] - \frac{1}{2}(\nabla f)^2 \right\}, \end{aligned} \quad (7)$$

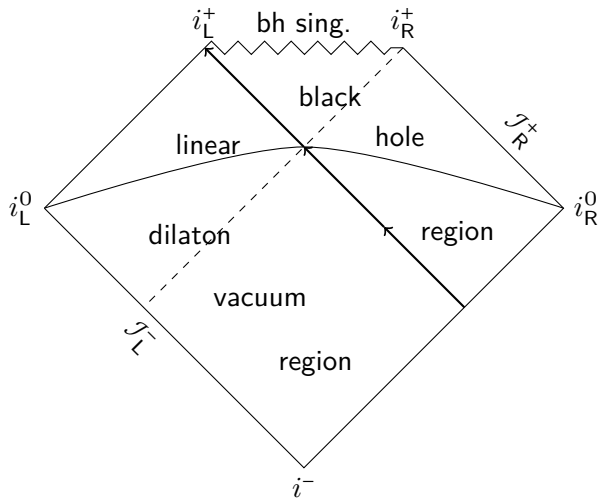
where in eq. (7), the **kinetic term** of the dilaton field  $\phi$  is eliminated by substituting  $\phi = e^{-2\bar{\phi}}$  and  $g_{\alpha\beta} = e^{-2\bar{\phi}} \bar{g}_{\alpha\beta}$ .

- Has a solution which **resembles** the collapsing body in (3 + 1)d Einstein gravitation



# (1 + 1)-dimensional Dilaton Gravity Model

Classical theory 2/2 [Cal+92; DK96; APR11]



# (1 + 1)-dimensional Dilaton Gravity Model

Quantum theory 1/2 [DK96]

- ▶ Can formally be canonically quantised as a constraint system
- ▶ Insert the ansatz  $\Psi[g, \phi, f] = e^{i(G^{-1}S_0 + S_1 + GS_2 + \dots)}$ 
  - ▶ Order  $G^{-1}$ : Hamilton–Jacobi equation for pure gravity
  - ▶ Order  $G^0$ : functional **Schrödinger equation for matter**

$$i \frac{\partial \chi}{\partial t} = \widehat{H}_m \chi, \quad (7)$$

$$\widehat{H}_m \equiv \frac{1}{2} \int_0^{+\infty} dk \left( -\frac{\delta^2}{\delta f^2(k)} + k^2 f^2(k) \right). \quad (8)$$

# (1 + 1)-dimensional Dilaton Gravity Model

Quantum theory 2/2 [DK96]

- ▶ At early time, the **ground-state** solution to eq. (7) is

$$\chi_0[f_e] \propto \exp\left\{-\frac{1}{2} \int_0^{+\infty} dk k f_e(k)^2\right\}, \quad (7)$$

while at late time it evolves to

$$\chi_b[f_l] \propto \exp\left\{-\int_{-\infty}^{+\infty} dp p \coth\left(\frac{\pi p}{2\lambda}\right) |f_l(p)|^2\right\}, \quad (8)$$

where  $f_e(k)$  and  $f_l(p)$  are the Fourier transform of the matter field at early and late time, respectively.

- ▶ At late time, **particle-number expectations** are

$$\langle \hat{n}_b(p) \rangle_{\chi_b} = \left( e^{2\pi|p|/\lambda} - 1 \right)^{-1}, \quad (9)$$

leading to a Hawking-like **black-body temperature**

$$T_{\text{HD}} := \lambda/2\pi.$$

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# Correlation of Fourier Modes

## The discrepancy

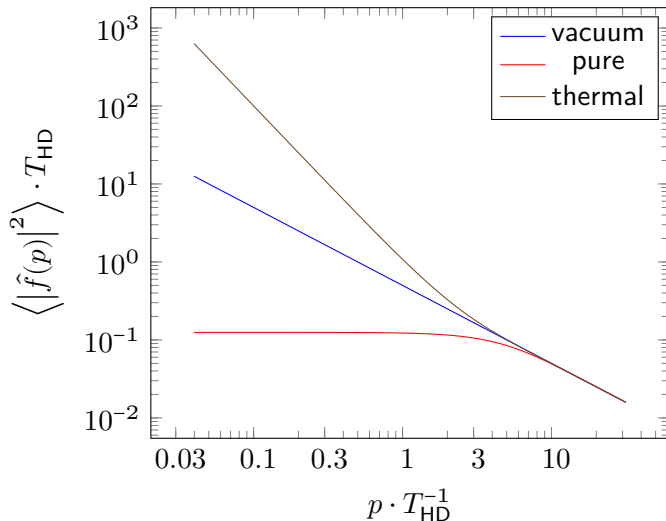
The Fourier-mode correlators of different states can be calculated,

$$\langle \hat{f}^\dagger(p_1) \hat{f}(p_2) \rangle = \frac{1}{T_{\text{HD}}} \delta(p_1 - p_2) \cdot \begin{cases} \frac{1}{2} \frac{1}{q}, & \text{vacuum;} \\ \frac{1}{8} \frac{\tanh \frac{q}{4}}{\frac{q}{4}}, & \chi_b; \\ \frac{1}{4} \frac{\coth \frac{q}{2}}{\frac{q}{2}}, & \hat{\rho}_{\text{BE}}(T_{\text{HD}}), \end{cases} \quad (11)$$

where  $q := p_1/T_{\text{HD}}$ .

# Correlation of Fourier Modes

Fluctuation of the Fourier modes: diagram in log-log scale



# Correlation of Fourier Modes

## Fluctuation of the Fourier modes: interpretation

- Vacuum fluctuation

$$\langle |\hat{f}|^2 \rangle_{\text{vac}} \sim O(q^{-1}) \quad (12)$$

- A black hole does **not** alter the **high**-energy processes.

$$\langle |\hat{f}|^2 \rangle_{\chi_b} \approx \langle |\hat{f}|^2 \rangle_{\text{th}} \approx \langle |\hat{f}|^2 \rangle_{\text{vac}} \sim O(q^{-1}), \quad |p| \gg T_{\text{HD}} \quad (13)$$

- A black hole **suppresses** low-energy fluctuation of the **pure** state, while **enhancing** that of the **thermal** state.

$$O(1) \sim \langle |\hat{f}|^2 \rangle_{\chi_b} \ll \langle |\hat{f}|^2 \rangle_{\text{vac}} \ll \langle |\hat{f}|^2 \rangle_{\text{th}} \sim O(q^{-2}), \quad |p| \ll T_{\text{HD}} \quad (14)$$

- Critical scale  $|p| \sim T_{\text{HD}}$

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# Trace Distance [Wil09, ch. 9]

## Definitions

- ▶ Trace distance between Hermitian operators  $\widehat{M}$  and  $\widehat{N}$

$$T(\widehat{M}, \widehat{N}) := \frac{1}{2} \operatorname{tr} \sqrt{(\widehat{M} - \widehat{N})^\dagger (\widehat{M} - \widehat{N})} \quad (15)$$

- ▶ For **density operators**  $\hat{\rho}$  and  $\hat{\sigma}$ ,
  - ▶  $0 \leq T(\hat{\rho}, \hat{\sigma}) \leq 1$ .
  - ▶ Controlled by **fidelity** in [FG99]

$$1 - F(\hat{\rho}, \hat{\sigma}) \leq T(\hat{\rho}, \hat{\sigma}) \leq \sqrt{1 - F^2(\hat{\rho}, \hat{\sigma})}, \quad (16)$$

where we only need

$$F(|\alpha\rangle, \hat{\sigma}) := \sqrt{\langle \alpha | \hat{\sigma} | \alpha \rangle}. \quad (17)$$

- ▶ In our application,  $T$  is difficult while  $F$  can be obtained.

# Distances between the Density Operators

## Single-mode trace distances

- ▶ The pure wave functional can sloppily be decomposed  $\chi_b[g] \sim \sum_p \chi_b^{(p)}(g_p) \equiv \sum_p \langle g_p | \chi_b^{(p)} \rangle$ , where  $g_p := g(p)$
- ▶ So does the thermal density operator  $\hat{\rho}_{\text{th}}(T) \sim \bigotimes_p \hat{\rho}_{\text{th}}^{(p)}(T)$
- ▶ Fidelity in eq. (16) can be factorised as well

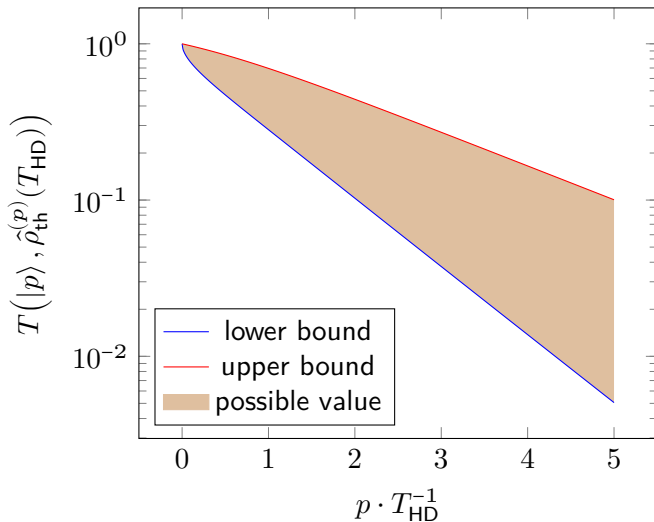
$$F \equiv \langle \chi_b | \hat{\rho}_{\text{th}} | \chi_b \rangle^{\frac{1}{2}} \sim \prod_p \langle \chi_b^{(p)} | \hat{\rho}_{\text{th}}^{(p)} | \chi_b^{(p)} \rangle^{\frac{1}{2}} =: \prod_p F^{(p)}; \quad (18)$$

- ▶  $F^{(p)}$  can be computed in order to find bounds of  $T$

$$F^{(p)}(|p\rangle, \hat{\rho}_{\text{th}}^{(p)}(T_{\text{HD}})) = \frac{\sqrt{u-1}}{\sqrt[4]{u^2+u+1}}, \quad u := e^q \equiv e^{|p|/T_{\text{HD}}}. \quad (19)$$

# Distances between the Density Operators

Single-mode trace distances, bounds set by fidelity and eq. (16)



# Distances between the Density Operators

## All-mode trace distance

- ▶ 'Go to the continuum limit':  $\Lambda$  dimension regulator

$$\sum_p g(p) \rightarrow \frac{1}{2\pi\Lambda} \int \mathrm{d}p g(p), \quad (18)$$

- ▶ Analogously, to regularise a product

$$\prod_p f(p) \equiv \exp \left\{ \sum_p \ln f(p) \right\} \rightarrow \exp \left\{ \frac{1}{2\pi\Lambda} \int \mathrm{d}p \ln f(p) \right\} \quad (19)$$

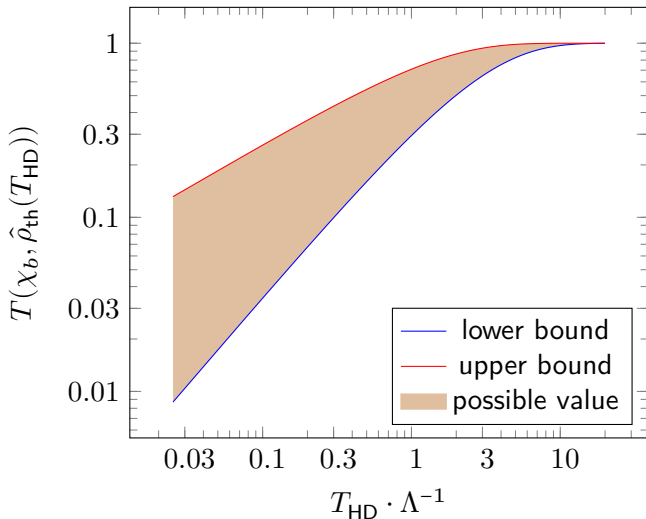
- ▶ Regularised  $F$  can be calculated **in order to set bounds of  $T$**

$$F(\chi_b, \hat{\rho}_{\text{th}}) = \exp \left\{ \frac{2}{2\pi\Lambda} \int_0^{+\infty} \mathrm{d}p \ln F^{(p)} \right\} = \exp \left( -\frac{\pi}{9} \frac{T_{\text{HD}}}{\Lambda} \right)$$



# Distances between the Density Operators






All-mode trace distance, bounds set by fidelity and eq. (16)







# Summary

- ▶ Compared the **pure** and the **thermal** descriptions of the radiation within the solvable dilaton gravity model
- ▶ Fourier-mode fluctuation: that of the thermal state **diverges** faster than the vacuum case does at low energy, while the pure-state fluc. remains **finite**; at high energies they **converge**.
- ▶ Trace distance: goes exponentially **small** with black hole **temperature** going to zero.
- ▶ Outlook
  - ▶ Understand the Fourier-mode fluctuation
  - ▶ Evaluate the real space correlator
  - ▶ Understand the regulator in total trace distance
  - ▶ Evaluate the exact trace distance

# For Further Reading I

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# For Further Reading II

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