538 Riddler: 20 May 2022

Colin Parker

May 20, 2022

1 Express

When Amare is at a radius r from the center of the web, crawling radially away puts him at a distance $r+1$ and crawling tangentially creates a right triangle with legs 1 and r, putting him a distance $\sqrt{r^2+1}$ from the center. As this is nonlinear, we can't simply put in the expected distance for n steps in order to get the expected distance for $n+1$ steps; we have to enumerate the possibilities. Luckily for us, Amare must be 1 inch away from the center after the first step, then there are 3 more steps to take for a total of only $2^3 = 8$ possibilities. These are, in increasing order, $\{2, \sqrt{6}, \sqrt{4+2\sqrt{2}}, \sqrt{10}, 1+\sqrt{3}, 1+\sqrt{5}, 2+\sqrt{2}, 4\}$, and averaging them gives an expected distance of 2.9509 inches from the center of the web.

2 Classic

We can imagine that our point of view relative to Sagittarius A* corresponds to a point on a sphere with the black hole at its center. The accretion disk would be at its "equator" and the line perpendicular to it through the black hole hitting the sphere at its "poles". Then, because our point is a random variable under a uniform distribution on the surface of the sphere, the probability that it is within 10° of one of the poles is the fraction of these areas divided by the total sphere area. This is given by

$$\frac{2A}{4\pi r^2} = \frac{\Omega}{2\pi} = 2\sin^2\left(\frac{10^\circ}{2}\right) = 2\sin^2(5^\circ) \approx 0.01519$$

Where A is the area of one of these two "polar caps" and Ω is the solid angle generated by the 10° central angle (Steradian on Wikipedia for reference).