Find the finite transformation

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May 18, 2017

Given a lagrangian of a relativistic particle with an auxiliary einbein

$$L := \frac{1}{2}\left(\frac{1}{e(t)}\eta_{\mu\nu}\dot{x}^\mu\dot{x}^\nu - m^2 e(t)\right), \tag{0.1}$$

where $\eta_{\mu\nu} := \mathrm{diag}(-,+,+,\ldots)$, which is adapted from Blumenhagen, Lüst, and Theisen 2013, sec. 2.1. This is classically equivalent to

$$L = -m\sqrt{-\eta_{\mu\nu}\dot{x}^\mu\dot{x}^\nu} \tag{0.2}$$

for $m > 0$ (in the sense that the equations of motion for x^μ's are the same, when $e(t)$ eliminated), but also works for $m = 0$.

One can use method in H. J. Rothe and K. D. Rothe 2010, sec. 2 to *solve for* the only gauge symmetry, whose *infinitesimal* form reads

$$\delta x^\mu = \dot{x}^\mu\epsilon, \tag{0.3}$$

$$\delta e = \dot{e}\epsilon + e\dot{\epsilon}, \tag{0.4}$$

where $\epsilon = \epsilon(t)$ parametrises the transformation. The question is, how to solve the *finite* transformation?

It is important to solve the transform, instead of guessing an answer, because there are more complicated scenarios where intuition is not enough.

References

Blumenhagen, Ralph, Dieter Lüst, and Stefan Theisen (2013). *Basic Concepts of String Theory*. Theoretical and Mathematical Physics. Springer. ISBN: http://id.crossref.org/isbn/978-3-642-29497-6. DOI: `10.1007/978-3-642-29497-6`. URL: `http://dx.doi.org/10.1007/978-3-642-29497-6`.

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