Integrable Cosmological Models with Liouville Fields

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Outline

- 1. Introduction
- 2. Classical model and the implicitised trajectories Lagrangian formalism
- 3. Quantised model and the wave packets
 Dirac quantisation
 Semi-classical approximation
 Inner product and wave packet
- 4. Conclusions





Introduction

Introduction





The Friedmann-Lemaître model 123

- Flat Robertson–Walker metric $\mathrm{d} s^2 = -N^2(t)\,\mathrm{d} t^2 + \varkappa^{-1}\mathrm{e}^{2\alpha(t)}\,\mathrm{d} \Omega_3^2$
 - $\varkappa := 8\pi G$; $d\Omega_3^2$ dimensionless spacial metric
- Real Klein–Gordon field with potential $Ve^{\lambda\phi}$ (Liouville), where $\lambda, V \in \mathbb{R}$, and kinetic term with sign $\ell = \pm 1$ (quintessence / phantom model).
- Total action $\mathcal{S} \coloneqq S_{\rm EH} + S_{\rm GHY} + S_{\rm L} = \int \mathrm{d} \varOmega_3^2 \int \mathrm{d} t \, L,$

$$L := \varkappa^{3/2} N e^{3\alpha} \left(-\frac{3}{\varkappa} \frac{\dot{\alpha}^2}{N^2} + \mathcal{E} \frac{\dot{\phi}^2}{2N^2} - V e^{\lambda \phi} \right), \tag{1}$$

in which dot means d/dt and $\ell = \pm 1$.



Decoupling the variables

Via rescaled special orthogonal transformation

• Setting $\overline{N} := N e^{-3\alpha}$, eq. (1) transforms to

$$L = \kappa^{3/2} \overline{N} \left(-\frac{3}{\varkappa} \frac{\dot{\alpha}^2}{\overline{N}^2} + \ell \frac{\dot{\phi}^2}{2\overline{N}^2} - V e^{\lambda \phi + 6\alpha} \right)$$
 (2)

• Defining $\Delta := \lambda^2 - 6\ell \varkappa$, $\beta := \operatorname{sgn} \Delta$ and $g := \beta \sqrt{|\Delta|} \equiv \beta \sqrt{\delta \Delta}$, the rescaled special orthogonal transformation

$$\begin{pmatrix} \alpha \\ \phi \end{pmatrix} = \frac{s}{g} \begin{pmatrix} \lambda & -\ell \kappa \\ -6 & \lambda \end{pmatrix} \begin{pmatrix} s_{\beta} \beta \\ s_{\chi} \chi \end{pmatrix} \quad \text{where } s_{\beta}, s_{\chi} = \pm 1 \tag{3}$$

gives the decoupled Lagrangian

$$L = \varkappa^{3/2} \overline{N} \left(-\beta \frac{3}{\varkappa} \frac{\dot{\beta}^2}{\overline{N}^2} + \ell \beta \frac{\dot{\chi}^2}{2\overline{N}^2} - V e^{\beta_{\chi} g \chi} \right). \tag{4}$$

• The Euler–Lagrange equations w.r.t. \overline{N} , β and χ will be called the trsfed. 1st, 2nd Friedmann eqs. and the Klein–Gordon eq., respectively.



Possible generalisation of the decoupling technique ²³³

- Beyond isotropy
 - Bianchi Type-I: under investigation
- Beyond one Liouville field
 - ullet Two exponential potentials: $V_1=V_2$ and special λ_i
 - Multiple Liouville fields: mixing kinetic terms¹
- Beyond classic matter
 - Non-Hermitian, PT-symmetric Liouville fields²: may avoid big-rip etc.

² A. A. Andrianov et al. In: *International Journal of Modern Physics D* 19.01 (2010), pp. 97–111, A. A. Andrianov, C. Lan, and O. O. Novikov. In: *Springer Proceedings in Physics*. Springer International Publishing, 2016, pp. 29–44.



¹ A. A. Andrianov, O. O. Novikov, and C. Lan. In: *Theoretical and Mathematical Physics* 184.3 (2015), pp. 1224–1233.

Implicitised integration

General integral for $p_{\beta} \neq 0$

• Since β is cyclic in eq. (4), the trsfed. 2nd Friedmann eq. can be integrated³

$${\rm const.} \equiv p_{\beta} \coloneqq \frac{\partial L}{\partial \dot{\beta}} = -6 \Im \varkappa^{1/2} \frac{\dot{\beta}}{\overline{N}} \equiv -6 \Im \Im_{\beta} \frac{\varkappa^{1/2}}{g} \frac{\lambda \dot{\alpha} + \ell \varkappa \dot{\phi}}{\overline{N}}. \tag{5}$$

• For $p_{\beta} \neq 0$, fixing the *implicitising gauge* $\overline{N} = -63\sqrt{\varkappa}\dot{\beta}/p_{\beta}$, the trsfed. 1st Friedmann equation can be integrated

$$e^{\beta_{\chi}g\chi} = \frac{p_{\beta}^2}{12\varkappa^2|V|} S^2 \left(\beta_{\beta} \sqrt{\frac{3}{2\varkappa}} g\beta + C\right),\tag{6}$$

in which $v := \operatorname{sgn} V$, $(\operatorname{sgn}, \operatorname{sgn})$ means $(\ell, \mathfrak{s}v)$, and

$$(+,+)S(\gamma) := \operatorname{sech}(\gamma), \qquad (+,-)S(\gamma) := \operatorname{csch}(\gamma),$$

$$(-,+)S(\gamma) := \operatorname{sec}(\gamma), \qquad (-,-)S(\gamma) := \operatorname{icsc}(\gamma).$$

$$(7)$$

³The same first integral in Hamiltonian formalism has been found in C. Lan. PhD thesis. Saint Petersburg State University, 2016, A. A. Andrianov, O. O. Novikov, and C. Lan. In: *Theoretical and Mathematical Physics* 184.3 (2015), pp. 1224–1233.



Trajectories

Quintessence (+,+)



Trajectories

Quintessence (+, -)



Trajectories Phantom (-, +)





Integration

- Discussions
 - The integrals are consistent with the trsfed. Klein-Gordon equation.
 - The integral for (+,+)
 - has two asymptotes
 - The implicitised integral for (+, -)
 - · contains two distinct solutions
 - has three asymptotes
 - The implicitised integral for (-, +)
 - is β -even for C=0
 - · contains infinite distinct solutions
 - · has infinite asymptotes, which are pairwise parallel
 - The integral for (-,-)
 - is not real
 - The implicitised integral enables one to compare trajectories with wave functions, see below.



Implicitised integration

Specific integral for $p_\beta=0$

- For $p_{\beta}=0$, one has $\beta\equiv\beta_0$ or $\phi-\phi_0=-\ell\lambda\alpha/\kappa$, which is the familiar power-law special solution⁴.
- Further integrating the first Friedmann equation demands (+,-) or (-,+) to guarantee $\overline{N}>0$, and the result is automatically consistent with the trsfed. Klein–Gordon equation.
- Fixing $\overline{N} = (2\varkappa^2|V|)^{-1/2}$ yields

$$e^{gs_{\chi}\chi} = \left(\frac{2\kappa}{g(t-t_0)}\right)^2.$$

⁴ M. P. Dabrowski, C. Kiefer, and B. Sandhöfer. In: *Physical Review D* 74.4 (2006).



Trajectories

Quintessence (+,+)



Trajectories

Quintessence (+, -)



Trajectories Phantom (-, +)



Dirac quantisation and the mode functions 233

The primary Hamiltonian and the Hamiltonian constraint reads

$$H^{\mathsf{p}} = \overline{N}H_{\perp} + v^{\overline{N}}p_{\overline{N}},\tag{9}$$

$$H_{\perp} := -\beta \frac{p_{\beta}^2}{12\varkappa^{1/2}} + \ell \beta \frac{p_{\chi}^2}{2\varkappa^{3/2}} + \varkappa^{3/2} V e^{g\beta_{\chi}\chi}. \tag{10}$$

• Applying the Dirac quantisation rules with Laplace–Beltrami opeator, one gets the mss. Wheeler–DeWitt eq. with (β,χ)

$$0 = \widehat{H}_{\perp} \Psi(\beta, \chi) := \left(s \frac{\hbar^2}{12\varkappa^{1/2}} \partial_{\beta}^2 - \ell' s \frac{\hbar^2}{2\varkappa^{3/2}} \partial_{\chi}^2 + \varkappa^{3/2} V e^{g^3 \chi^{\chi}} \right) \Psi. \tag{11}$$

• Equation (11) is KG-like, hyperbolic for $\ell=+1$ and elliptic for $\ell=-1$.



Separation of the variables and mode functions 233

• Writing $\Psi(\beta,\chi)=\varphi(\beta)\psi(\chi)$, eq. (11) can be separated into

$$\partial_{\beta}^{2}\varphi(\beta) = k_{\beta}^{2}\varphi(\beta); \tag{12}$$

$$\mathbb{D}\psi(\chi) = k_{\beta}^{2}\psi(\chi), \qquad \mathbb{D} := -\ell \frac{6}{\varkappa} \partial_{\chi}^{2} + \vartheta v \frac{12\varkappa^{2}|V|}{\hbar^{2}}. \tag{13}$$

• Equation (13) turns out to be Besselian, and the mode functions are

$$\Psi_{\nu}(\beta,\chi) \coloneqq \sum_{i=1}^{2} c_{i} \varphi_{\nu}^{(i)}(\gamma) \sum_{i=1}^{2} a_{j} \mathbf{B}_{\nu}^{(j)}(\sigma), \quad \nu \ge 0; \quad (14)$$

$$\nu \coloneqq \sqrt{\frac{2\varkappa}{3}} \frac{1}{g} k_{\beta}, \quad \gamma \coloneqq \sqrt{\frac{3}{2\varkappa}} \frac{g}{1} \beta, \qquad \sigma^2 \coloneqq \frac{8\varkappa^3 |V| e^{g^{\jmath} \chi^{\chi}}}{\hbar^2 g^2}, \qquad \text{(15)}$$

$${}_{(+,+)} B^{(i)}_{\nu}(\sigma) \coloneqq \text{K and } I_{!\nu}(\sigma), \qquad {}_{(+,-)} B^{(i)}_{\nu}(\sigma) \coloneqq \text{F and } G_{!\nu}(\sigma),$$

$$_{(-,+)}B_{\nu}^{(i)}(\sigma)\coloneqq J \text{ and } Y_{\nu}(\sigma), \qquad _{(-,-)}B_{\nu}^{(i)}(\sigma)\coloneqq K \text{ and } I_{\nu}(\sigma).$$

- Adapted to imaginary order, $F_{\nu}(\sigma)$ and $G_{\nu}(\sigma)$ are defined in $^5;$ also see below.
- ⁵ T. M. Dunster. In: SIAM Journal on Mathematical Analysis 21.4 (1990), pp. 995–1018.



Physical mode functions 233

- Physical mode functions are expected to be regular on the boundary.
- (+,+): $|I_{i\nu}(\sigma)| \to +\infty$ as $\alpha \to +\infty$
- (-,+):
 - $\bullet \ \ \forall n \in \mathbb{N} \text{, } |\mathbf{Y}_n(\sigma)| \to +\infty \text{ as } \alpha \to -\infty.$
 - $\forall \nu \in \mathbb{R}^+ \setminus \mathbb{N}$, choose $J_{-\nu}$ instead of Y_{ν} , since $J_{\pm \nu}$ are also linearly independent.
 - $\bullet \ \forall \nu \in \mathbb{R}^+ \backslash \mathbb{N}, \ |J_{-\nu}(\sigma)| \to +\infty \ \text{as} \ \alpha \to -\infty.$
- $\bullet \ \ (-,-) \colon \left| \mathrm{K}_{\nu}(\sigma) \right| \to +\infty \ \text{as} \ \alpha \to -\infty; \ \left| \mathrm{I}_{\nu}(\sigma) \right| \to +\infty \ \text{as} \ \alpha \to +\infty$
- These are not to be included in the space of physical wave functions. $\forall \nu \geq 0$,
 - (+,+): $K_{\dagger\nu}(\sigma)$ survives
 - (+,-): F and $G_{i\nu}(\sigma)$ survives
 - (-,+): $J_{\nu}(\sigma)$ survives
 - (-, -): drops out



Matching quantum number with classical first integral

Principle of constructive interference

- Baustelle
- In order to match the quantum number k_{β} (or linearly, ν) with the classical first integral p_{β} , one may apply the principle of constructive interference⁶.
- Writing the mode function in the WKB form

$$\Psi_{k_{\beta}}(\beta,\chi) \sim \sqrt{\rho} \, \mathrm{e}^{\mathrm{i} S/\hbar}, \qquad S/\hbar \gg 1 \, \mathrm{and} \, k_{\beta} \gg 1, \qquad \qquad (16)$$

the principle demands that $\partial S/\partial k_{\beta}=0$ be equivalent to the classical trajectory.



⁶ C. Kiefer. In: *Nuclear Physics B* 341.1 (1990), pp. 273–293.

Matching quantum number with classical first integral

(+,+) as exemplar

• Fixing $\nu/\sigma > 1$, the asymptotic expansion reads

$$K_{i\nu}(\sigma) \sim \frac{\sqrt{2\pi}\cos\left(\sqrt{\nu^2 - \sigma^2} - \nu\arccos\frac{\nu}{\sigma} - \frac{\pi}{4}\right)}{(\nu^2 - \sigma^2)^{1/4}e^{\pi\nu/2}} + O\left(\frac{1}{\sigma}\right). \tag{17}$$

• There are two phases with opposite signs. Assuming c_i , a_j 's are real and applying the principle to $\Psi_{\nu}(\sigma)$, one has $\sigma/\nu = \mathrm{sech}(s_{\beta}\gamma)$, which matches the trajectory with C=0 if

$$\hbar k_{\beta} \equiv \hbar \sqrt{\frac{3}{2\pi}} g \hbar \nu = p_{\beta}, \tag{18}$$

- ullet Non-vanishing C can be compensated by the phase of c_i and a_j 's.
- Fixing $\nu/\sigma < 1$, the asymptotic expansion is not oscillatory, but exponential; it is not within the WKB regime.
- The conclusions also hold for $F_{"\nu}(\sigma)$, $G_{"\nu}(\sigma)$ for (+,-), and $J_{\nu}(\sigma)$ for (-,+).



Inner product for wave functions

Schrödinger product

- To make sense of amplitude and wave packet, an inner product is necessary
- A common starting point is the Schrödinger product

$$\left(\Psi_1,\Psi_2\right)_{\mathsf{S}} \coloneqq \int \mathrm{d}\chi \, \Psi_1^*(\beta,\chi) \Psi_2(\beta,\chi); \tag{19}$$

- $(\Psi,\Psi)_{\mathsf{S}}$ is positive-definite, and the integrand $\rho_{\mathsf{S}}(\beta,\chi)$ is non-negative
- The corresponding Schrödinger current is real but does not satisfy continuity equation $\dot{\rho}_{S} + \nabla \cdot \vec{j}_{S} = 0$, because eq. (11) is KG-like.
- $K_{\parallel\nu}^{-7}$ for (+,+), $F_{\parallel\nu}$ and $G_{\parallel\nu}$ for (+,-) can be proved to be orthogonal and complete among themselves, as well as can be normalised.
 - J_{ij} 's for (+, -) are not orthogonal

⁷ S. B. Yakubovich. In: Opuscula Math. 26.1 (2006), pp. 161–172, A. Passian et al. In: Journal of Mathematical Analysis and Applications 360.2 (2009), pp. 380–390, R. Szmytkowski and S. Bielski. In: Journal of Mathematical Analysis and Applications 365.1 (2010), pp. 195–197.

Peculiarity of the phantom model (-,+)

Orthogonality for mode functions; Hermiticity for operators

 $\bullet~J_{\nu}(\sigma)$'s are not orthogonal under the Schrödinger product

$$(\mathbf{J}_{\nu}, \mathbf{J}_{\tilde{\nu}})_{\mathsf{S}} \propto \int_{-\infty}^{+\infty} \mathrm{d}x \, \mathbf{J}_{\nu}^{*}(\mathbf{e}^{x}) \mathbf{J}_{\tilde{\nu}}(\mathbf{e}^{x}) = \frac{2 \mathrm{sin}(\pi(\nu - \tilde{\nu})/2)}{\pi(\nu^{2} - \tilde{\nu}^{2})},\tag{20}$$

therefore D in eq. (13) is not Hermitian (though we do not need it so far)

• \hat{p}_{χ}^2 is not Hermitian for $\{J_{\nu}(\sigma)\}$ under the Schrödinger product

$$\int_{-\infty}^{+\infty} \mathrm{d}x \, \mathrm{J}_{\nu}^* \left(-\partial_x^2 \mathrm{J}_{\tilde{\nu}} \right) - \int_{-\infty}^{+\infty} \mathrm{d}x \, \left(-\partial_x^2 \mathrm{J}_{\nu} \right)^* \mathrm{J}_{\tilde{\nu}} = \frac{2}{\pi} \sin \frac{\pi (\nu - \tilde{\nu})}{2} \,. \tag{21}$$

• In order to save Hermiticity for p_χ^2 and $\mathbb D$ and orthogonality of the modes under Schrödinger product, one can restrict

$$\nu = 2n + \nu_0, \qquad n \in \mathbb{N}, \quad \nu_0 \in [0, 2).$$
 (22)

• The classical trajectory is β -even; imposing the same condition fixes $\nu_0=1$.



Discritisation of the phantom model (-,+)

Levels of the phantom model are discretised

...if one imposes Hermiticity of squared momenta under the Schrödinger product

- This kind of subtlety on Hermiticity is named as self-adjoint extension, which
 arises already for infinite square well⁸.
- It also applies to x^{-2} potentials⁹, which is of cosmological relevance¹⁰.



⁸ G. Bonneau, J. Faraut, and G. Valent. In: American Journal of Physics 69.3 (2001), pp. 322-331.

⁹ A. M. Essin and D. J. Griffiths. In: American Journal of Physics 74.2 (2006), pp. 109–117,

V. S. Araujo, F. A. B. Coutinho, and J. F. Perez. In: *American Journal of Physics* 72.2 (2004), pp. 203–213.

M. Bouhmadi-López et al. In: *Physical Review D* 79.12 (2009).

Further inner products for wave functions

Klein-Gordon and Mostafazadeh product

• Since eq. (11) is KG-like, another popular choice is the KG product

$$\left(\boldsymbol{\varPsi}_{1},\boldsymbol{\varPsi}_{2}\right)_{\mathsf{KG}}^{g} \coloneqq \mathbf{\mathring{g}}\Big\{ \left(\boldsymbol{\varPsi}_{1},\dot{\boldsymbol{\varPsi}}_{2}\right)_{\mathsf{S}} - \left(\dot{\boldsymbol{\varPsi}}_{1},\boldsymbol{\varPsi}_{2}\right)_{\mathsf{S}} \Big\}, \qquad g > 0. \tag{23}$$

- $(\Psi,\Psi)^g_{\mathrm{KG}}$ is real but not positive-definite, so does the integrand ρ_{KG} ;
- The corresponding \vec{J}_{KG} is conserved $\dot{\rho}_{\text{KG}} + \nabla \cdot \vec{J}_{\text{KG}} = 0$ and real.
- Mostafazadeh¹¹ found a product for Hermitian D with positive spectrum:

$$(\varPsi_1,\varPsi_2)^\kappa_\mathsf{M} \coloneqq \kappa \Big\{ \big(\varPsi_1, \mathbb{D}^{+1/2}\varPsi_2\big)_\mathsf{S} + \big(\dot{\varPsi}_1, \mathbb{D}^{-1/2}\dot{\varPsi}_2\big)_\mathsf{S} \Big\}, \qquad \kappa > 0. \tag{24}$$

- $(\Psi,\Psi)^{\kappa}_{\mathrm{M}}$ is positive-definite, but the integrand $\rho^{\kappa}_{\mathrm{M}}$ is complex
- The corresponding $\vec{J}_{\mathsf{M}}^{\kappa}$ is conserved $\dot{\rho}_{\mathsf{M}}^{\kappa} + \nabla \cdot \vec{J}_{\mathsf{M}}^{\kappa} = 0$ but also complex.

A. Mostafazadeh. In: Classical and Quantum Gravity 20.1 (2002), pp. 155–171.



Mostafazadeh inner product and the corresponding density

- The real power of $\mathbb D$ is defined by spectral decomposition $\mathbb D^\gamma \coloneqq \sum_{\nu} \nu^{2\gamma} \mathbf P_{\nu}, \ \mathbf P_{\nu} \Psi \coloneqq \Psi_{\nu} (\Psi_{\nu}, \Psi)_{\mathsf S}.$
- It can be shown¹² that the density

$$\varrho_{\mathsf{M}}^{\kappa} := \kappa \left\{ \left| \mathbb{D}^{+1/4} \Psi \right|^2 + \left| \mathbb{D}^{-1/4} \dot{\Psi} \right|^2 \right\} \tag{25}$$

- is equivalent to ho_{M}^{κ} up to a boundary term $\int \mathrm{d}\chi \, \varrho_{\mathsf{M}}^{\kappa} = \int \mathrm{d}\chi \, \rho_{\mathsf{M}}^{\kappa} \equiv (\varPsi_1, \varPsi_2)_{\mathsf{M}}^{\kappa}$;
- is non-negative.
- The corresponding current $\vec{\mathcal{J}}_{\mathsf{M}}^{\kappa}$ is real but not conserved¹³.

B. Rosenstein and L. P. Horwitz. In: Journal of Physics A: Mathematical and General 18.11 (1985), pp. 2115–2121.



¹² A. Mostafazadeh and F. Zamani. In: Annals of Physics 321.9 (2006), pp. 2183–2209.

Wave packets with Gaussian amplitude for continuous spectrum

- It is difficult to find an amplitude such that the wave packet is Gaussian
- Instead, one can choose a Gaussian amplitude

$$A(\nu; \overline{\nu}, \sigma) := \left(\frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(\nu - \overline{\nu})^2}{2\sigma^2}\right)\right)^{1/2} \tag{26}$$

• \ln^{14} , $A(\nu; \overline{\nu}, \sigma/\sqrt{2})$ was chosen.

¹⁴ M. P. Dabrowski, C. Kiefer, and B. Sandhöfer. In: *Physical Review D* 74.4 (2006).



Wave packets with Poissonian amplitude for discrete spectrum

• One can choose a Poissonian amplitude

$$A_n(\overline{n}) := \left(\mathrm{e}^{-\overline{n}} \frac{\overline{n}^n}{n!}\right)^{1/2}$$

• \ln^{15} , $A_n(\overline{n}/\sqrt{2})$ was chosen.

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(27)

Phantom model

C. Kiefer. In: Nuclear Physics B 341.1 (1990), pp. 273-293.

Issues 233

• A normalising κ for $(\cdot,\cdot)^{\kappa}_{\mathsf{M}}$ has not yet been able to be evaluated, hence a quantitative comparison of $(\cdot,\cdot)_{\mathsf{S}}$ and $(\cdot,\cdot)^{\kappa}_{\mathsf{M}}$ is not yet possible.



Outlook 233

- PT-symmetric instead of phantom field
- Cosmological perturbation





Integration of the transformed first Friedmann equation $p_{\beta} \neq 0$

In order to integrate the equation under the implicitising gauge

$$s \frac{p_{\beta}^2}{12} \left(-\mathcal{E} \frac{\varkappa^{1/2}}{6} \left(\frac{\mathrm{d}\chi}{\mathrm{d}\beta} \right)^2 + \varkappa^{-1/2} \right) - \varkappa^{3/2} V e^{gs_{\chi}\chi} = 0, \tag{28}$$

one can substitute

$$\gamma \coloneqq \sqrt{\frac{3}{2\varkappa}} g\beta, \qquad \tilde{\sigma}^2 \coloneqq \frac{p_\beta^2}{12\varkappa^2 |V|} e^{-g s_\chi \chi}, \tag{29}$$

to get

$$\left(\frac{\mathrm{d}\tilde{\sigma}}{\mathrm{d}\gamma}\right)^{2} + \ell\left(\beta v - \tilde{\sigma}^{2}\right) = 0 \quad \Rightarrow \quad \frac{\mathrm{d}\gamma}{\mathrm{d}\tilde{\sigma}} = \pm \frac{1}{\sqrt{\ell\left(-\beta v + \tilde{\sigma}^{2}\right)}},\tag{30}$$

which is of the standard inverse hyperbolic / trigonometric form for (+,+), (+,-) and (-,+).



Integration of the separated mss. Wheeler-DeWitt equation 233

In order to integrate the separated minisuperspace Wheeler-DeWitt equation

$$\mathbb{D}\psi(\chi) = k_{\beta}^2 \psi(\chi), \qquad \mathbb{D} \coloneqq -\ell \frac{6}{\varkappa} \partial_{\chi}^2 + \mathfrak{z} v \frac{12\varkappa^2 |V|}{\hbar^2}, \tag{13 rev.}$$

one can transform

$$\nu \coloneqq \sqrt{\frac{2\varkappa}{3}} \frac{k_\beta}{q}, \qquad \sigma^2 \coloneqq \frac{8\varkappa^3 |V| \mathrm{e}^{g_{J_\chi \chi}}}{\hbar^2 q^2}, \tag{15 rev.}$$

to get

$$\sigma^2 \psi''(\sigma) + \sigma \psi'(\sigma) + \ell(\nu^2 - \beta \nu \sigma^2) \psi(\sigma) = 0, \tag{31}$$

which is of the standard Besselian form





Allgemeines

- Mit diesem beamer theme ist es möglich, Präsentationen in LATEX mit der Beamer-Klasse zu erstellen, die dem Corporate Design der Universität zu Köln entsprechen
- Auf die Beamer-Klasse wird in diesem Dokument nicht n\u00e4her eingegangen, n\u00e4here Informationen finden Sie unter http://latex-beamer.sourceforge.net/



Laden des Themes

Das Theme kann mit den folgenden Optionen geladen werden

Die Fußzeile

- Es stehen verschiedene Fußzeilen zur Auswahl, die als Option beim Laden des themes übergeben werden:
 - Balken mit allen Fakultätsfarben (Option uk)
 - Balken in jeweils einer Fakultätsfarbe (Optionen wiso, jura, medizin, philo, matnat, human, verw)¹⁶
- "'Universität zu Köln" sowie der Name der Fakultät sind im Theme definiert, das Institut oder Seminar kann mit dem Befehl \institute{} festgelegt werden
- Die Optionen sind im Quellcode dieser Präsentation dokumentiert

¹⁶Es werden die offiziellen RGB-Werte aus dem 2-D Handbuch Corporate Design verwendet.



Englische Präsentationen

- Der Universitäts- sowie die Fakultätsnamen werden standardmäßig auf Deutsch angezeigt.
- Übergeben Sie dem Paket babel die Option english, so werden diese Namen entsprechen angepasst.
- Die Übersetzungen können in der Theme-Datei beamerthemeUzK.sty geändert werden



block-Umgebungen

Standard (block)

Verwendet die Farbe "Blaugrau Mittel" als Blocktitel-Hintergrund

exampleblock

Bei Verwendung der Fußzeile mit allen Fakultätsfarben Titelhintergrund in Wiso-Grün, sonst in der jeweiligen Fakultätsfarbe

alertblock

Verwendet das Rot der Folientitel



Installation

- Das Theme besteht aus den Dateien beamerthemeUzK.sty und beamercolorthemeUzK.sty sowie den Grafikdateien logo.pdf und logo-small.pdf.
- Das Theme kann auf zwei Arten verwendet werden:
 - Die vier Dateien werden in den selben Ordner wie die zu erstellende Präsentation gelegt
 - 2. Die vier Dateien werden im lokalen texmf-Baum abgelegt
- Die zweite Variante ist der ersten vorzuziehen, da das Theme so an einem zentralen Ort vorliegt



ToDo

Was noch zu tun ist...

- Erstellen einer eigenen Titelseite
- ...

