Question 9

1. Let $X = \{x_1 = 5, x_2 = 3, x_3 = 9, x_4 = 3, x_5 = 8, x_6 = 4, x_7 = 7\}$

$$P(X|A) = \frac{1 \cdot 3 \cdot 1 \cdot 3 \cdot 1 \cdot 2}{20^7} = \frac{18}{20^7} \qquad P(X|B) = \frac{2 \cdot 2 \cdot 2 \cdot 1 \cdot 2 \cdot 2 \cdot 2}{20^7} = \frac{64}{20^7}$$

$$P(X) = \frac{18+64}{20^7} \cdot \frac{1}{2} = \frac{41}{20^7}$$

Z is RV for die

$$P(Z = \text{die A} | X) = \frac{P(X|A) \cdot P(A)}{P(X)} = \frac{\frac{18}{20^7} \cdot \frac{1}{2}}{\frac{41}{20^7}} = \frac{9}{41}$$

2. From part 1. $P(X|A) = \frac{18}{20^7}$ and $P(X|B) = \frac{64}{20^7}$

$$p(X|C) = \frac{1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1}{20^7} = \frac{1}{20^7}$$

$$P(X) = \frac{1}{3} \cdot \frac{83}{20^7}$$

$$p(Z = \text{die C} | X) = \frac{\frac{1}{20^7} \cdot \frac{1}{3}}{\frac{83}{20^7} \cdot \frac{1}{3}} = \frac{1}{83}$$

$$p(Z = \text{die B} | X) = \frac{\frac{64}{20^7} \cdot \frac{1}{3}}{\frac{83}{20^7} \cdot \frac{1}{3}} = \frac{64}{83}$$

$$p(Z = \text{die A} | X) = \frac{\frac{18}{20^7} \cdot \frac{1}{3}}{\frac{83}{20^7} \cdot \frac{1}{3}} = \frac{18}{83}$$

3.