

Q9) ① $P(\text{Die} = A \mid X = (5, 3, 9, 3, 8, 4, 7)) = ?$

$$P(\text{Die} \mid X) = \frac{P(X \mid \text{Die}) \cdot P(\text{Die})}{\sum_{\text{Die}} P(X \mid \text{Die}) \cdot P(\text{Die})} P(X)$$

$$P(X \mid \text{Die} = A) = P(X_1 \mid A) \cdot P(X_2 \mid A) \cdot \dots \cdot P(X_7 \mid A)$$

$$X = (X_1, \dots, X_7)$$

$$P(X = (5, 3, 9, 3, 8, 4, 7) \mid \text{Die} = A) = \frac{1}{20} \cdot \frac{3}{20} \cdot \frac{1}{20} \cdot \frac{3}{20} \cdot \frac{1}{20} \cdot \frac{2}{20} \cdot \frac{1}{20}$$

$$P(X = \quad \quad \quad \mid \text{Die} = B) = \frac{2}{20} \cdot \frac{2}{20} \cdot \frac{1}{20} \cdot \frac{2}{20} \cdot \frac{2}{20} \cdot \frac{2}{20} \cdot \frac{2}{20}$$

$$\text{Assume } P(\text{Die} = A) = P(\text{Die} = B) = 1/2$$

$$P(\text{Die} = A \mid X) = \frac{\left(\frac{1}{20}\right)^7 \cdot 18 \cdot \frac{1}{2}}{\left(\frac{1}{20}\right)^7 \cdot \left(18 \cdot \frac{1}{2} + 64 \cdot \frac{1}{2}\right)} = \frac{18}{18+64} = \frac{18}{82} = 0.2195$$

② $P(\text{Die} = A) = P(\text{Die} = B) = P(\text{Die} = C) = 1/3$

$$P(X = (3, 5, 4, 8, 3, 9, 7) \mid \text{Die} = A) = \frac{3}{20} \cdot \frac{1}{20} \cdot \frac{2}{20} \cdot \frac{1}{20} \cdot \frac{3}{20} \cdot \frac{1}{20} \cdot \frac{1}{20}$$

$$P(X = \quad \quad \quad \mid \text{Die} = B) = \frac{2}{20} \cdot \frac{2}{20} \cdot \frac{2}{20} \cdot \frac{2}{20} \cdot \frac{2}{20} \cdot \frac{1}{20} \cdot \frac{2}{20}$$

$$P(X = \quad \quad \quad \mid \text{Die} = C) = \left(\frac{1}{20}\right)^7$$

$$P(\text{Die} = A \mid X) = \frac{\left(\frac{1}{20}\right)^7 \cdot 18 \cdot \frac{1}{3}}{\left(\frac{1}{20}\right)^7 \cdot \left(18 \cdot \frac{1}{3} + 64 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3}\right)} = \frac{18}{83} = 0.2169$$

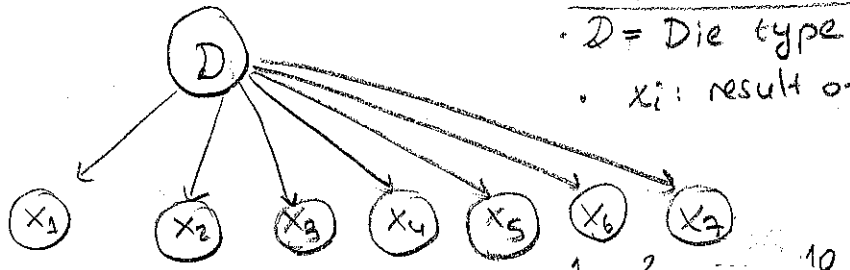
$$P(\text{Die} = B \mid X) = \frac{\left(\frac{1}{20}\right)^7 \cdot 64 \cdot \frac{1}{3}}{\left(\frac{1}{20}\right)^7 \cdot \left(18 \cdot \frac{1}{3} + 64 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3}\right)} = \frac{64}{83} = 0.7711$$

$$P(\text{Die} = C \mid X) = \frac{\left(\frac{1}{20}\right)^7 \cdot 1 \cdot \frac{1}{3}}{\left(\frac{1}{20}\right)^7 \cdot \left(18 \cdot \frac{1}{3} + 64 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3}\right)} = \frac{1}{83} = 0.0120$$

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Problem 1:

Random variables:
• D = Die type, $D = \{A, B\}$
• x_i : result of throw i ,
 $x_i = \{1, 2, \dots, 10\}$



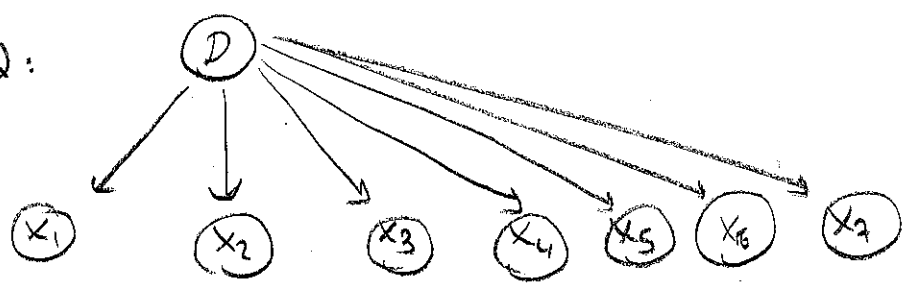
$P(D) =$

| | |
|---|-----|
| A | ... |
| B | ... |

$P(x_i | D) =$

| | | | | |
|---|---|---|-----|----|
| A | 1 | 2 | ... | 10 |
| B | | | | |

Problem 2:



Random variables:

• D = die type, $D = \{A, B, C\}$
• x_i = result of throw i , $x_i = \{1, 2, \dots, 20\}$

$P(D) =$

| | |
|---|----|
| A | .. |
| B | .. |
| C | .. |

$P(x_i | D) =$

| | | | | |
|---|---|---|-----|----|
| A | 1 | 2 | ... | 20 |
| B | | | | |
| C | | | | |