(0.1) The Boulware state

 $|B\rangle$ retarded propagator is given by

$$\begin{split} E_{\mathrm{B}}^{+}(x,x') &= \mathbb{i} \sum_{lm} \int_{0}^{+\infty} \frac{\mathrm{d}\omega}{4\pi\omega} \mathrm{e}^{-\mathbb{i}\omega(t-t')} Y_{lm}(\theta,\phi) Y_{lm}^{*}(\theta',\phi') \\ &\qquad \qquad \cdot (R_{l}^{\rightarrow}(\omega|r) R_{l}^{\rightarrow *}(\omega|r') + R_{l}^{\leftarrow}(\omega|r) R_{l}^{\leftarrow *}(\omega|r')) \end{split} \tag{0.1}$$

$$\begin{split} E_{\mathrm{H}}^{+}(x,x') &= \mathbb{i} \sum_{lm} \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{4\pi\omega} \\ &\cdot \left(\mathbb{e}^{-\mathbb{i}\omega(t-t')} Y_{lm}(\theta,\phi) Y_{lm}^{*}(\theta',\phi') \frac{R_{l}^{\rightarrow}(\omega|r) R_{l}^{\rightarrow*}(\omega|r')}{1 - \mathbb{e}^{-2\pi\omega/\kappa}} \right. \\ &\left. + \mathbb{e}^{+\mathbb{i}\omega(t-t')} Y_{lm}^{*}(\theta,\phi) Y_{lm}(\theta',\phi') \frac{R_{l}^{\leftarrow*}(\omega|r) R_{l}^{\leftarrow}(\omega|r')}{\mathbb{e}^{+2\pi\omega/\kappa} - 1} \right) \end{split}$$