Notes on Singular Dynamics

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April 6, 2017

1 Lagrangian formalism

2 Hamiltonian formalism

$$L^v = L|_{\dot{q}=v} \tag{2.1}$$

$$S = \int \mathbb{d}t \left(L^v + \sum_i p_i(\dot{q}_i - v_i) \right) \tag{2.2}$$

$$p_i = \frac{\partial L}{\partial v_i} \tag{2.3}$$

$v^a = \overline{v}^a(q,p)$ can be solved, $a = 1, 2, \dots, R_W$; v^α cannot be solved, $\alpha = R_W + 1, \dots, n$.

Example:

$$L^v = \frac{1}{2}\sum_{i,j} W_{ij}(q) v_i v_j + \sum_i \eta_i(q) v_i - V(q). \tag{2.4}$$

$$p_i = \frac{\partial L^v}{\partial v_i} = \sum_{i,j} W_{ij} v_j + \eta_i. \tag{2.5}$$

Let

$$\sum_j W_{ij} e_j^{(a)} = \lambda^{(a)} e_i \neq 0, \tag{2.6}$$

$$\sum_j W_{ij} e_j^{(\alpha)} = 0. \tag{2.7}$$

3 Examples