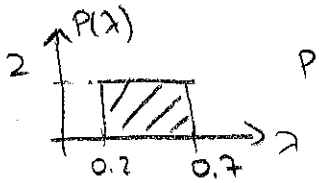
CMPE 547
HW 1

Class question

Suppose $\lambda \sim U[0.2, 0.7]$



$P(\lambda) = 2$

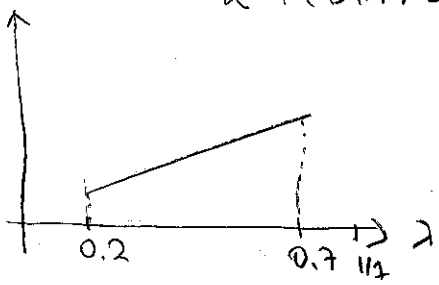
Update $P(\lambda \mid D)$ given $D = (T, T, T, T, T, Y, T, T)$

1. $D = T$

$$P(\lambda \mid D) = \frac{P(D \mid \lambda) \cdot P(\lambda)}{P(D)} \propto P(D \mid \lambda) \cdot P(\lambda) \propto P(D \mid \lambda)$$

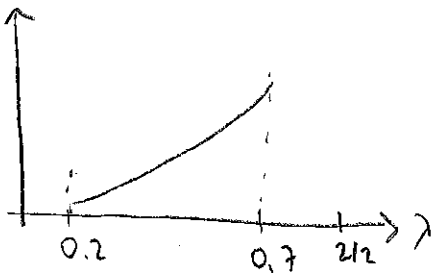
($P(\lambda)$: flat)

$$\propto P(D \mid \lambda) = \lambda$$



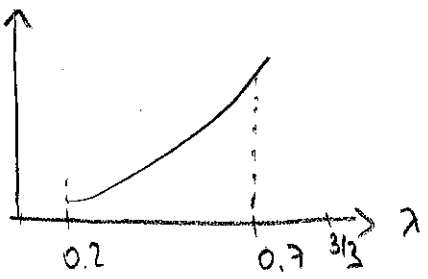
2. $D = (T, T)$

$$P(\lambda \mid D) \propto P(D \mid \lambda) = P(d_1 \mid \lambda) \cdot P(d_2 \mid \lambda) = \lambda^2$$



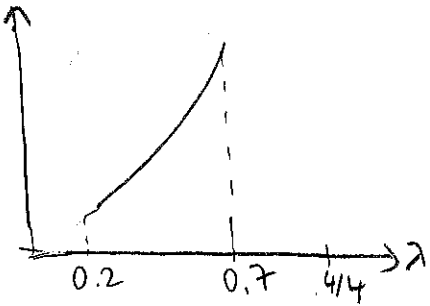
3. $D = (T, T, T)$

$$P(\lambda \mid D) \propto \lambda^3$$



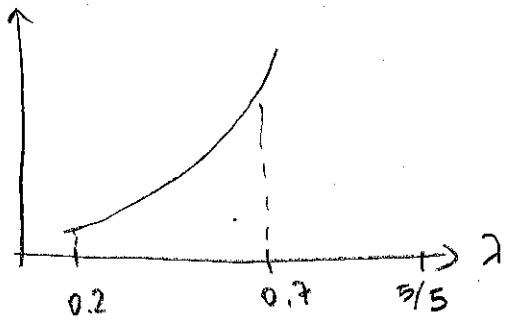
4. $D = (T, T, T, T)$

$$P(\lambda \mid D) \propto \lambda^4$$



5. $D = (T, T, T, T, T)$

$P(\lambda \mid D) \propto \lambda^5$



6. $D = (T, T, T, T, T, Y)$

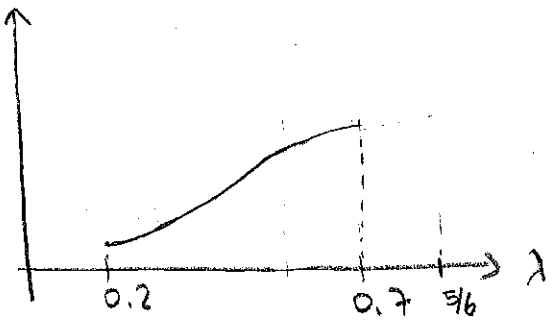
$P(\lambda \mid D) \propto \lambda^5 \cdot (1-\lambda) = \lambda^5 - \lambda^6$

$\frac{d(\lambda^5 - \lambda^6)}{d\lambda} = 5\lambda^4 - 6\lambda^5 = 0$

$\lambda^4(5 - 6\lambda) = 0$ $\quad \lambda^{ML} = 5/6 = 0.83$

$\frac{d^2(\lambda^5 - \lambda^6)}{d\lambda^2} = 20\lambda^3 - 30\lambda^4$

$= 10\lambda^3(2 - 3\lambda) = 0$ $\quad \lambda/3 = 0.66$ inflection point



7. $D = (T, T, T, T, T, Y, T)$

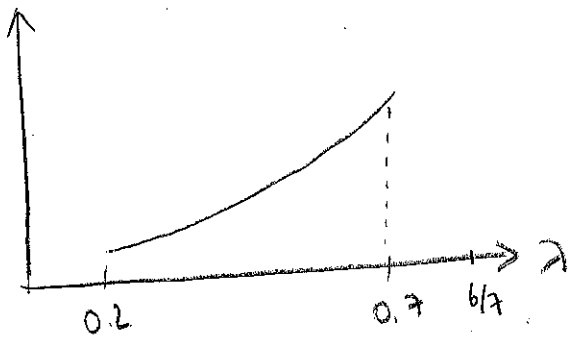
$P(\lambda \mid D) \propto \lambda^6(1-\lambda) = \lambda^6 - \lambda^7$

$\frac{d(\lambda^6 - \lambda^7)}{d\lambda} = 6\lambda^5 - 7\lambda^6 = 0$

$\lambda^5(6 - 7\lambda) = 0$ $\quad \lambda^{ML} = 6/7 = 0.85$

$\frac{d^2(\lambda^6 - \lambda^7)}{d\lambda^2} = 30\lambda^4 - 42\lambda^5 = 0$

$\lambda^4(30 - 42\lambda) = 0 \rightarrow \lambda = 0.71$



8. $D = (T, T, T, T, T, Y, T, T)$

$P(\lambda \mid D) \propto \lambda^7(1-\lambda) = \lambda^7 - \lambda^8$

$\frac{d(\lambda^7 - \lambda^8)}{d\lambda} = 7\lambda^6 - 8\lambda^7 = 0$

$\lambda^6(7 - 8\lambda) = 0$ $\quad \lambda^{ML} = 7/8 = 0.875$

$\frac{d^2(\lambda^7 - \lambda^8)}{d\lambda^2} = 42\lambda^5 - 56\lambda^6 = 0$

$\lambda^5(42 - 56\lambda) = 0 \rightarrow \lambda = 0.75$