

Integrable Cosmological Models with Liouville Fields

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Outline

1. Introduction
2. Classical model and the implicitised trajectories
Lagrangian formalism
3. Quantised model and the wave packets
Dirac quantisation
Semi-classical approximation
Inner product and wave packet
4. Conclusions



Introduction

Introduction



- Flat Robertson–Walker metric $ds^2 = -N^2(t) dt^2 + \varkappa^{-1} e^{2\alpha(t)} d\Omega_3^2$
 - $\varkappa := 8\pi G$; $d\Omega_3^2$ dimensionless spacial metric
- Real Klein–Gordon field with potential $V e^{\lambda\phi}$ (Liouville), where $\lambda, V \in \mathbb{R}$, and kinetic term with sign $\ell = \pm 1$ (quintessence / phantom model).
- Total action $\mathcal{S} := S_{\text{EH}} + S_{\text{GHY}} + S_{\text{L}} = \int d\Omega_3^2 \int dt L$,

$$L := \varkappa^{3/2} N e^{3\alpha} \left(-\frac{3}{\varkappa} \frac{\dot{\alpha}^2}{N^2} + \ell \frac{\dot{\phi}^2}{2N^2} - V e^{\lambda\phi} \right), \quad (1)$$

in which dot means d/dt and $\ell = \pm 1$.



Decoupling the variables

Via *rescaled* special orthogonal transformation

- Setting $\overline{N} := N e^{-3\alpha}$, eq. (1) transforms to

$$L = \kappa^{3/2} \overline{N} \left(-\frac{3}{\kappa} \frac{\dot{\alpha}^2}{\overline{N}^2} + \ell \frac{\dot{\phi}^2}{2\overline{N}^2} - V e^{\lambda\phi+6\alpha} \right) \quad (2)$$

- Defining $\Delta := \lambda^2 - 6\ell\kappa$, $\jmath := \text{sgn } \Delta$ and $g := \jmath\sqrt{|\Delta|} \equiv \jmath\sqrt{\jmath\Delta}$, the *rescaled* special orthogonal transformation

$$\begin{pmatrix} \alpha \\ \phi \end{pmatrix} = \frac{\jmath}{g} \begin{pmatrix} \lambda & -\ell\kappa \\ -6 & \lambda \end{pmatrix} \begin{pmatrix} \jmath_\beta \beta \\ \jmath_\chi \chi \end{pmatrix} \quad \text{where } \jmath_\beta, \jmath_\chi = \pm 1 \quad (3)$$

gives the decoupled Lagrangian

$$L = \kappa^{3/2} \overline{N} \left(-\jmath \frac{3}{\kappa} \frac{\dot{\beta}^2}{\overline{N}^2} + \ell \jmath \frac{\dot{\chi}^2}{2\overline{N}^2} - V e^{\jmath_\chi g \chi} \right). \quad (4)$$

- The Euler–Lagrange equations w.r.t. \overline{N} , β and χ will be called the trsfed. 1st, 2nd Friedmann eqs. and the Klein–Gordon eq., respectively.



- Beyond isotropy
 - Bianchi Type-I: under investigation
- Beyond one Liouville field
 - Two exponential potentials: $V_1 = V_2$ and special λ_i
 - Multiple Liouville fields: mixing kinetic terms¹
- Beyond classic matter
 - Non-Hermitian, PT -symmetric Liouville fields²: may avoid big-rip etc.

¹ A. A. Andrianov, O. O. Novikov, and C. Lan. In: *Theoretical and Mathematical Physics* 184.3 (2015), pp. 1224–1233.

² A. A. Andrianov et al. In: *International Journal of Modern Physics D* 19.01 (2010), pp. 97–111, A. A. Andrianov, C. Lan, and O. O. Novikov. In: *Springer Proceedings in Physics*. Springer International Publishing, 2016, pp. 29–44.



Implicitised integration

General integral for $p_\beta \neq 0$

- Since β is cyclic in eq. (4), the trsfed. 2nd Friedmann eq. can be integrated³

$$\text{const.} \equiv p_\beta := \frac{\partial L}{\partial \dot{\beta}} = -6\mathfrak{s}\kappa^{1/2} \frac{\dot{\beta}}{\bar{N}} \equiv -6\mathfrak{s}\mathfrak{s}_\beta \frac{\kappa^{1/2}}{g} \frac{\lambda\dot{\alpha} + \ell\kappa\dot{\phi}}{\bar{N}}. \quad (5)$$

- For $p_\beta \neq 0$, fixing the *implicitising gauge* $\bar{N} = -6\mathfrak{s}\sqrt{\kappa}\dot{\beta}/p_\beta$, the trsfed. 1st Friedmann equation can be integrated

$$\mathrm{e}^{\mathfrak{s}_\chi g \chi} = \frac{p_\beta^2}{12\kappa^2|V|} S^2 \left(\mathfrak{s}_\beta \sqrt{\frac{3}{2\kappa}} g\beta + C \right), \quad (6)$$

in which $\mathfrak{v} := \text{sgn } V$, (sgn, sgn) means $(\ell, \mathfrak{s}\mathfrak{v})$, and

$$\begin{aligned} (+,+)S(\gamma) &:= \text{sech}(\gamma), & (+,-)S(\gamma) &:= \text{csch}(\gamma), \\ (-,+)S(\gamma) &:= \sec(\gamma), & (-,-)S(\gamma) &:= \mathfrak{i} \csc(\gamma). \end{aligned} \quad (7)$$

³The same first integral in Hamiltonian formalism has been found in C. Lan. PhD thesis. Saint Petersburg State University, 2016, A. A. Andrianov, O. O. Novikov, and C. Lan. In: *Theoretical and Mathematical Physics* 184.3 (2015), pp. 1224–1233.



Trajectories

Quintessence (+, +)



Trajectories

Quintessence (+, -)



Trajectories

Phantom $(-, +)$



Integration

Discussions

- The integrals are consistent with the trsfed. Klein–Gordon equation.
- The integral for $(+, +)$
 - has two asymptotes
- The implicitised integral for $(+, -)$
 - contains two distinct solutions
 - has three asymptotes
- The implicitised integral for $(-, +)$
 - is β -even for $C = 0$
 - contains infinite distinct solutions
 - has infinite asymptotes, which are pairwise parallel
- The integral for $(-, -)$
 - is not real
- The implicitised integral enables one to compare trajectories with wave functions, see below.



Implicitised integration

Specific integral for $p_\beta = 0$

- For $p_\beta = 0$, one has $\beta \equiv \beta_0$ or $\phi - \phi_0 = -\ell\lambda\alpha/\kappa$, which is the familiar power-law special solution⁴.
- Further integrating the first Friedmann equation demands $(+, -)$ or $(-, +)$ to guarantee $\bar{N} > 0$, and the result is automatically consistent with the transferred Klein–Gordon equation.
- Fixing $\bar{N} = (2\kappa^2|V|)^{-1/2}$ yields

$$e^{g_{\mathcal{J}}\chi} = \left(\frac{2\kappa}{g(t-t_0)} \right)^2. \quad (8)$$

⁴ M. P. Dąbrowski, C. Kiefer, and B. Sandhöfer. In: *Physical Review D* 74.4 (2006).



Trajectories

Quintessence (+, +)



Trajectories

Quintessence (+, -)



Trajectories

Phantom $(-, +)$



- The primary Hamiltonian and the Hamiltonian constraint reads

$$H^p = \overline{N} H_\perp + v^{\overline{N}} p_{\overline{N}}, \quad (9)$$

$$H_\perp := -\imath \frac{p_\beta^2}{12\kappa^{1/2}} + \ell \imath \frac{p_\chi^2}{2\kappa^{3/2}} + \kappa^{3/2} V e^{g_\imath \chi}. \quad (10)$$

- Applying the Dirac quantisation rules with Laplace–Beltrami operator, one gets the mss. Wheeler–DeWitt eq. with (β, χ)

$$0 = \widehat{H}_\perp \Psi(\beta, \chi) := \left(\imath \frac{\hbar^2}{12\kappa^{1/2}} \partial_\beta^2 - \ell \imath \frac{\hbar^2}{2\kappa^{3/2}} \partial_\chi^2 + \kappa^{3/2} V e^{g_\imath \chi} \right) \Psi. \quad (11)$$

- Equation (11) is KG-like, hyperbolic for $\ell = +1$ and *elliptic* for $\ell = -1$.



Separation of the variables and mode functions

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- Writing $\Psi(\beta, \chi) = \varphi(\beta)\psi(\chi)$, eq. (11) can be separated into

$$\partial_\beta^2 \varphi(\beta) = k_\beta^2 \varphi(\beta); \quad (12)$$

$$\mathbb{D}\psi(\chi) = k_\beta^2 \psi(\chi), \quad \mathbb{D} := -\ell^2 \frac{6}{\varkappa} \partial_\chi^2 + \imath v \frac{12\varkappa^2 |V|}{\hbar^2}. \quad (13)$$

- Equation (13) turns out to be Besselian, and the mode functions are

$$\Psi_\nu(\beta, \chi) := \sum_{i=1}^2 c_i \varphi_\nu^{(i)}(\gamma) \sum_{j=1}^2 a_j B_\nu^{(j)}(\sigma), \quad \nu \geq 0; \quad (14)$$

$$\nu := \sqrt{\frac{2\varkappa}{3}} \frac{1}{g} k_\beta, \quad \gamma := \sqrt{\frac{3}{2\varkappa}} \frac{g}{1} \beta, \quad \sigma^2 := \frac{8\varkappa^3 |V| e^{g\imath \chi}}{\hbar^2 g^2}, \quad (15)$$

$$_{(+,+)} B_\nu^{(i)}(\sigma) := K \text{ and } I_{i\nu}(\sigma), \quad _{(+,-)} B_\nu^{(i)}(\sigma) := F \text{ and } G_{i\nu}(\sigma),$$

$$_{(-,+)} B_\nu^{(i)}(\sigma) := J \text{ and } Y_\nu(\sigma), \quad _{(-,-)} B_\nu^{(i)}(\sigma) := K \text{ and } I_\nu(\sigma).$$

- Adapted to imaginary order, $F_\nu(\sigma)$ and $G_\nu(\sigma)$ are defined in⁵; also see below.

⁵ T. M. Dunster. In: *SIAM Journal on Mathematical Analysis* 21.4 (1990), pp. 995–1018.



- Physical mode functions are expected to be regular on the boundary.
- $(+, +)$: $|I_{i\nu}(\sigma)| \rightarrow +\infty$ as $\alpha \rightarrow +\infty$
- $(-, +)$:
 - $\forall n \in \mathbb{N}$, $|Y_n(\sigma)| \rightarrow +\infty$ as $\alpha \rightarrow -\infty$.
 - $\forall \nu \in \mathbb{R}^+ \setminus \mathbb{N}$, choose $J_{-\nu}$ instead of Y_ν , since $J_{\pm\nu}$ are also linearly independent.
 - $\forall \nu \in \mathbb{R}^+ \setminus \mathbb{N}$, $|J_{-\nu}(\sigma)| \rightarrow +\infty$ as $\alpha \rightarrow -\infty$.
- $(-, -)$: $|K_\nu(\sigma)| \rightarrow +\infty$ as $\alpha \rightarrow -\infty$; $|I_\nu(\sigma)| \rightarrow +\infty$ as $\alpha \rightarrow +\infty$
- These are not to be included in the space of physical wave functions.
 $\forall \nu \geq 0$,
 - $(+, +)$: $K_{i\nu}(\sigma)$ survives
 - $(+, -)$: F and $G_{i\nu}(\sigma)$ survives
 - $(-, +)$: $J_\nu(\sigma)$ survives
 - $(-, -)$: drops out



Matching quantum number with classical first integral

Principle of constructive interference

- Baustelle
- In order to match the quantum number k_β (or linearly, ν) with the classical first integral p_β , one may apply the *principle of constructive interference*⁶.
- Writing the mode function in the WKB form

$$\Psi_{k_\beta}(\beta, \chi) \sim \sqrt{\rho} e^{iS/\hbar}, \quad S/\hbar \gg 1 \text{ and } k_\beta \gg 1, \quad (16)$$

the principle demands that $\partial S/\partial k_\beta = 0$ be equivalent to the classical trajectory.

⁶ C. Kiefer. In: *Nuclear Physics B* 341.1 (1990), pp. 273–293.



Matching quantum number with classical first integral

(+, +) as exemplar

- Fixing $\nu/\sigma > 1$, the asymptotic expansion reads

$$K_{i\nu}(\sigma) \sim \frac{\sqrt{2\pi} \cos\left(\sqrt{\nu^2 - \sigma^2} - \nu \arccos \frac{\nu}{\sigma} - \frac{\pi}{4}\right)}{(\nu^2 - \sigma^2)^{1/4} e^{\pi\nu/2}} + O\left(\frac{1}{\sigma}\right). \quad (17)$$

- There are two phases with opposite signs. Assuming c_i, a_j 's are real and applying *the principle* to $\Psi_\nu(\sigma)$, one has $\sigma/\nu = \operatorname{sech}(j_\beta \gamma)$, which matches the trajectory with $C = 0$ if

$$\hbar k_\beta \equiv \hbar \sqrt{\frac{3}{2\mathcal{N}}} g \hbar \nu = p_\beta, \quad (18)$$

- Non-vanishing C can be compensated by the phase of c_i and a_j 's.
- Fixing $\nu/\sigma < 1$, the asymptotic expansion is not oscillatory, but exponential; it is not within the WKB regime.
- The conclusions also hold for $F_{i\nu}(\sigma)$, $G_{i\nu}(\sigma)$ for $(+, -)$, and $J_\nu(\sigma)$ for $(-, +)$.



Inner product for wave functions

Schrödinger product

- To make sense of amplitude and wave packet, an inner product is necessary
- A common starting point is the *Schrödinger product*

$$(\Psi_1, \Psi_2)_S := \int d\chi \Psi_1^*(\beta, \chi) \Psi_2(\beta, \chi); \quad (19)$$

- $(\Psi, \Psi)_S$ is **positive-definite**, and the integrand $\rho_S(\beta, \chi)$ is **non-negative**
- The corresponding Schrödinger current is **real** but **does not satisfy continuity equation** $\dot{\rho}_S + \nabla \cdot \vec{j}_S = 0$, because eq. (11) is KG-like.
- $K_{i\nu}$ ⁷ for $(+, +)$, $F_{i\nu}$ and $G_{i\nu}$ for $(+, -)$ can be proved to be **orthogonal** and **complete** among themselves, as well as **can be normalised**.
 - $J_{i\nu}$'s for $(+, -)$ are **not orthogonal**

⁷ S. B. Yakubovich. In: *Opuscula Math.* 26.1 (2006), pp. 161–172, A. Passian et al. In: *Journal of Mathematical Analysis and Applications* 360.2 (2009), pp. 380–390, R. Szmytkowski and S. Bielski. In: *Journal of Mathematical Analysis and Applications* 365.1 (2010), pp. 195–197.



Peculiarity of the phantom model $(-, +)$

Orthogonality for mode functions; Hermiticity for operators

- $J_\nu(\sigma)$'s are **not orthogonal** under the Schrödinger product

$$(J_\nu, J_{\tilde{\nu}})_S \propto \int_{-\infty}^{+\infty} dx J_\nu^*(e^x) J_{\tilde{\nu}}(e^x) = \frac{2 \sin(\pi(\nu - \tilde{\nu})/2)}{\pi(\nu^2 - \tilde{\nu}^2)}, \quad (20)$$

therefore \mathbb{D} in eq. (13) is **not Hermitian** (though we do not need it so far)

- \hat{p}_χ^2 is **not Hermitian** for $\{J_\nu(\sigma)\}$ under the Schrödinger product

$$\int_{-\infty}^{+\infty} dx J_\nu^*(-\partial_x^2 J_{\tilde{\nu}}) - \int_{-\infty}^{+\infty} dx (-\partial_x^2 J_\nu)^* J_{\tilde{\nu}} = \frac{2}{\pi} \sin \frac{\pi(\nu - \tilde{\nu})}{2}. \quad (21)$$

- In order to save Hermiticity for p_χ^2 and \mathbb{D} and orthogonality of the modes under Schrödinger product, one can restrict

$$\nu = 2n + \nu_0, \quad n \in \mathbb{N}, \quad \nu_0 \in [0, 2). \quad (22)$$

- The classical trajectory is β -even; imposing the same condition fixes $\nu_0 = 1$.



Levels of the phantom model are discretised

...if one imposes Hermiticity of squared momenta under the Schrödinger product

- This kind of subtlety on Hermiticity is named as *self-adjoint extension*, which arises already for infinite square well⁸.
- It also applies to x^{-2} potentials⁹, which is of cosmological relevance¹⁰.

⁸ G. Bonneau, J. Faraut, and G. Valent. In: *American Journal of Physics* 69.3 (2001), pp. 322–331.

⁹ A. M. Essin and D. J. Griffiths. In: *American Journal of Physics* 74.2 (2006), pp. 109–117,
V. S. Araujo, F. A. B. Coutinho, and J. F. Perez. In: *American Journal of Physics* 72.2 (2004),
pp. 203–213.

¹⁰ M. Bouhmadi-López et al. In: *Physical Review D* 79.12 (2009).



Further inner products for wave functions

Klein-Gordon and Mostafazadeh product

- Since eq. (11) is KG-like, another popular choice is the KG product

$$(\Psi_1, \Psi_2)_{\text{KG}}^g := ig \left\{ (\Psi_1, \dot{\Psi}_2)_S - (\dot{\Psi}_1, \Psi_2)_S \right\}, \quad g > 0. \quad (23)$$

- $(\Psi, \Psi)_{\text{KG}}^g$ is **real** but **not positive-definite**, so does the integrand ρ_{KG} ;
- The corresponding \vec{J}_{KG} is **conserved** $\dot{\rho}_{\text{KG}} + \nabla \cdot \vec{J}_{\text{KG}} = 0$ and **real**.
- Mostafazadeh¹¹ found a product *for Hermitian \mathbb{D} with positive spectrum*:

$$(\Psi_1, \Psi_2)_{\text{M}}^{\kappa} := \kappa \left\{ (\Psi_1, \mathbb{D}^{+1/2} \Psi_2)_S + (\dot{\Psi}_1, \mathbb{D}^{-1/2} \dot{\Psi}_2)_S \right\}, \quad \kappa > 0. \quad (24)$$

- $(\Psi, \Psi)_{\text{M}}^{\kappa}$ is **positive-definite**, but the integrand ρ_{M}^{κ} is **complex**
- The corresponding $\vec{J}_{\text{M}}^{\kappa}$ is **conserved** $\dot{\rho}_{\text{M}}^{\kappa} + \nabla \cdot \vec{J}_{\text{M}}^{\kappa} = 0$ but also **complex**.

¹¹ A. Mostafazadeh. In: *Classical and Quantum Gravity* 20.1 (2002), pp. 155–171.



Mostafazadeh inner product and the corresponding density

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- The real power of \mathbb{D} is defined by spectral decomposition $\mathbb{D}^\gamma := \sum_\nu \nu^{2\gamma} \mathbf{P}_\nu$,
 $\mathbf{P}_\nu \Psi := \Psi_\nu (\Psi_\nu, \Psi)_S$.

- It can be shown¹² that the density

$$\varrho_M^\kappa := \kappa \left\{ |\mathbb{D}^{+1/4} \Psi|^2 + |\mathbb{D}^{-1/4} \dot{\Psi}|^2 \right\} \quad (25)$$

- is equivalent to ρ_M^κ up to a boundary term $\int d\chi \varrho_M^\kappa = \int d\chi \rho_M^\kappa \equiv (\Psi_1, \Psi_2)_M^\kappa$;
- is non-negative.
- The corresponding current $\vec{\mathcal{J}}_M^\kappa$ is real but not conserved¹³.

¹² A. Mostafazadeh and F. Zamani. In: *Annals of Physics* 321.9 (2006), pp. 2183–2209.

¹³ B. Rosenstein and L. P. Horwitz. In: *Journal of Physics A: Mathematical and General* 18.11 (1985), pp. 2115–2121.



Wave packets with Gaussian amplitude for continuous spectrum

Quintessence models

- It is difficult to find an amplitude such that the wave packet is Gaussian
- Instead, one can choose a Gaussian amplitude

$$A(\nu; \bar{\nu}, \sigma) := \left(\frac{1}{\sqrt{2\pi} \sigma} \exp \left(-\frac{(\nu - \bar{\nu})^2}{2\sigma^2} \right) \right)^{1/2} \quad (26)$$



Wave packets with Poissonian amplitude for discrete spectrum

Phantom model

- One can choose a Poissonian amplitude

$$A_n(\bar{n}) := \left(e^{-\bar{n}} \frac{\bar{n}^n}{n!} \right)^{1/2} \quad (27)$$

- In¹⁴, $A_n(\bar{n}/\sqrt{2})$ was chosen

¹⁴ C. Kiefer. In: *Nuclear Physics B* 341.1 (1990), pp. 273–293.



- A normalising κ for $(\cdot, \cdot)_M^\kappa$ has not yet been able to be evaluated, hence a quantitative comparison of $(\cdot, \cdot)_S$ and $(\cdot, \cdot)_M^\kappa$ is not yet possible.



- PT -symmetric instead of phantom field
- Cosmological perturbation



Integration of the transformed first Friedmann equation

$$p_\beta \neq 0$$

In order to integrate the equation under the implicitising gauge

$$\mathcal{J} \frac{p_\beta^2}{12} \left(-\ell \frac{\kappa^{1/2}}{6} \left(\frac{d\chi}{d\beta} \right)^2 + \kappa^{-1/2} \right) - \kappa^{3/2} V e^{g\mathcal{J}_\chi \chi} = 0, \quad (28)$$

one can substitute

$$\gamma := \sqrt{\frac{3}{2\kappa}} g\beta, \quad \tilde{\sigma}^2 := \frac{p_\beta^2}{12\kappa^2 |V|} e^{-g\mathcal{J}_\chi \chi}, \quad (29)$$

to get

$$\left(\frac{d\tilde{\sigma}}{d\gamma} \right)^2 + \ell(\mathcal{J}v - \tilde{\sigma}^2) = 0 \quad \Rightarrow \quad \frac{d\gamma}{d\tilde{\sigma}} = \pm \frac{1}{\sqrt{\ell(-\mathcal{J}v + \tilde{\sigma}^2)}}, \quad (30)$$

which is of the standard inverse hyperbolic / trigonometric form for $(+, +)$, $(+, -)$ and $(-, +)$.



Integration of the separated mss. Wheeler–DeWitt equation

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In order to integrate the separated minisuperspace Wheeler–DeWitt equation

$$\mathbb{D}\psi(\chi) = k_\beta^2 \psi(\chi), \quad \mathbb{D} := -\ell^2 \frac{6}{\kappa} \partial_\chi^2 + \imath \nu \frac{12\kappa^2 |V|}{\hbar^2}, \quad (13 \text{ rev.})$$

one can transform

$$\nu := \sqrt{\frac{2\kappa}{3}} \frac{k_\beta}{g}, \quad \sigma^2 := \frac{8\kappa^3 |V| e^{g\imath_\chi \chi}}{\hbar^2 g^2}, \quad (15 \text{ rev.})$$

to get

$$\sigma^2 \psi''(\sigma) + \sigma \psi'(\sigma) + \ell(\nu^2 - \imath \nu \sigma^2) \psi(\sigma) = 0, \quad (31)$$

which is of the standard Besselian form.





- Mit diesem *beamer theme* ist es möglich, Präsentationen in \LaTeX mit der Beamer-Klasse zu erstellen, die dem Corporate Design der Universität zu Köln entsprechen
- Auf die Beamer-Klasse wird in diesem Dokument nicht näher eingegangen, nähere Informationen finden Sie unter <http://latex-beamer.sourceforge.net/>



Das Theme kann mit den folgenden Optionen geladen werden

```
\usetheme[%  
% uk,      %% Farben aller Fakultaeten  
wiso,      %% Wiso-Fakultaet  
% jura,    %% Rechtswissenschaftliche Fakultaet  
% medizin, %% Medizinische Fakultaet  
% philo,   %% Philosophische Fakultaet  
% matnat,  %% Mathematisch-Naturwissenschaftliche Fakultaet  
% human,   %% Humanwissenschaftliche Fakultaet  
% verw,    %% Universitaetsverwaltung  
{UzK}
```



- Es stehen verschiedene Fußzeilen zur Auswahl, die als Option beim Laden des *themes* übergeben werden:
 - Balken mit allen Fakultätsfarben (Option uk)
 - Balken in jeweils einer Fakultätsfarbe (Optionen wiso, jura, medizin, philo, matnat, human, verw)¹⁵
- "'Universität zu Köln"' sowie der Name der Fakultät sind im Theme definiert, das Institut oder Seminar kann mit dem Befehl `\institute{}` festgelegt werden
- Die Optionen sind im Quellcode dieser Präsentation dokumentiert

¹⁵Es werden die offiziellen RGB-Werte aus dem 2-D Handbuch Corporate Design verwendet.



- Der Universitäts- sowie die Fakultätsnamen werden standardmäßig auf Deutsch angezeigt.
- Übergeben Sie dem Paket `babel` die Option `english`, so werden diese Namen entsprechen angepasst.
- Die Übersetzungen können in der Theme-Datei `beamerthemeUzK.sty` geändert werden



block-Umgebungen

Standard (block)

Verwendet die Farbe "Blaugrau Mittel" als Blocktitel-Hintergrund

exampleblock

Bei Verwendung der Fußzeile mit allen Fakultätsfarben Titelhintergrund in Wiso-Grün, sonst in der jeweiligen Fakultätsfarbe

alertblock

Verwendet das Rot der Folientitel



Installation

- Das Theme besteht aus den Dateien `beamerthemeUzK.sty` und `beamercolorthemeUzK.sty` sowie den Grafikdateien `logo.pdf` und `logo-small.pdf`.
- Das Theme kann auf zwei Arten verwendet werden:
 1. Die vier Dateien werden in den selben Ordner wie die zu erstellende Präsentation gelegt
 2. Die vier Dateien werden im lokalen *texmf*-Baum abgelegt
- Die zweite Variante ist der ersten vorzuziehen, da das Theme so an einem zentralen Ort vorliegt



Was noch zu tun ist...

- Erstellen einer eigenen Titelseite
- ...

