Tool-kit for the coframe formalism

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1 Introduction

This note is intended to be pragmatic, collecting a set of toolkits for the calculation of field theories without and with gravitation. The physical motivation will be discussed elsewhere.

2 Complex scalar field

The action reads

$$S = \int -\mathbb{d}\phi^* \wedge \star\mathbb{d}\phi - m^2\phi^* \wedge \star\phi \tag{1}$$

$$= \int -\frac{1}{2}(\mathbb{d}\phi^* \wedge \star\mathbb{d}\phi + \mathbb{d}\phi \wedge \star\mathbb{d}\phi^*) - \frac{1}{2}m^2(\phi^* \wedge \star\phi + \phi \wedge \star\phi^*) \tag{2}$$

Variation commutes with exterior derivative

$$[\delta, \mathbb{d}]\phi = 0\,. \tag{3}$$

In the absence of gravitation, variation also commutes with Hodge star. A generic variation of the kinetic terms therefore reads

$$\begin{aligned}\delta(\mathrm{d}\phi^* \wedge \star\mathrm{d}\phi) = \mathrm{d}(\delta\phi^* \wedge \star\mathrm{d}\phi + \delta\phi \wedge \star\mathrm{d}\phi^*)\\ + \delta\phi^* \wedge \star\mathrm{d}^\dagger\mathrm{d}\phi + \delta\phi \wedge \star\mathrm{d}^\dagger\mathrm{d}\phi^*\,,\end{aligned} \tag{4}$$

where the codifferential is defined in eq. (52). A generic variation of the action reads

$$\begin{aligned}\delta S = \int \mathrm{d}(-\delta\phi^* \wedge \star\mathrm{d}\phi - \delta\phi \wedge \star\mathrm{d}\phi^*)\\ - \delta\phi^* \wedge \star(\mathrm{d}^\dagger\mathrm{d} + m^2)\phi - \delta\phi \wedge \star(\mathrm{d}^\dagger\mathrm{d} + m^2)\phi^*\,.\end{aligned} \tag{5}$$

2.1 $\mathsf{U}(1)$ Noether current

The action is invariant

$$\delta_\lambda S \equiv 0 \tag{6}$$

under the rigid transformation

$$\phi \to \mathsf{e}^{-\mathrm{i}e\lambda}\phi\,, \qquad \phi^* \to \mathsf{e}^{+\mathrm{i}e\lambda}\phi^*\,. \tag{7}$$

When the equations of motion are satisfied, infinitesimal transformation leads to

$$\begin{aligned}0 &= \int \lambda\, \mathrm{d}\mathfrak{J}_0\,,\\ \mathfrak{J}_0 &:= \mathrm{i}e(\phi^* \wedge \star\mathrm{d}\phi - \phi \wedge \star\mathrm{d}\phi^*)\,,\end{aligned} \tag{8}$$

which is the Noether current, a twisted 3-form, satisfying the continuity equation

$$\mathrm{d}\mathfrak{J}_0 = 0\,. \tag{9}$$

2.2 $\mathsf{T}(D)$ Noether current

The action is invariant up to a total differential

$$\begin{aligned}\delta_\lambda S &= \int -\lambda\, \mathrm{d}(z \lrcorner \mathfrak{L})\\ &= \int \lambda\, \frac{1}{2}\, \mathrm{d}\{+z \lrcorner [\mathrm{d}\phi^* \wedge \star\mathrm{d}\phi + \mathrm{d}\phi \wedge \star\mathrm{d}\phi^* + m^2(\phi^* \star \phi + \phi \star \phi^*)]\}\,,\end{aligned} \tag{10}$$

under the rigid infinitesimal transformation

$$\delta\phi = -\lambda\, \pounds_z\phi = -\lambda\, z \lrcorner \mathrm{d}\phi\,, \qquad \delta\phi^* = -\lambda\, \pounds_z\phi^* = -\lambda\, z \lrcorner \mathrm{d}\phi^*\,. \tag{11}$$

Note the minus sign, which has to do with our mostly-positive metric convention $(-,+,\dots,+)$.

When the equations of motion are satisfied, infinitesimal transformation leads to

$$\delta_\lambda S = \int \lambda\, \mathrm{d}\{+(z \lrcorner \mathrm{d}\phi^*) \wedge \star\mathrm{d}\phi + (z \lrcorner \mathrm{d}\phi) \wedge \star\mathrm{d}\phi^*\}\,. \tag{12}$$

Taking the difference with eq. (10) and setting $z = e_\alpha$ yield

$$0 = \int \lambda \, \mathrm{d}\mathfrak{T}_\alpha , \tag{13}$$

$$\begin{aligned} 2\mathfrak{T}_\alpha := &+ (e_\alpha \lrcorner \, \mathrm{d}\phi^*) \wedge \star\mathrm{d}\phi + \mathrm{d}\phi^* \wedge (e_\alpha \lrcorner \star\mathrm{d}\phi) \\ &+ (e_\alpha \lrcorner \, \mathrm{d}\phi) \wedge \star\mathrm{d}\phi^* + \mathrm{d}\phi \wedge (e_\alpha \lrcorner \star\mathrm{d}\phi^*) \\ &- m^2(\phi^* \, e_\alpha \lrcorner \star\phi + \phi \, e_\alpha \star \phi^*) \, . \end{aligned} \tag{14}$$

The continuity equation reads

$$\mathrm{d}\mathfrak{T}_\alpha = 0 \, . \tag{15}$$

In components,

$$\mathfrak{T}_\alpha = + \left[e_\alpha(\phi^*) e^\beta(\phi) + e_\alpha(\phi) e^\beta(\phi^*) \right] \epsilon_\beta - \left[e^\beta(\phi^*) e_\beta(\phi^*) + m^2 \phi^* \phi \right] \epsilon_\alpha \, . \tag{16}$$

3 Pure electromagnetic field

The action reads

$$S = \int -\frac{1}{2} F \wedge \star F \, . \tag{17}$$

Variation of the Lagrangian reads

$$\delta(F \wedge \star F) = 2 \, \delta F \wedge \star F \tag{18}$$

$$= 2 \, \mathrm{d}(\delta A \wedge \star F) + 2 \, \delta A \wedge \mathrm{d}\star F \, . \tag{19}$$

Note that this is not gauge invariant.

3.1 Lorenz gauge

The Laplace–de Rham operator, or in our Lorentzian metric signature the d'Alembertian

$$\Box^2 := \left(\mathrm{d} + \mathrm{d}^\dagger \right)^2 = \mathrm{d}\mathrm{d}^\dagger + \mathrm{d}^\dagger \mathrm{d} \, . \tag{20}$$

$$\mathrm{d}\star F = \mathrm{d} \star \mathrm{d}A = \star(-)^2 \star^{-1} \mathrm{d} \star \mathrm{d}A = \star \mathrm{d}^\dagger \mathrm{d}A = \star(\Box^2 - \mathrm{d}\mathrm{d}^\dagger) A \, . \tag{21}$$

One would like to have $\mathrm{d}\mathrm{d}^\dagger A = 0$, or $\mathrm{d}^\dagger A = \text{const}$. This would be fulfilled if

$$\mathrm{d}^\dagger A = 0 \, , \tag{22}$$

which is the Lorenz gauge[2, 3, 6].

3.2 $\mathsf{T}(D)$ Noether current

The action is invariant up to a total differential

$$\delta_\lambda S = - \int \lambda \, \mathrm{d}(z \lrcorner \, \mathfrak{L}) = \int \lambda \, \frac{1}{2} \, \mathrm{d}\{ +z \lrcorner \, (F \wedge \star F) \} \, , \tag{23}$$

under the rigid infinitesimal transformation combined with a gauge transformation [8, eq. 3.46]

$$\delta A = -\lambda\{z \lrcorner \mathrm{d}(A - \mathrm{d}\Lambda) + \mathrm{d}[z \lrcorner (A - \mathrm{d}\Lambda)]\} . \tag{24}$$

Choosing

$$\mathrm{d}\Lambda = A(z)\, z^\flat \tag{25}$$

makes the second term vanish, yielding

$$\delta A = -\lambda\, z \lrcorner \mathrm{d}A = -\lambda\, z \lrcorner F . \tag{26}$$

When the equations of motion are satisfied, infinitesimal transformation leads to

$$\delta_\lambda S = \int \lambda\, \mathrm{d}\{+(z \lrcorner F) \wedge \star F\} . \tag{27}$$

Taking the difference with eq. (23) and setting $z = e_\alpha$ yield

$$0 = \int \lambda\, \mathrm{d}\mathfrak{T}_\alpha, \tag{28}$$

$$2\mathfrak{T}_\alpha := +(e_\alpha \lrcorner F) \wedge \star F - F \wedge (e_\alpha \lrcorner \star F) . \tag{29}$$

The continuity equation reads

$$\mathrm{d}\mathfrak{T}_\alpha = 0 . \tag{30}$$

In components, one needs

$$e_\alpha \lrcorner F = F_{\alpha\beta}\, \vartheta^\beta , \tag{31}$$

$$e_\alpha \lrcorner \star F = \frac{1}{2} F^{\beta\gamma}\, \epsilon_{\beta\gamma\alpha} ; \tag{32}$$

equipped with eqs. (58) to (58), one arrives at

$$(e_\alpha \lrcorner F) \wedge \star F = -F_{\alpha\gamma} F^{\gamma\beta}\, \epsilon_\beta , \tag{33}$$

$$F \wedge (e_\alpha \lrcorner \star F) = \frac{1}{2} F_{\beta\gamma} F^{\beta\gamma}\, \epsilon_\alpha + F_{\alpha\gamma} F^{\gamma\beta}\, \epsilon_\beta . \tag{34}$$

One finally has

$$\mathfrak{T}_\alpha = -F_{\alpha\gamma} F^{\gamma\beta}\, \epsilon_\beta - \frac{1}{4} F_{\beta\gamma} F^{\beta\gamma}\, \epsilon_\alpha . \tag{35}$$

4 U(1)-gauged complex scalar field theory

The action reads

$$S = \int -\mathbb{D}\phi^* \wedge \star \mathbb{D}\phi - m^2 \phi^* \wedge \star\phi - \frac{1}{2} F \wedge \star F , \tag{36}$$

where the exterior covariant derivative is defined in eq. (63).

Variation does *not* commute with exterior covariant derivative.

$$[\delta, \mathbb{D}]\phi = -\mathrm{i}e\delta A\phi\,. \tag{37}$$

Variation of the covariant kinetic term reads

$$\begin{aligned}\delta(\mathbb{D}\phi^* \wedge \star\mathbb{D}\phi) &= \mathbb{d}(\delta\phi^* \star \mathbb{D}\phi + \delta\phi \star \mathbb{D}\phi^*) \\ &\quad + \delta\phi^* \wedge \star\mathbb{D}^\dagger\mathbb{D}\phi + \delta\phi \wedge \star\mathbb{D}^\dagger\mathbb{D}\phi^* + \delta A \wedge \mathfrak{J}_A\,,\end{aligned} \tag{38}$$

where the covariant codifferential is defined in eq. (66), and

$$\mathfrak{J}_A := \mathrm{i}e(\phi^* \star \mathbb{D}\phi - \phi \star \mathbb{D}\phi^*)\,. \tag{39}$$

A generic variation of the action then reads

$$\begin{aligned}\delta S = \int & \mathbb{d}(-\delta\phi^* \wedge \star\mathbb{D}\phi - \delta\phi \wedge \star\mathbb{D}\phi^* - \delta A \wedge \star F) \\ & - \delta\phi^* \wedge \star(\mathbb{D}^\dagger\mathbb{D} + m^2)\phi - \delta\phi \wedge \star(\mathbb{D}^\dagger\mathbb{D} + m^2)\phi^* \\ & - \delta A \wedge (\mathbb{d}\star F - \mathfrak{J}_A).\end{aligned} \tag{40}$$

4.1 Noether's invariances

The action is invariant under the generic transformation

$$\phi \to \mathrm{e}^{-\mathrm{i}e\Lambda}\phi\,, \quad \phi^* \to \mathrm{e}^{+\mathrm{i}e\Lambda}\phi^*\,, \quad A \to A - \mathbb{d}\Lambda\,. \tag{41}$$

There are two scenarios [1].

If the transformation is rigid, $\mathbb{d}\Lambda = 0$, one obtains $\mathfrak{J}_A$ as the Noether current from the boundary term as before, which satisfies the continuity equation $\mathbb{d}\mathfrak{J}_A = 0$.

If the transformation is gauge with a compact support, the boundary term can be dropped, and one obtains a Noether identity, which is under construction.

[1]

A Differential geometry

A.1 Basic operations

Following the style of [4], it is attempted to define the operations *inductively*, instead of giving explicit expressions, which is much less effective than the former in practice.

Interior product Let z be a vector, ω be a 1-form, χ be a k-form. The *interior product* is defined inductively as the bilinear map satisfying

$$z \lrcorner 1 = 0\,, \tag{42}$$

$$z \lrcorner \omega := \omega(z)\,, \tag{43}$$

$$z \lrcorner (\omega \wedge \chi) := (z \lrcorner \omega) \wedge \chi - \omega \wedge (z \lrcorner \chi)\,. \tag{44}$$

Equation (44) is also known as the anti-product rule.

By induction one can show that for a p-form ϕ,

$$z \lrcorner (\phi \wedge \chi) = (z \lrcorner \phi) \wedge \chi + (-)^p \chi \wedge (z \lrcorner \chi)\,. \tag{45}$$

Hodge star Let ω be an 1-form, χ be a k-form. The Hodge star $\star$ is defined inductively as the linear map

$$\star 1 := \mathrm{vol}\,, \tag{46}$$

$$\star(\chi \wedge \omega) := \omega^\sharp \lrcorner \star\chi\,. \tag{47}$$

Non-gravitational theories features $[\delta, \star] = 0$, which means [7, sec. 3.2]

$$\delta g_{\mu\nu} = -2\omega_{(\mu\nu)}, \qquad \delta\vartheta^\mu = \omega_\nu{}^\mu\,\vartheta^\nu\,; \tag{48}$$

for an orthonormal coframe, the allowed variations are $\omega_{(\alpha\beta)} = 0$.

Identity Inspired by [9, eq. (3.167)], for a 1-form ω, k-form χ, one can derive

$$\begin{aligned}
\omega \wedge \star\chi &= (-)^{D-k} \star \chi \wedge \omega = (-)^{D-k} \star^{-1} \left(\omega^\sharp \lrcorner \star\star\,\chi\right) \\
&= (-)^{D-k} (-)^{(D-k-1)(k+1)+s} \star \left(\omega^\sharp \lrcorner (-)^{k(D-k)+s} \star^{-1} \star\chi\right) \\
&= (-)^{k+1} \star \left(\omega^\sharp \lrcorner\, \chi\right).
\end{aligned} \tag{49}$$

This will be useful in eqs. (58) to (60).

Codifferential The *codifferential* $\mathbb{d}^\dagger$ is defined as follows. Let ϕ be a k-form, χ be a $(k-1)$-form.

$$\mathbb{d}(\chi^* \wedge \star\phi) \equiv \mathbb{d}\chi^* \wedge \star\phi - (-)^k \chi^* \wedge \mathbb{d} \star \phi =: \mathbb{d}\chi^* \wedge \star\phi - \chi^* \wedge \star\mathbb{d}^\dagger\phi \tag{50}$$

$$= \mathbb{d}\chi^* \wedge \star\phi - \chi^* \wedge \star(-)^k \star^{-1} \mathbb{d} \star \phi\,. \tag{51}$$

In other words,

$$\boxed{\mathbb{d}^\dagger\phi = (-)^k \star^{-1} \mathbb{d} \star \phi\,.} \tag{52}$$

If $\int \mathbb{d}(\chi^* \wedge \star\phi) = 0$, then $\mathbb{d}^\dagger$ is the adjoint of the exterior derivative $\mathbb{d}$ in functional sense.

A.2 Coframe bases

Coframes come into play if an explicit expression in components is wanted, or if gravitation comes into play.

For a detailed discussion about the meaning of untwisted (describing extensions) and twisted (describing densities) differential forms, see e.g. [4, sec. 22, 28].

Untwisted orthonormal k-cobases Let $\{\vartheta^\alpha\}$ be an orthonormal coframe. The orthonormal basis for untwisted k-form is defined inductively as

$$1\,, \tag{53}$$

$$\vartheta^{\alpha_1\alpha_2\cdots\alpha_k} := \vartheta^{\alpha_1} \wedge \vartheta^{\alpha_2\cdots\alpha_k}\,. \tag{54}$$

Twisted orthonormal k-cobases Let $\{\vartheta^\alpha\}$ be an orthonormal coframe. The orthonormal basis for twisted $(D-k)$-form is defined inductively as

$$\epsilon := \mathrm{vol}\,, \tag{55}$$

$$\epsilon_{\alpha_1\alpha_2\cdots\alpha_k} := e_{\alpha_k} \rfloor \epsilon_{\alpha_1\cdots\alpha_{k-1}}\,. \tag{56}$$

By using eq. (47) and induction, one can show that

$$\epsilon_{\alpha_1\alpha_2\cdots\alpha_k} = \star\vartheta_{\alpha_1\alpha_2\cdots\alpha_k}\,. \tag{57}$$

Identities The following identities come from eq. (49)

$$\vartheta^\alpha \wedge \epsilon_{\beta\gamma} = -\delta^\alpha_\beta\,\epsilon_\gamma + \delta^\alpha_\gamma\,\epsilon_\beta\,, \tag{58}$$

$$\vartheta^\alpha \wedge \epsilon_{\beta\gamma\delta} = \delta^\alpha_\beta\,\epsilon_{\gamma\delta} - \delta^\alpha_\gamma\,\epsilon_{\beta\delta} + \delta^\alpha_\delta\,\epsilon_{\beta\gamma}\,. \tag{59}$$

One can further deduce that

$$\begin{aligned}\vartheta^{\alpha\beta} \wedge \epsilon_{\gamma\delta\epsilon} = &+\left(\delta^\alpha_\gamma\delta^\beta_\delta - \delta^\alpha_\delta\delta^\beta_\gamma\right)\epsilon_\epsilon\\ &-\left(\delta^\alpha_\gamma\delta^\beta_\epsilon - \delta^\alpha_\epsilon\delta^\beta_\gamma\right)\epsilon_\delta\\ &+\left(\delta^\alpha_\delta\delta^\beta_\epsilon - \delta^\alpha_\epsilon\delta^\beta_\delta\right)\epsilon_\gamma\,.\end{aligned} \tag{60}$$

Equations (58) to (60) are useful in constructing explicit expressions in components.

Translation Consider the rigid infinitesimal transformation

$$\delta_\lambda\chi = \lambda\,\pounds_z\chi = \lambda[z \rfloor \mathrm{d}\chi + \mathrm{d}(z \rfloor \chi)]\,; \tag{61}$$

'rigid' means

$$\delta_\lambda\vartheta^\alpha = 0\,. \tag{62}$$

A.3 Connection on the principal bundle

Exterior covariant derivative Let χ be a $\mathbb{C}$-valued k-form. The *exterior covariant derivative* of χ reads

$$\mathbb{D}\chi := (\mathrm{d} - \mathrm{i}eA)\chi, \qquad \mathbb{D}\chi^* := (\mathrm{d} + \mathrm{i}eA)\chi^*\,, \tag{63}$$

where A is a $\mathfrak{u}(1)$-valued *connection form.*

Covariant codifferential The *covariant codifferential* $\mathbb{D}^\dagger$ is defined as follows. Let ϕ be a $\mathbb{C}$-valued k-form, χ be a $\mathbb{C}$-valued $(k-1)$-form.

$$\mathrm{d}(\chi^* \wedge \star\phi) \equiv \mathrm{d}\chi^* \wedge \star\phi - (-)^k\chi^* \wedge \mathrm{d}\star\phi =: \mathbb{D}\chi^* \wedge \star\phi - \chi^* \wedge \star\mathbb{D}^\dagger\phi \tag{64}$$

$$\begin{aligned}&= \mathbb{D}\chi^* \wedge \star\phi - \mathrm{i}eA \wedge \chi^* \wedge \star\phi - (-)^k\chi^* \wedge \mathrm{d}\star\phi\\ &= \mathbb{D}\chi^* \wedge \star\phi + \chi^* \wedge (-)^k\mathrm{i}eA\star\phi - (-)^k\chi^* \wedge \mathrm{d}\star\phi\\ &= \mathbb{D}\chi^* \wedge \star\phi - \chi^* \wedge \star(-)^k\star^{-1}(\mathrm{d} - \mathrm{i}eA)\star\phi\,.\end{aligned} \tag{65}$$

In other words,

$$\boxed{\mathbb{D}^\dagger\phi = (-)^k\star^{-1}(\mathrm{d} - \mathrm{i}eA)\star\phi\,.} \tag{66}$$

If $\int \mathrm{d}(\chi^* \wedge \star\phi) = 0$, $\mathbb{D}^\dagger$ becomes the adjoint of the exterior covariant derivative $\mathbb{D}$ in the functional sense.

A.4 Variation of coframe bases

Upon variation of ϑ^α, $\vartheta^{\alpha_1\alpha_2\dots\alpha_k}$ goes under

$$\delta\vartheta^{\alpha_1\alpha_2\dots\alpha_k} = \delta\vartheta^\alpha \wedge (e_\alpha \rfloor \vartheta^{\alpha_1\alpha_2\dots\alpha_k})\,, \tag{67}$$

which can be proved by induction.

Upon variation of ϑ^α, $\epsilon_{\alpha_1\alpha_2\dots\alpha_k}$ goes under [7, sec. A.2]

$$\delta\epsilon_{\alpha_1\alpha_2\dots\alpha_k} = \delta\vartheta^\alpha \wedge \left(e_\alpha \rfloor \epsilon_{\alpha_1\alpha_2\dots\alpha_k}\right). \tag{68}$$

Variation of Hodge star In gravitational theories [7, sec. 3.2] with an orthonormal cobasis,

$$[\delta,\star]\phi = \delta\vartheta^\alpha \wedge (e_\alpha \rfloor \star\phi) - \star(\delta\vartheta^\alpha \wedge (e_\alpha \rfloor \phi))\,. \tag{69}$$

Let χ be a p-form, ϕ another form [5, sec. 5].

$$\delta(\chi\wedge\star\phi) = \delta\chi\wedge\star\phi + \delta\phi\wedge\star\chi - \delta\vartheta^\alpha \wedge \Sigma_\alpha\,, \tag{70}$$

$$\Sigma_\alpha := \chi \wedge \left\{\star(e_\alpha \rfloor \phi) - (-)^p(e_\alpha \rfloor \star\phi)\right\}. \tag{71}$$

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