Black Hole Thermodynamics

And Hawking Radiation

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Advanced Seminar on Relativity and Cosmology

Outline

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1. Motivations
2. Particle creation in curved space-time
3. Gravitational collapse
4. Eternal black hole
5. Information loss problem

Motivations for the Radiation

Developments in classical physics

- (Speculative) entropy and temperature of black holes [Bek73; Bek80]
- Penrose process and super-radiance

Motivations for the Radiation

Developments in quantum physics

- Arguments from fundamental principles
  - Uncertainty principle (black holes “might split up” [Sus08])
  - Vacuum polarisation (e.g. Schwinger Effect, where electron-positron pairs are pulled out from vacuum in strong external electric field), see e.g. [FN98]
- Arguments from field theory in curved space-time: definition of particle ambiguous [Ful73], leading to particle creations
  - in dynamical universes, see e.g. [PT09]
  - for accelerated observers [Ful73; Dav75; Unr76]

Scalar Field in Special Relativity

Warm-up

- Diagonalising Hamiltonian: *Fourier* kernel & '*momentum*' p

$$\tilde{\phi}(p) \propto \int \mathrm{d}x\, \mathrm{e}^{-\mathrm{i}px} \phi(x), \quad \tilde{\pi}(p) \propto \int \mathrm{d}x\, \mathrm{e}^{+\mathrm{i}px} \pi(x).$$

- Trsfing field between inertial obs. A and B: Poincaré $\mathcal{P}$

$$x \to \mathcal{P}x = \Lambda x + x_0, \quad p \to \mathcal{P}p = p + p_0;$$

$$\tilde{\phi}_A(p) \to \tilde{\phi}_A(\mathcal{P}^{-1}p) \propto \tilde{\phi}_B(p'), \quad \text{etc.}$$

- Canonical quantisation: ladder operators

$$\hat{a}_\phi(p) \propto E_p^{+\frac{1}{2}} \hat{\tilde{\phi}}(p) + \mathrm{i} E_p^{-\frac{1}{2}} \hat{\tilde{\pi}}^\dagger(p), \quad \text{etc.}$$

- Trsfing ladder optrs btw. inertial obs.: particle remains p.

$$\hat{b}(p') = \widehat{U}_\mathcal{P} \hat{a}(p) \widehat{U}_\mathcal{P}^\dagger \propto \hat{a}(\mathcal{P}p), \quad \text{etc.}$$

Scalar Field in General Relativity

The technical advance we just mentioned

- Diagonalising Hamiltonian: *general* kernel & *parameter* k

$$\tilde{\phi}(k) \propto \int \mathbb{d}x\, K(k;x)\phi(x), \quad \tilde{\pi}(k) \propto \int \mathbb{d}x\, K^*(k;x)\pi(x)$$

- Trsfing field between (time-like) obs.: general diffeom. $\mathcal{D}$

$$x \to \chi = f(x), \quad k \to \kappa = g(k);$$

$$\tilde{\phi}_A(k) \to \tilde{\phi}_A(g^{-1}(\kappa)) = \int \mathbb{d}\kappa\, T(k;\kappa)\tilde{\phi}_B(\kappa), \quad \text{etc.}$$

- Canonical quantisation: ladder operators

$$\hat{a}_\phi(k) \propto E_k^{+\frac{1}{2}}\hat{\tilde{\phi}}(k) \pm \mathbb{i}E_k^{-\frac{1}{2}}\hat{\tilde{\pi}}^\dagger(k), \quad \text{etc.}$$

- Transforming ladder optrs: Bogolyubov, mixing p. & anti-p.

$$\hat{b}(\kappa) = \widehat{U}_{\mathcal{D}}\hat{a}(k)\widehat{U}_{\mathcal{D}}^\dagger \propto \int \mathbb{d}k\, \alpha(\kappa;k)\hat{a}(k) + \beta(\kappa;k)\hat{a}^\dagger(k), \quad \text{etc.}$$

Scalar Field in Special and General Relativity

Particle 'creation'

- Defining the $\hat{a}$-vacuum as being annihilated by all $\hat{a}$'s

$$\hat{a}(p \text{ or } k)\,|\Omega\rangle \equiv 0$$

- Number operators transform as

$$\begin{aligned}\hat{a}^\dagger(p)\hat{a}(p) &\to \hat{b}^\dagger(p')\hat{b}(p') \\ &\propto \hat{a}^\dagger(\mathcal{P}p)\hat{a}(\mathcal{P}p);\end{aligned} \qquad \begin{aligned}\hat{a}^\dagger(k)\hat{a}(k) &\to \hat{b}^\dagger(\kappa)\hat{b}(\kappa) \\ &= \ldots\hat{a}^\dagger\hat{a} + \ldots\hat{a}\hat{a} + \ldots\hat{a}^\dagger\hat{a}^\dagger \\ &\quad + \int \mathbb{d}k\,|\beta(\kappa;k)|^2\hat{a}(k)\hat{a}^\dagger(k).\end{aligned}$$

- $\langle\hat{n}_a\rangle_\Omega$'s transform as

$$\begin{aligned}\left\langle\hat{a}^\dagger(p)\hat{a}(p)\right\rangle &\to \left\langle\hat{b}^\dagger(p')\hat{b}(p')\right\rangle \\ &= 0;\end{aligned} \qquad \begin{aligned}\left\langle\hat{a}^\dagger(k)\hat{a}(k)\right\rangle &\to \left\langle\hat{b}^\dagger(\kappa)\hat{b}(\kappa)\right\rangle \\ &= \int \mathbb{d}k\,|\beta(\kappa;k)|^2.\end{aligned}$$

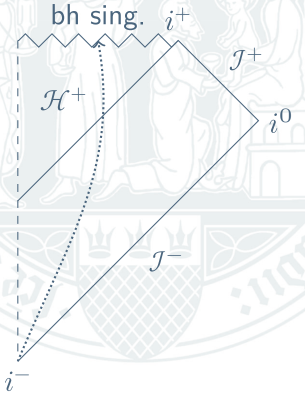
Spherically gravitational collapse

The model and its quantisation

- Massless neutral scalar field on the given background space-time w/ in- and out-going eigen-modes

$$f_{\omega' lm} \propto r^{-1} F_{\omega'} \mathbb{e}^{\mathbb{i}\omega' v} Y_{lm}(\theta, \phi),$$
$$p_{\omega lm} \propto r^{-1} P_{\omega} \mathbb{e}^{\mathbb{i}\omega u} Y_{lm}(\theta, \phi)$$

- Quantising at Cauchy surfaces
  1. Early time: $\hat{a}$'s defined on $\mathcal{J}^-$
  2. Late time: $\hat{b}$'s on $\mathcal{J}^+$, $\hat{c}$'s on $\mathcal{H}^+$
- *Physical* vacuum: $\hat{a}\,|H\rangle \equiv 0$
- Concerned with $\langle H\,|\,\hat{n}_b\,|\,H\rangle$



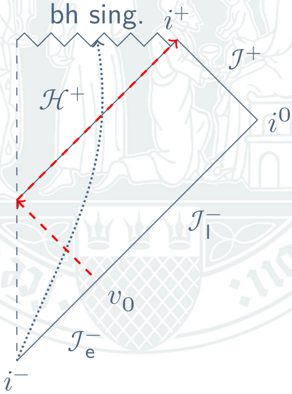
Spherically gravitational collapse

Hawking's construction [Haw74; Haw75]

- Consider p_ω on $\mathcal{J}^+$ propagating backwardly to $\mathcal{J}^-$, ending up with
  - The same freq. $p_\omega^{(1)}$: only gives a $\delta(\omega - \omega')$ term in $\alpha(\omega; \omega')$
  - The rest $p_\omega^{(2)}$: contributing to β as well; of interest
- On $\mathcal{J}^-$, approximately $p_\omega^{(2)} \propto$

$$\begin{cases} 0, & v > v_0; \\ r^{-1} P_\omega \left(\frac{v_0 - v}{CD}\right)^{-\mathrm{i}\frac{\omega}{\kappa_\mathsf{S}}}, & v \lesssim v_0 \end{cases}$$

where v_0 is the latest time a null geodesic could leave $\mathcal{J}^-$, C and D const.



Results and Interpretation

Hawking temperature

- Expectation value of particle number

$$\langle \hat{n}_b(\omega) \rangle_H = \int \mathrm{d}\omega' \left|\beta(\omega;\omega')\right|^2 \approx \Gamma_\omega \left(\mathrm{e}^{\frac{2\pi\omega}{\kappa_S}} - 1 \right)^{-1} \tag{1}$$

- Comparing eq. (1) with Bose–Einstein dist. (*black* body) $\langle \hat{n}(\omega) \rangle_{\mathrm{BE}} = (\mathrm{e}^{\frac{\omega}{T}} - 1)^{-1}$, one may conclude that the physical system concerned is a *grey* body, with a temperature of

$$T_{\mathrm{H}} = \frac{\kappa_{\mathrm{S}}}{2\pi} \approx \left(\frac{M_\odot}{M} \right) \cdot 6.169 \times 10^{-8}\,\mathrm{K}.$$

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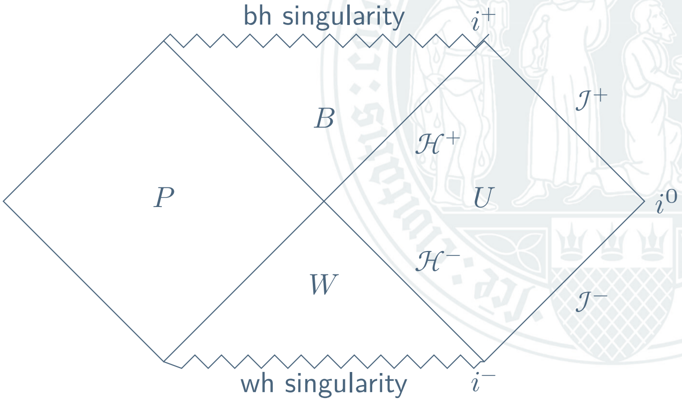
- $\hat{\rho}_H = |H\rangle\langle H|$ pure, while the thermal $\hat{\rho}_{\mathsf{BE}} = \frac{1}{Z}\mathbb{e}^{-\widehat{H}/T}$ mixed?

Eternal Schwarzschild black hole

The conformal diagram



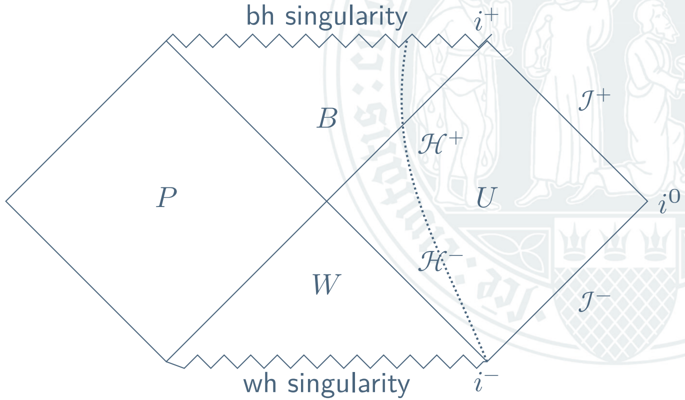
Eternal Schwarzschild black hole

[Can80; FN98]

- Clean calculation; $\left\langle \widehat{\phi}^2 \right\rangle_{\text{ren}}$, $\left\langle \widehat{T}_{\mu\nu} \right\rangle_{\text{ren}}$ etc. obtainable
- Different ‘vacua’ can be defined
  - Boulware: No flux, $\left\langle \widehat{\phi}^2 \right\rangle_{\text{ren}}$ blows up at $\mathcal{H}^- \cup \mathcal{H}^+$
    - Miracle near the horizons
  - Israel–Hartle–Hawking: I/O flux, $\left\langle \widehat{\phi}^2 \right\rangle_{\text{ren}}$ finite at $\mathcal{H}^- \cup \mathcal{H}^+$
    - Black hole in a heat bath
    - Mathematically more desirable
  - Unruh: O flux, $\left\langle \widehat{\phi}^2 \right\rangle_{\text{ren}}$ blows up at $\mathcal{H}^-$

Eternal Schwarzschild black hole

The conformal diagram, with cut



Eternal Schwarzschild black hole

[Can80; FN98]

- Comparing the space-time with that of the collapsing body: cut along the dots and stick the right to an interior solution
- Clean calculation; $\left\langle\widehat{\phi}^2\right\rangle_{\text{ren}}$, $\left\langle\widehat{T}_{\mu\nu}\right\rangle_{\text{ren}}$ etc. obtainable
- Different 'vacua' can be defined
  - Boulware: No flux, $\left\langle\widehat{\phi}^2\right\rangle_{\text{ren}}$ blows up at $\mathscr{H}^- \cup \mathscr{H}^+$
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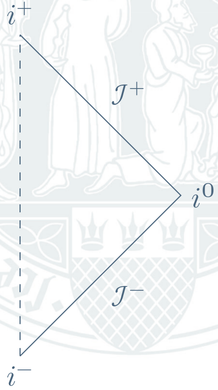
  - Unruh: O flux, $\left\langle\widehat{\phi}^2\right\rangle_{\text{ren}}$ blows up at $\mathscr{H}^-$
    - $\mathscr{H}^-$ and the divergence therein can be hidden
    - Similar to the (physical) collapsing case

Information loss problem

See e.g. [Mat09; Man15]

- Minkowski: particles from $\mathcal{J}^-$ to $\mathcal{J}^+$



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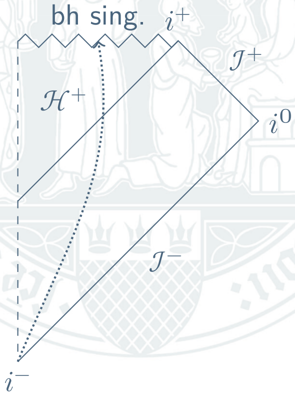
- Minkowski: particles from $\mathscr{I}^-$ to $\mathscr{I}^+$
- Collapsing body: can also go to $\mathscr{H}^+$
- Obs. able to set up input at $\mathscr{I}^-$ and collect output at $\mathscr{I}^+$, but not at $\mathscr{H}^+$



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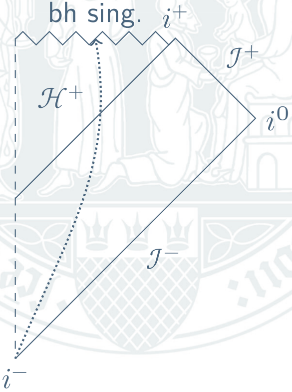
- $|\alpha\rangle$ on $\mathscr{I}^-$ evolves to $|\beta\rangle$ on $\mathscr{H}^+ \cup \mathscr{I}^+$
- Pure $\hat{\rho}_{\mathsf{I}} = |\alpha\rangle \langle\alpha|$ turns mixed $\hat{\rho}_{\mathsf{O}} = \mathrm{tr}_{\mathscr{H}^+} |\beta\rangle \langle\beta|$! Something is lost



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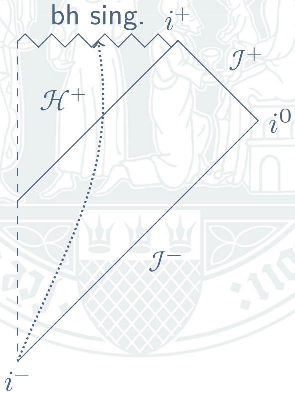
- Non-conservation arguments [Haw76]
- Conservation arguments [Pag93]



Summary

- Robust calculation of outgoing particle flux for the space-time of collapsing body
- Controversial interpretations and extrapolations

- Back-reaction to the metric: no time

- Seeks combination with
  - Quantum gravitation
  - Quantum information

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