

Q9

1.) $P(\text{die} = A | X_{4:7} = \{5, 3, 9, 3, 8, 4, 7\}) = ?$
 call this X

$$P(\text{die} = A | X) = \frac{P(X | \text{die} = A) P(\text{die} = A)}{P(X)}$$

$$P(X | \text{die} = A) = \frac{1}{20} \cdot \frac{3}{20} \cdot \frac{1}{20} \cdot \frac{3}{20} \cdot \frac{1}{20} \cdot \frac{2}{20} \cdot \frac{1}{20} = \frac{18}{20^7}$$

$$P(\text{die} = A) = \frac{1}{2}$$

$$P(X | \text{die} = B) = \frac{2}{20} \cdot \frac{2}{20} \cdot \frac{1}{20} \cdot \frac{2}{20} \cdot \frac{2}{20} \cdot \frac{2}{20} \cdot \frac{2}{20} = \frac{64}{20^7}$$

$$P(\text{die} = B) = \frac{1}{2}$$

$$P(X) = P(X, \text{die} = A) + P(X, \text{die} = B) = \frac{18}{20^7} \times \frac{1}{2} + \frac{64}{20^7} \times \frac{1}{2} = \frac{82}{20^8}$$

$$P(\text{die} = A | X) = \frac{\frac{18}{20^7} \times \frac{1}{2}}{\frac{82}{20^8}} = \frac{9}{41}$$

2.) Since only order has changed, $P(X | \text{die} = A)$ and $P(X | \text{die} = B)$ will remain same.

$$P(\text{die} = A) = P(\text{die} = B) = P(\text{die} = C) = \frac{1}{3}$$

$$P(X | C) = \frac{1}{20} \times \frac{1}{20} \times \frac{1}{20} \times \frac{1}{20} \times \frac{1}{20} \times \frac{1}{20} \times \frac{1}{20} = \frac{1}{20^7}$$

$$\text{New } P(X) = \frac{1}{3} \left(\frac{18}{20^7} + \frac{64}{20^7} + \frac{1}{20^7} \right) = \frac{83}{20^7 \cdot 3}$$

since uniform random.

$$P(\text{die} = C | X) = \frac{\frac{1}{20^7} \cdot \frac{1}{3}}{\frac{83}{20^7 \cdot 3}} = \frac{1}{83}$$

$$P(\text{die} = B | X) = \frac{\frac{64}{20^7} \cdot \frac{1}{3}}{\frac{83}{20^7 \cdot 3}} = \frac{64}{83}$$

$$P(\text{die} = A | X) = \frac{\frac{18}{20^7} \cdot \frac{1}{3}}{\frac{83}{20^7 \cdot 3}} = \frac{18}{83}$$

3) $\text{dom}(\text{die}) = \{A, B, C\}$, selected die
 $\text{dom}(X_i) = \{1, 2, 3, \dots, 20\}$, outcomes, of die

